

THE FUNDAMENTALS OF LOGIC

BY

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PREFACE

It was not without hesitation that I gave my consent for putting into print in book-form my lectures on the Fundamentals of Logic which were first delivered to my students in the Intermediate classes in the year 1935-36. What dissuaded me at first was the thought that nothing original could be achieved, the nature of the subject being so narrowly circumscribed. And what made me give my consent finally for the publication of this book was the hope that the presentation of the subject here attempted may be found helpful by a wider circle of students and even by others who may be inclined to study the elements of Logic.

My indebtedness to the various writers on Logic is acknowledged in the footnotes where references to their works are given. Special mention must, however, be made of three books which helped me considerably: H. W. B. Joseph's *Introduction to Logic*, Robert Latta and Alexander Macbeath's *Elements of Logic* and J. E. Creighton and H. R. Smart's *Introductory Logic*.

My thanks are due to Mr. V. A. Devasenapathy, M.A., Lecturer in Philosophy, Pachaiyappa's College, for valuable assistance rendered by way of reading through the proofs and preparing the Questions and Model Exercises.

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CHAPTER I

THE SCOPE OF LOGIC

i. Introduction

Man is a *rational* animal. The ability to *judge* is what distinguishes him from the other animate beings. It is *reason* that places him at the top of evolution and gives him a position of vantage over the other members of the animal kingdom. In the matter of behaviour he is not much different from the animals. Like them he acts on and is acted upon by his environment. But while the others merely behave in certain ways, man is able to evaluate his behaviour by virtue of his power of judge and decide as to the propriety or impropriety of his conduct. Knowledge, and not physical might, is his power. But for this element of reason, he would be one with the beasts of the wild, adhering to the codes of the jungle, without science and poetry, art and morality, and without the distinctions of right and wrong.

Man's behaviour consists of thinking, feeling and willing which together constitute the functions of his mind. When an individual is faced with an intellectual problem, he searches in the armoury of thought for such weapons as would help him in solving his difficulty. When he is presented with a work of art, he brings into use his ability to appreciate beauty and excellence of form. And when he is caught in a moral crisis, he decides as to the course of action he should pursue: Thinking, feeling and willing are mental functions which enable man to distinguish the right behaviour from the wrong. And the ability to judge what is right and what is wrong depends upon the development and training of the human mind. The inquiry into right thinking is logic or the science of thought. The investigation of what is right feeling is aesthetics or the science of emotions. And the study of what is right willing is ethics or the science of conduct.

As students of logic our prime concern is to study the science of thought. Thinking is the instrument of knowledge. What comes to us through knowledge is the

be called 'knowledge'. Real knowledge is that which is reached as a result of our own intellectual activity. That two and two make four is not a belief for the man who knows the import of numbers. It is a piece of definite knowledge.

Logic, then, is the science which helps us in understanding the operations of the human mind in its search for truth. It treats of the principles which should guide thought in this quest. The actual process of thought is not its subject-matter. An analysis and description of the thought-content is not its task. Logic studies the process of thought, not as it is, but as it ought to be regulated for the attainment of truth which is the *norm* or ideal of the thought-process. It may be defined as *the normative science of thought, a systematic inquiry into the principles which govern valid thinking.*

ii. Logic and psychology

The nature of logic may be better understood by contrasting it with psychology. Both of these sciences study mental functions or consciousness. But the standpoint of psychology is *positive*. It deals with the thought-process *as it is* without any reference to ideals or their attainment. Logic, on the contrary, studies thought *as it ought to be*. Its outlook is *normative*. Psychology is interested in the *process* of thought, and logic in the *product*. To the former what is important is the *nature* of thought, and to the latter what is essential is the *significance* of thought. Every thought has not only an *existence* but also a *meaning*; and it is with the meaning side of ideas or thoughts that logic has to do. The difference between psychology and logic is analogous to that which obtains between morphology or the science of the structure of living organisms and physiology or the study of the various acts and functions which these organisms discharge in fulfilling the ends of life. While psychology treats of the actual structure of mental processes, logic concerns itself with the part which they play in giving us knowledge.

Logic differs from psychology in another respect. It is the science of thought alone whereas the other is the science of behaviour in general which includes, besides thought, feeling and willing as well. Psychology describes pleasures and pains, acts of will, and the association of ideas, as well as

logical thinking. All these processes which constitute the content of consciousness are studied by Psychology *for their own sake*, and just as they stand. But logic investigates into thinking alone, and that too with a definite ideal, *viz.*, the attainment of truth. Thus there is a twofold difference between logic and psychology. In method, logic is normative and psychology is positive, and with regard to material, logic concerns itself with thinking alone whereas psychology has to cover the entire field of behaviour.

This distinction, however, is not absolute. It is only convenient for the purposes of investigation. In the first place, mental life cannot be compartmentalised. And secondly, the difference between normative and natural sciences is not fixed. Many sciences which are mainly positive have in them elements that are of a normative character; and even purely normative sciences may be said, in a sense, to deal with what actually is. Logic cannot satisfactorily study the function of thought without knowing its nature and structure; and psychology cannot take ideas simply as existing conscious processes devoid of meaning or importance.

iii. Logic and aesthetics

We have seen that there are certain values which serve as the standards or norms of human behaviour. Truth, beauty and goodness are norms respectively of thinking, feeling, and willing. We noticed that logic is the science of correct thinking with truth as its goal. The ideal of beauty is the basis of artistic appreciation; and the science which studies emotions with reference to this ideal is aesthetics. It will be easily seen that the method adopted by logic and aesthetics is the same. Both of them are normative sciences; and they differ in their method from psychology which is the positive science of mind.' But there is a difference between logic and aesthetics in that they have to do with different aspects of behaviour. Thought is the subject-matter of logic. Feeling is the material of aesthetics.

iv. Logic and ethics

Just as there are normative sciences for thinking and feeling, there is also a normative science for the purpose of study-

ing conduct or willed activity. Man's actions are judged to be right or wrong by referring them to the standard of goodness. The science, which enables us to decide whether an action is good or bad and sets forth the method by which the ideal of goodness may be attained, is ethics. Ethics is the science of the ideal involved in conduct, as logic is the science of the ideal involved in thinking. They agree in method but differ in scope.

v. Logic and metaphysics

The different sciences we have considered so far are all of them philosophical sciences. But the philosophical science *par excellence* is metaphysics. Though the nature of the present work precludes any detailed discussion of the relation between logic and metaphysics, it would tend to make the scope of logic clear, if it is shown at the outset how logic stands in need of metaphysics. Logic, aesthetics and ethics assume the existence and value of certain ideals—truth, beauty and goodness. They do not question their ultimate value. Whether these ideals have a rightful place or not in a rational conception of the universe they do not help us to discern. Metaphysics inquires into such questions that are not solved by them. It is the science of the nature of reality as a whole. The meaning of truth and the way in which we can distinguish true knowledge from erroneous knowledge are problems of metaphysics. The task of logic is to know how truth is attained. The function of metaphysics is to discover "What is truth?" "Is truth possible?"

vi. Sciences, philosophical and objective

Philosophical sciences differ from objective sciences in that they do not treat of things which are of the external world. Objective sciences like physics, chemistry and biology study the different aspects of the external world. The philosophical sciences, on the other hand, have to do with what is 'within.' Our knowledge of the external world, and not the world itself, comes within the scope of a philosophical science. In a sense, it may be said that philosophy is the basis of all other sciences; for it is the business of philosophy to unravel things which are most fundamental and vital. Philosophical sciences are of this nature. They penetrate beyond the sen-

suous. Not without justification is it said that the postulates of the objective sciences become the problems of philosophy.

vii. Logic as the science of sciences

Logic which is a philosophical science inquires into the fundamentals of other sciences. All sciences assume the possibility of thinking. Thinking is the instrument through which they attain knowledge. Science is an expression of thinking at its best. But in science we *use* thought without thinking about it. "The scientist, though he thinks well, does not necessarily think about thinking itself. He takes it for granted: he uses it without examining it or inquiring into its nature and laws. Thus he speaks of methods of observation, experiment, induction and deduction, etc.... These are his tools or instruments and he does not necessarily inquire into their nature, any more than the gardener studies the chemistry of his spade or the dynamics of digging."* Logic as the science of thought, investigates into the nature of the common tool of science, *viz.*, thinking. In studying logic we are thinking about thought. Here there is a double process of thinking. In this sense logic may be described as *scientia scientiarum*, the science of the sciences. It is entitled to be called so, because its subject-matter is thinking which is the fundamental presupposition of the possibility of all science.

Some overzealous logicians push this claim to absurd limits. They consider logic to be a very necessary prelude to all other inquiry and to form part of every other science. The names of some sciences are unfortunately responsible for this misconception about the position of logic. The termination "logy" in biology, geology, zoology, etc. is said to indicate the subservience of these sciences to logic. It is asserted that these are particular sciences of logic, that geology is the logic of the earth, biology the logic of life, zoology the logic of animals and so on. But it will be evident that this is a wrong conception of the function of logic. Logic is not interested in this or that particular branch of knowledge. It evaluates knowledge *per se*. It is not its aim to give us any detailed knowledge of the earth or of the animals. Its purpose is only

* Latta and Macbeath: *The Elements of Logic*, p. 5

to make known how thinking is to be guided in the attainment of knowledge. It is the scientific method that lies within the scope of logic, and not the subject-matter of the special sciences. These are not "logics"; they are themselves objects of logical study, just as the earth's crust is an object of geological study.

viii. Is logic an art?

We have defined logic as the science of correct thinking. Logic is a science in so far as it seeks to systematise knowledge. Is it also an art? It has sometimes been called "the art of arguing." Is such a claim valid?

Before we can answer this question it is necessary to distinguish between science and art. Science is interested in the systematic study of anything without a thought as to its utility. Art, on the contrary, gives practical guidance and direction for some course of action. "A science teaches us to know, and an art teaches us to do." The progress of science would be impossible, if it is tied down to considerations of practical utility. An art would lose its character if it is not useful in producing tangible results. Science and art, however, are not independent of each other. Every art presupposes a certain amount of knowledge or science. Theory and practice, knowledge, and action, determine each other. The art of healing, for instance, is founded upon the sciences of chemistry, physiology and anatomy. An art depends on science for its growth and perfection.

Logic which is the science of thought, no doubt, influences the mode of thinking. But this is no sufficient reason for saying that logic is an art. A study of logic is not necessary to reason correctly. Even before logic began to be taught as a science people were found thinking aright. "God did not make man barely a two-legged creature and leave it to Aristotle to make him rational." Nor is a student of logic immune from erroneous thinking. He is as likely to reason incorrectly as the untutored man. Hence, logic is not an art.

What then, it may be asked, is the advantage of studying logic? Logic, certainly, cannot make its votaries infallible reasoners. But when those who are skilled in the science of logic err, they can easily detect their error. The

student of logic will be able to understand the "how" and the "why" of his mistakes, and this will serve to put him on his guard in future. By giving him a clear grasp of the conditions of the validity of thought, logic "helps to form a critical habit of mind and to develop a fine scent for fallacies."

ix. Logic as a formal science

Logic studies thought. In logic we think about thought and not about things; thinking is made its own object. Because of this fact some logicians have held that logic investigates the form of thought to the exclusion of the matter. "A form," according to Jevons, "is something which may remain uniform and unaltered, while the matter thrown into that form may be varied. Medals struck from the same die have exactly the same form, but they may be of various matter, as bronze, copper, gold or silver". Thought has its form as well as matter. The form of thought is the way in which we think of things; and the matter of thought is the various particular objects we think of. The objective sciences are concerned with the things about which we think and not with the ways in which we think. The matter of thought varies from science to science. But the ways of thinking through which knowledge is obtained are the same. It is with these ways of thinking or forms of thought that logic has to do. Therefore logic has been called a "formal science".

While agreeing with this view that the object of logical study is the form of thought, we must, however, take note of the inseparability of form from matter. It is impossible to have form without matter, as it is impossible to have formless matter. And so it cannot be said that logic attends to the form of thought alone without having anything to do with the matter of thought. We may distinguish between the form and the matter but we cannot separate them. As Latta and Macbeath observe, "There is no such thing as 'form without matter', except in *Alice in Wonderland*, where the grin remained after the face of the Cheshire cat had vanished."*

x. Logic and language

Words are the vehicles of thought. They are counters in the market of thought-exchange. Language is the instru-

* **The Elements of Logic**, p. 7.

ment of expression and preservation of thought. The function of words is to fix ideas, to *impress* meanings on us who think, and to *express* our meanings to others. The permanence and progress of thought would be impossible but for the use of language. The application of the same term 'logos' by the Greeks to denote both 'thought' and 'word' or 'discourse' is significant of the vital connection that there is between thought and its expression in language.

Grammar studies words and their right use in significant speech. Correct forms of expression constitute the objects of its investigation, as correct forms of thought are the content of logical study. Grammar is the normative science of language, as logic is the normative science of thought. Hence it is sometimes said that logic is the grammar of thought, and grammar the logic of language.

Rhetoric is the study of language with a view to bring about effectiveness of expression. To produce picturesqueness of description, vivacity in narration, lucidity in exposition, vehemence in persuasion or literary charm, cold grammatical language is not enough. Not mere correctness but beauty and excellence of form is the aim of rhetoric in language. If grammar be described as the logic of language, rhetoric may be regarded as the aesthetics of language. The right use of words with a view to correct expression is the aim of grammar; and the right use of words with a view to persuasion and creating an artistic effect is the aim of rhetoric.

xi. The material of logic

The business which logic sets for itself is rather difficult. It has to think about thought and discover the laws regulative of thinking in the search for truth. But how is this accomplished? Whose thought are we to take as the pattern for logical study? Can we regard our own thinking as providing the material for logic? Definitely we cannot. It is not possible to know all about thought and what it is capable of by an analysis of our ways of thinking. In the first place, it is difficult to observe thought when it is actually in the process of functioning. Secondly, we cannot take our thinking as the pattern of all thinking. Nor is it possible to observe in what manner another person thinks. But we can

study the products and results of the thoughts of others. We have a heritage of knowledge to our credit. Humanity, in its adventures of ideas, has achieved successes and failures. Principles of truth have been discovered in the course of history. The history of knowledge is the starting point of the science of logic. We find certain standards already set; and in the light of them we extend the province of knowledge. The story of the progress of science is the story of the conquest of ignorance. The various sciences have built up knowledge; and they provide logic with material to study the method by which truth is to be attained.

xii. The utility of logic

We have learned that logic is not an art, that it cannot claim to make its students correct reasoners, and that people are found to think correctly even without logical training. What then, it may be asked, is the practical utility of logic? Quick returns of profit should not be expected from a study of logic. In spite of learning logic one may go wrong in thinking. But logic will help its votary to recognise his errors and guard himself against them in the future. In the words of Minto, "Logic does not beckon him on to the right path as beckon him back from the wrong."* Thus the use of logic is indirect or negative.

Since logic is a formal or abstract subject, it gives its student intellectual discipline. The distinctive feature of man is his thinking; and a study of the principles of correct thinking must be of utmost importance to him. As Hamilton says, "In the world there is nothing great but man and in man there is nothing great but mind." Since there is nothing so valuable to man as the development of his mind, logic which disciplines his mind is of very great use to him. A study of logic is essential for understanding philosophical principles. Its chief value lies in its bearing upon ultimate problems, concerning the nature of reality, and man's place in the scheme of things. "Logic," says J. S. Mill, "is common ground on which the partisans of Hartley and of Reid, of Locke and of Kant may meet and join hands."

CHAPTER II

THE DOCTRINE OF TERMS

i. Introduction

Logic is the science of correct thinking. The unit of thought is a judgment. Judgment is a single pulsation of intelligence. It is "the simplest product of thought that can claim to be true, the minimum vehicle of truth (or falsity).*" When expressed in language, judgment is called a proposition, which is composed of words. Words are simpler than the proposition which is a sentence. But it must not be supposed that ideas expressed in words, which are used in judgments, are more elementary than the judgments. There can be no simpler act of thought than judgment. Ideas which are used in it are themselves the result of prior judgments. Before we come to have the idea 'man' we have to make such judgments as 'he speaks,' 'he reasons' etc. The judgment is therefore the most elementary act of thought. And in a study of logic it is but proper that we should consider judgments first. But it has been the tradition to begin the logical study of thought with an investigation into the ideas which enter into judgments. For the sake of convenience we shall stick to this traditional procedure and examine the nature of ideas that constitute the component parts of the judgment.

ii. The structure of propositions

The proposition is the verbal expression of the judgment. The ideas that enter into it are called *terms*. The proposition is composed of two terms and a *copula*. The terms, which are the termini or extremes of the proposition, are named the *subject* and the *predicate*. Thus in the proposition 'the weather is pleasant', 'the weather' is the subject, 'is' is the copula, and 'pleasant' is the predicate. *The subject is that about which something is affirmed or denied; the predicate is that which is affirmed or denied of the subject; and the copula is the sign of relation between the subject and the predicate.* For the sake of convenience all propositions are reduced to the logical form wherein the two terms are related by some part of the verb 'to be', preferably 'is', 'is not', 'are'

*F. C. S. Schiller: *Formal Logic*, p. 12

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i. Introduction

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quite vividly', 'at a certain time', etc., cannot be used as terms; and they are called *syncategorematic* words. Those words which can be employed independently as terms are known as *categorematic* words. All substantives, adjectives and participles may be used categorically, e.g., 'the sun is warm', 'the moon is shining', etc. But when words that are syncategorematic are themselves spoken of and are used substantively, they become categoric. In the sentence 'and is a conjunction,' 'and' is categoric.

Sometimes names are regarded as equivalent to terms. But in so far as names may be used by themselves independent of their functioning as subject or predicate of a proposition, distinction must be made between the two. In the words of Schiller, terms carry meaning, and are subjects for logical study, only *in* the proposition. It is possible for such names like 'pound' and 'balance' to have a plurality of meanings; but that is impossible in the case of terms which have significance only in propositions, where they have definite meanings.

v. Classification of terms

(a) *Concrete and Abstract Terms*

Terms are classified into *concrete* or *abstract*. A term is concrete when it serves to indicate (1) individual existing things, places, persons or events, or (2) an attribute or a relation concretely manifested in individuals, e.g., in "Crows are black", 'crows' and 'black' are concrete terms. A term is abstract when it indicates (1) an attribute or a relation in abstraction from the individuals to which it belongs, or (2) the attribute of an abstract object, e.g., in "Beauty is a human value", 'Beauty' and 'human value' are abstract terms.

There are two ways of understanding the meaning of the word *abstract*. We have dealt with one of them above. We said that a term is *abstract* when it refers to something which cannot directly be sensed; that it "stands for an attribute or group of attributes conceived *in abstracto*, i.e., without reference to any concrete exemplification of them."* The other sense in which the term *abstract* is used is this. A term

* Mellone

is regarded as abstract when it signifies any object treated in abstraction from the whole to which it belongs. Thus man would be an abstract term, and 'society' a concrete term. From this standpoint sense-perception is more abstract than thought. The senses give us particulars, whereas through thought we arrive at universals. While thought systematises, the senses represent things in isolation. In this way we find the whole position reversed. The same terms which will have to be regarded as abstract from the prior point of view appear to be concrete if the word 'concrete' stands for wholeness or completeness.

(b) *Singular, General and Collective Terms*

Concrete terms are classified into *singular* and *general*. 'Devadatta' is a singular term, while 'man' is a general term. A singular term is one that is predicable of one individual only in the same sense. A general term is one that is predicable of a number of individuals in the same sense. Singular terms, again, may be either proper names or significant singular terms. Proper names are those which are permanently assigned to particular individuals, e.g., Asoka, Harischandra, the Himalayas, etc. Significant singular terms serve to indicate individuals otherwise than by names of their own, e.g., 'the greatest man living in the world', 'the highest peak in Asia,' etc. Proper names, no doubt, are sometimes used to denote a class of individuals, e.g., 'a Daniel', 'a Hercules', etc. But in such usages they lose their character as proper names.

A *collective* term is the name of a group of similar but distinct units taken as a whole, e.g., 'a regiment,' 'a crowd'. A general term refers equally to each individual of a group. It can be used *distributively*. But a collective term cannot be so used. The term 'regiment' cannot be applied to a soldier. Collective terms, however, may be either general or singular. Army may signify any one of the armies in the world, and so it is both collective and general. The term 'the British army' refers only to one group of soldiers, and hence it is singular. The distinction is not between collective and general terms, but between the collective and distributive use of terms.

(c) *Positive, Negative and Privative Terms*

A distinction is made between *positive* and *negative* terms. A positive term implies the presence of an attribute or group of attributes in the object which it signifies. A negative term indicates the opposite, *viz.*, the absence of a quality or qualities from the object it denotes. Goodness, happy, order, etc., are positive terms; bad, unhappy, disorder, etc., are negative terms. Very often we decide whether a particular term is negative or not by looking at its form. If it has any one of the prefixes or suffixes such as—*un*, *in*, *dis*, *a*, *anti*, *less*—we say it is negative. But this method is not always safe. Words positive in form may be negative in meaning. 'Ignorant' is the negative of 'learned'; 'darkness' is the negation of 'light'. To avoid possible ambiguities and errors the logical negative (which is also the contradictory) of a particular term is formed by prefixing to it a *not* or *non*.

We referred above to terms which are positive in form but negative in meaning. Some of these are *privative* terms. A privative term is used to indicate the absence of a quality which the object might be expected to have. It implies the deprivation of a quality, *e.g.*, dumb, blind, deaf, etc.

The incompatibility of terms may be due to contradiction or contrariety. Contradictory terms exhaust the universe of discourse and exclude a middle ground, *e.g.*, white and non-white. Contraries, on the other hand, though they express a great difference of degree between the objects denoted by them, admit of a middle ground. Darkness and light are contraries because they do not exhaust all the possibilities. 'Twilight' is neither of the two. Of the contradictories one must be true and the other false. Of the contraries, if one is true, the other is false; but if one of them is false, we cannot say that the other is true, for both of them may be false.

(d) *Relative and Absolute Terms*

Terms are classified by logicians into relative and absolute. An *absolute* term refers to a thing whose meaning is complete when taken by itself, *e.g.*, book, table, tree, etc. A *relative* term is a term which is intelligible only in relation to something else. Thus the term 'parent' has no significance

except in relation to 'child.' Pairs of names thus mutually dependent are called *correlatives*.

vi. The Significance of Terms

Every term has a double significance. It refers (1) to an object or group of objects, and (2) to a quality or set of qualities. The objects to which the term is applied go to form the *denotation* or *extension* of the term. The attributes which it implies constitute the *connotation* or *intension* of the term. A term, then, denotes objects and connotes qualities. The connotation of a term is also known as its intension because it is what we *intend* by the term. The denotation is called extension because it refers to the various objects over which the predication of the term may extend. Thus, in the proposition 'All lions are carnivorous,' the term 'lions' has extension or denotation in so far as it refers to the various individual lions or to the different varieties of lions like the African and the Asiatic; and it has intension or connotation in so far as it refers to the qualities or attributes of lions like being quadrupeds, mammals, feline, etc. As denoting or naming objects terms are said to be employed in extension; and when terms are used to define or describe things, they are said to be employed in intension. Denotation and connotation are different aspects of the significance of terms; and every term must have both these aspects.

vii. Limits of Connotation

The connotation of a term, we have seen, is the attribute or attributes which that term implies. Now, what are the limits of connotation? Should the connotation or intension of a term include all the attributes belonging to the thing or things which the term denotes? Or are we to include in the connotation of a term only some of the qualities? Some logicians regard the connotation of a term as the sum total of qualities actually possessed in common by members of the class. This is called the *objective view*. All the attributes of an object known and unknown constitute, according to this view, the connotation of that object. But, for the purposes of logic, the unknown attributes are of no use, since they do not come under the purview of reason. To avoid this difficulty it may be said that the connotation of a term refers to

wise follow. Independently they calculated the whereabouts of this unknown planet.

J. C. Adams took his calculations to the Astronomer Royal at Greenwich; but as the latter was at dinner at that time, he could not be disturbed. Disheartened, Adams left a note for the Astronomer Royal, which read as follows: "According to my calculations, the observed irregularities in the movements of the planet Uranus may be accounted for by supposing the existence of an exterior body, the orbit of which is as follows." The Astronomer Royal did not take the note seriously. All that he did was to write to Adams, asking him what he regarded as a test question. Adams was disappointed, thought that his calculations must have gone wrong, and did not take further steps to verify them.

Meanwhile, Leverrier, who had become a famous astronomer by that time, unlike Adams, published several papers in which he explained his calculations and ended up by pointing to the place near which the new planet must be. The place of his calculation was very nearly the same as that of Adams. Leverrier set Galle, a German astronomer, to work, and on the very first night Galle found the planet.*

xxii. Limitations of the Method

The limitations of the Method of Residues we have already pointed out during the course of our explanation of the method. In the first place, the method will not be useful at the initial stages of an inductive inquiry. Only after a major portion of a complex phenomenon has been explained, the Method of Residues can be used for discovering the cause of the residual factor of the phenomenon. Secondly, it has been shown that the Method of Residues is not an independent method. It is a variant of the Method of Difference. Thirdly, a good part of the calculations involved in the application of the Method is deductive in character. Lastly, in actual practice the cause of the residue of the phenomenon is not readily present before us to be connected with the residue; it has to be discovered; and what the Method of Residues does is to warn us that we should not stop in our investigation till every part of the phenomenon has been explained.

*For a fuller account see H. H. Turner, *A Voyage in Space*, pp. 158-164.

who bear that name are men; for even dogs and other pets have been named John. Even in cases where proper names seem to have been given for a reason they have subsequently grown independent of their reason. Dartmouth is a place which is situated at the mouth of the river Dart; but if the Dart should change its course, the town would still continue to be called Dartmouth. Mont Blanc means white mountain. But the name is applied only to one of the mountains in the world and not to every peak that is white. Such are the arguments advanced by Mill to prove that proper names are non-connotative.

Proper names taken by themselves signify, no doubt, no definite meaning. But when they are used as terms in propositions, they have always some more or less definite connotation. If it is said that a proper name is not given to an individual because of certain attributes, the objection holds good even in the case of general names. Why should the word 'cat' be given to a particular creature? Was it not an arbitrary act of the builders of language? There is, of course, lack of definiteness about the meaning of proper names. But lack of definiteness does not imply lack of meaning. A proper name undoubtedly possesses meaning, in the sense of reference to attributes. "The proper name is only unmeaning *before it is given*: by being given and becoming a mark, it acquires a meaning. And the general name was equally unmeaning *before it was given*; but being general, it can be given to more things than one"* and it acquires a meaning which is instructive. Those who use a proper name use it with a knowledge of its significance. In the context in which the name is used it bears a definite meaning. The nature of general terms is also similar. Only those who have identity of interest and who use a particular word can understand its definite meaning. A word like 'balance' for example, may mean different things to different people. It can have a definite meaning only to those who have the same interests. Thus, proper names are not different in kind from general terms. If proper names have no connotation, why should every criminal assume an alias? As Professor Bosanquet points out, "The convention of the usage which

* H. W. B. Joseph: *An Introduction to Logic*, p. 136.

prevents a proper name from being general, *i.e.*, from being cut loose and used simply for its meaning, is always on the point of breaking down." Witness the use of proper names to denote certain types of individuals, 'a Daniel,' 'a Don Quixote,' 'a Caesar Borgia,' etc.

(2) Mill characterises abstract terms as non-connotative on two grounds. He urges that abstract terms have no connotation, because they cannot be qualified by other attributes. 'Whiteness' has no connotation, since it is not the subject of other attributes. And if it is qualified by other attributes, it would become concrete and cease to be abstract.

This argument advanced by Mill is pointless. The connotation of a term refers to the attributes signified by it, not to the attributes that may be predicated of it. Further, it is absurd to say that an abstract term becomes concrete when attributes are predicated of it. A term is concrete only when it refers to a concrete existent.

The second objection of Mill against regarding abstract terms as connotative is this. Abstract terms, because of their abstractness, cannot possess denotation. According to Mill, a term which does not denote but indicates only an attribute is non-connotative. If an abstract term is made to refer to existents, then it ceases to be abstract.

This contention is without any validity. We do not regard whiteness as existing in this or that particular form. But we do think about it as *real*. Reality which is the whole and which is not concrete in the sense in which individual things are, is the denotation of abstract terms. Every abstract term denotes reality without the risk of becoming concrete. Mill and his followers are not justified in dividing terms into connotative and non-connotative. Every term has a double function. It denotes an individual or class of individuals. It connotes a quality or set of qualities. In some terms the connotative aspect may be prominent, and in others the denotative function may be dominant.

ix. The Doctrine of Inverse Ratio

It is sometimes said that there is a quantitative relation between connotation and denotation. It is thought that as connotation increases denotation decreases and *vice versa*.

By increasing the connotation of the term 'ship' by prefixing the term 'steam' we decrease the denotation, as steam-ships are fewer than ships. Let us examine a series of related terms: 'ship,' 'steam-ship,' 'screw-steam-ship,' 'iron-screw-steam-ship,' 'British iron-screw-steam-ship.' Here the connotations form an increasing series and the denotations a diminishing series. Hence it is said 'As connotation increases denotation decreases.' Some logicians push this view to extremes and state, in the language of mathematics, that connotation and denotation vary in inverse ratio.

This view is defective for the following reasons:

(1) The mathematical terminology is unjustifiable. An inverse ratio is possible only between two calculable quantities. Though the denotation of a term may occasionally be a calculable quantity, the connotation of a term is incalculable. Connotation signifies qualities and they cannot be numbered. Intension and extension are in reality incommensurable. There are qualities like 'beautiful,' 'rational,' etc., which are not really single qualities. Each of them represents a variety of attributes.

(2) Even granting that there is some correspondence, it will be seen that the decrease of denotation is not in exact proportion to the increase of connotation. The addition of one quality may decrease the denotation much more than the addition of another. 'Red man' decreases the denotation of man much more than 'White man.'

(3) When we pass from man to red man or white man, we are really passing from one term to another. Hence it is not a ratio between the increase and decrease of the denotation and connotation of the same term.

(4) In certain cases the expansion of the connotation of a term through increase of knowledge does not bring about a decrease in denotation. The connotation of the term 'planet' has become richer with the advance of Astronomy; but the number of planets has not become less.

(5) It is possible to increase the denotation without in any way diminishing the connotation. For example, the increase of population does not change the meaning of 'man.'

(6) To talk of the connotation of a term being increased is logically meaningless. The conventional connotation of the term 'planet' was changed when the planets were found to be bodies moving in definite orbits round the sun. Though we can here say that there has been an increase in connotation, in the strictly logical sense there has been a change in the meaning or reference of the term. In changing the connotation we constitute really a different term.

(7) The doctrine of the inverse ratio is based on a wrong notion of the relation between genus and species—the notion that attributes are added when we go from the genus to the species and subtracted when we go from the species to the genus. No such change, however, takes place. The genus 'animal' is not less in intension than the species 'man.' The genus may be less definite in meaning, but in itself it does not have less meaning.

CHAPTER III THE PREDICABLES

i. Introduction

We have already seen that the proposition is the verbal expression of judgment and that it contains two terms, the subject about which something is asserted and the predicate which is the assertion made. Evidently, then, the two terms must be related. The relation in which a predicate stands to the subject of which it is predicated is called a *Predicable*.* According to Aristotle, in every proposition the predicate must be either the definition, the genus, the differentia, a property or an accident. But Porphyry (about 600 years after Aristotle) gave a revised classification of predicables. He substituted species for definition and sub-divided accidents into separable and inseparable. Thus he enumerates five predicables: (1) Genus, (2) Species, (3) Differentia, (4) Proprium, (5) Accidens, separable and inseparable. The first two refer to the denotative aspect of terms; and the last three to the aspect of the qualities signified by terms.

* "The Predicables are the possible relations (in extension and intension) which the predicate of a proposition may bear to its subject when it makes an affirmation of the subject." (Mellone.)

The difference between the Aristotelian and the Porphyrian accounts of the Predicables is this: Aristotle had in view the *definition* of a subject, while Porphyry was primarily concerned with the *division* of a class. Aristotle, whose aim was to define the subject, considered the various qualities of the subject. The qualities may be (1) the essential qualities (definition), (2) the essential qualities which the subject has in common with others (genus or differentia), (3) the peculiarities, qualities which always accompany the essential qualities (*propria*) and (4) the accidental qualities which are not essential to the subject (*accidens*). Porphyry, on the other hand, was having in view *division* rather than *definition*. Therefore, he begins with (1) the genus or the class to be divided, considers (2) species to be the sub-classes or subordinate divisions, (3) differentia to be the attribute or attributes by which each species is distinguished from the other species in the same genus, (4) *propria*, and (5) *accidens* to be other qualities which are irrelevant for the purposes of division and definition. He omits definition as a distinct predicable, and makes definition = genus + differentia, *e.g.*, man = rational (differentia) + animal (genus). Aristotle did not regard species as a predicable because the species was to him the thing to be defined, the subject.*

Porphyry's account of the Predicables has become the traditionally accepted one, and so we shall be mainly concerned with his list of Predicables: (1) Genus, (2) Species, (3) Differentia, (4) *Proprium* and (5) *Accidens*—(a) separable and (b) inseparable.

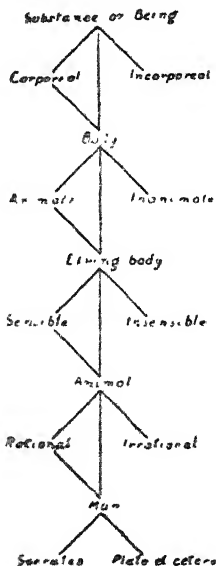
ii. Genus and Species

A genus is any class containing two or more subordinate classes or species. It is any wider class which is made up of narrower classes. 'Animal' is the genus of 'man'; and 'man' is a species of 'animal' which consists of other species like 'tiger,' 'lion,' etc. 'Triangle' is the genus of the species 'equilateral,' 'isosceles' and 'scalene.'

The terms 'genus' and 'species' when employed in Natural Sciences like Botany and Zoology, indicate certain relatively fixed divisions, or permanent ways of grouping the

* Latta and Macbeath: *The Elements of Logic*, p. 138.

various forms of plant and animal life. But in logic, as Creighton remarks, the terms 'genus' and 'species' are employed to indicate the relationship between any higher and lower class whatsoever. Genus and species, as logical predicates, are entirely relative to one another, capable of being moved up and down the ladder, any lower class being species in relation to any higher. That is to say, the same class may be a genus with reference to its sub-classes and a species in relation to a wider class under which it is subsumed. 'Animal' is the genus of 'man'; but it is a species of 'living being.' 'Triangle' is the genus of 'equilateral,' etc., and a species of 'rectilinear figure.' Thus logically there are no fixed genera or species. What is a genus in relation to a narrower class is a species in relation to a wider class.



A class which cannot be included as a species in any wider class is called the *sumum genus*. It is the highest genus which is never a species. Similarly, a species, which

cannot be the genus of sub-classes and which can only be divided into individuals, is known as the *infima species*. A line of division beginning with the *summum genus* and ending with the *infima species* was called by the scholastic logicians a 'predicamental line' (*linea predicamentalis*). The familiar instance of a predicamental line is 'Porphyry's tree,' which is given on opposite page.

Beginning with 'substance' as *summum genus*, and adding the differentia 'corporeal,' we frame the species 'Body.' Taking 'body' as the genus and adding the differentia 'animate,' we frame the species 'Living Body' and so on till 'Man' is reached, which being *infima species* is only sub-divisible into individuals.

In this illustration, then, 'substance' is the *summum genus* and 'man' is the *infima species*. The other classes which intervene between the *summum genus* and the *infima species* are called *subaltern genera* or *species*. Thus 'body,' 'living body,' and 'animal' are classes which intervene between 'substance' and 'man.' The class which is the nearest genus to another class is called its *proximum genus*; and the class which is the nearest species to another class is called its *proximum species*. If a class is removed from another class as its genus by one or more intermediate classes, it is termed a *remote genus*; and if a class is removed from another class as its species by one or more intervening classes, it is known as a *remote species*. Thus 'substance,' 'body' and 'living body' are the remote genera of 'man'; and 'living body,' 'animal' and 'man' are the remote species of 'substance.' The relation in which a species stands to its genus is that of subordination. 'Man' is subordinate to the genus 'animal,' 'equilateral' is subordinate to the genus 'triangle.' A genus contains a number of species. The species which are under the same genus are called *cognate* or *co-ordinate* species. Thus the relation of the different constituent species to each other is that of co-ordination. Man, tiger, lion, etc., are the co-ordinate species of the genus 'animal.' Equilateral, isosceles and scalene are the co-ordinate species of the genus 'triangle.'

The relation between genus and species has been likened to that between a container and its contents. We ourselves

have said that a genus is a class which contains two or more subordinate classes. Though the use of the word 'contain' is permissible, the genus cannot be viewed as the container in the literal sense of the term and the species as the contents. Even after the contents have perished the container may continue to exist. But the genus has no existence apart from the species. The genus is realised in the species. It is nothing over and above them. It is sometimes said that the genus possesses less connotation than the species. But even this is wrong. The connotation of the genus is not very definite whereas the connotation of the species is definite. The genus is extensive and indeterminate; the species is more restricted and determinate.

iii. **Differentia**

The differentia is a quality or group of qualities which distinguishes a species from other species in the same genus. The distinguishing attribute of the species, peculiar to it and differentiating it from other species of the same genus, is what is called the 'differentia.' Thus 'rationality' is the differentia of 'man,' since it distinguishes 'man' from 'tiger,' 'lion,' etc., which are its co-ordinate species under the same genus 'animal.' The attribute of having three equal sides is the differentia of equilateral triangle, since it differentiates it from the other species of triangles like the isosceles and the scalene.

iv. **Proprium**

According to Aristotle, a proprium is a property peculiar to its subject and is an inseparable concomitant of it. The propria which belong to the whole of a class, and are peculiar to it, do not have any important effect on its other characteristics. When we say "The Ethiopian cannot change his skin, nor the leopard his spots," we mean that the pigment of the Ethiopian's skin and the spots of the leopard are their respective properties.

Porphory made a change in the meaning of proprium. According to him, a proprium, is not necessarily peculiar to its subject, but it follows from the definition (essence or connotation) of its subject. Aristotle would regard the capacity to cook food as a proprium of man. But unless the

culinary art can be shown to be a necessary consequence of man's rationality, it would, for Porphyry, be an accident of man. Porphyry regarded proprium as an attribute which does not form part of the connotation (or definition) of a term, but which follows from it, either as effect from cause or a conclusion from premise. Thus 'the power of judging' predicated of 'man' is a property which follows as an effect from his 'rationality.' That the three angles of a triangle are together equal to two right angles is a property of a triangle which follows as a conclusion deducible from the essential properties of a triangle.

v. Accidens

An accidens is an attribute which neither forms part of the connotation of a term nor is deducible from the connotation. It is a feature accompanying the defining attributes without being deducible from them. The word 'accidens' may suggest that such an attribute is merely 'accidental' to its subject. But that is not the intended meaning here. 'Accidens' as a predicable means a quality whose connection with the essence *we* do not understand. Thus the colour of animals was once regarded as an accident for which no reason could be given. But now the colour of animals is considered to be an effect of their nature and habits. In some animals the colour is determined by its being advantageous for concealment. In others the determinant may be the advantage of advertising.

A distinction is made between inseparable and separable accidens. The inseparable accidens of a class is that quality which is present in all the members of the class, but whose presence is not known to be essential to the nature of the class. All crows are black. But we know of no reason why they should be black. It is not easy to distinguish an inseparable accidens, such as the black colour of the crow, from the proprium. The latter can be distinguished only by the test of deducibility from the essence. The inseparable accidens of an individual is that non-essential quality which pertains to the individual at all times, e.g., 'born in India,' 'six feet high,' etc. The separable accidens of a class is an attribute which belongs only to some members of the class, e.g.,

कृते संभाष्य पतितस्त्रेतायां स्पर्शनेन तु ।
द्रापरे धनमादाय कलौ पतति कर्मणा ॥

बृहस्पतिरपि.—

कृते यदब्दाद्यो धर्मस्स त्रेतायामृतुत्रयात् ।
द्रापरे तु त्रिपक्षेण कलावद्वा तु तद्भवेत् ॥

विष्णुपुराणेऽपि.—

यत्कृते दशभिर्वर्षैस्त्रेतायां हायनेन तु ।
द्रापरे तच्च¹ मासेन अहोरात्रेण तत्कलौ ॥

ब्रह्माण्डपुराणेऽपि—

त्रेतायामाब्दिको धर्मो द्रापरे मासिकस्मृतः ।
यथाशक्ति चरेत्प्राज्ञः तमद्वा प्राप्नुयात्कलौ ॥

विष्णुधर्मोत्तरेऽपि.—

पुष्करं तु कृते सेव्यं त्रेतायां नैमिशं तथा ।
द्रापरे तु कुरुक्षेत्रं कलौ गङ्गां समाश्रयेत् ॥

इति स्मृतिचन्द्रिकायां युगधर्माः.

कलियुगधर्माः.

अथ कलियुगधर्माः । तत्र व्यासः—

ध्यायन्कृते यजन्यज्ञैस्त्रेतायां द्रापरेऽर्चयन् ।

animal.' Here, 'animal' is the genus and 'rational' is the differentia. In defining a term we should relate it to its proximum genus and state its differentia. Otherwise the definition would be incomplete. 'Man is a rational being' is not a definition: for, 'being' is not the proximum genus of man and the definition omits the qualities of 'animal' which form part of the connotation of 'man.'

ii. Rules of Definition

Certain rules or principles of definition have been recognised by logicians. They state what the requirements of a logical definition are:

(1) *A definition should state the essential attributes of the things defined.* This is done by defining a term *per genus et differentiam*, i.e., by stating the immediate higher class and the differentiating quality or qualities.

(2) *It should state neither more nor less than the whole connotation of the term defined. The definition should be exactly equivalent to the class of objects defined.*

A definition which states more than the connotation of the term defined is *too narrow*. A definition can embrace more than the connotation of the term defined by including either some of the propria or some of its accidents. If we define an equilateral triangle as a triangle having three equal sides and three equal angles, we include in the definition, besides the differentia a property. In so defining we suggest that there are equilateral triangles which may have three equal sides and yet not have their angles equal. But this is not the case. The definition is too narrow because it suggests that the denotation is restricted, that triangles with three equal sides and three equal angles are a species of triangles with three equal sides. When an accident is included in the definition the same error is committed. The definition of a triangle as a plane rectilinear figure having three *equal* sides is too narrow. Having three *equal* sides is an accident to a triangle; what is essential is the possession of three sides. In including the accident the legitimate denotation of triangle is restricted. Hence the definition is too narrow.

If a definition contains less than the connotation of the name, it is *too wide*, for it is applicable to a greater number

of things than are included in the denotation of the term defined. If an equilateral triangle is defined as a plane rectilinear three sided figure, the definition really includes *all* triangles. The predicate is wider in denotation than the subject. Hence the definition is too wide.

(3) *A definition should not be expressed in obscure, figurative, or ambiguous language.* The aim of definition is to make distinct the meaning of a term. If the definition be as vague and obscure as the term itself, then, the purpose of definition would be defeated. When we violate the demand for clearness in a definition we are said to define *ignotum per ignotius* or *per acque ignotum*; that is, we are explaining the unknown by the more, or at least equally, unknown. An amusing instance of such a definition is Dr. Johnson's definition of a net as "a reticulated fabric, decussated at regular intervals, with interstices between the decussations." Here the words used in defining are less familiar than the term defined. This definition is as useless as the Irishman's conception of the net as a lot of holes tied together by a string. "Eccentricity is peculiar idiosyncrasy," and "Fluency is an exuberance of verbosity," are cases of definitions which do not make clear the terms defined. When we say "The lion is the king of beasts," or "Bread is the staff of life" or "Necessity is the mother of invention" or "Memory is the tablet of the mind" we are indulging in figurative language and not giving definitions.

In special sciences, however, definitions may contain words which, to the uninstructed man, will be obscure, e.g., Sodium is an element exhibiting line D in the spectrum. Such definitions are perfectly sound, though they may be unintelligible to one who is unacquainted with those sciences.

(4) *A definition should not contain the term to be defined, nor any word directly synonymous with it. It should not, directly or indirectly, define the term by itself. It should not be tautologous.*

When a definition violates this rule, it is called *circulus in definiendo* or *a circle in defining*. It is meaningless to define a term by means of the term itself, e.g., 'A conic section is a section of a cone'; 'Truth is veracity in speech and act'; 'Justice is the way of acting justly'; 'Life is the sum of vital

processes', etc. It is tautologous to define an archdeacon as one who exercises archidiaconal functions, or a viceroy as one who discharges viceregal duties. All these are glaring cases of circular definitions.

This rule is not violated when the name of the genus is repeated in defining a term which stands for a subordinate species without a distinct name of its own. The definition of an equilateral triangle as a triangle which has three equal sides is perfectly valid.

(5) *A definition should not be negative if it is possible to make it affirmative.*

A definition should state what a term signifies rather than what it does not signify. We must not, for example, define things by their contraries or contradictories. To say that 'knowledge is the opposite of ignorance' or 'sleeping is the opposite of waking' is not to define 'knowledge' and 'sleeping.' There are, however, terms which are best defined negatively, e.g., 'non-belligerent,' 'alien,' 'indivisible,' etc. We may define, for instance, a spiritual being as a being which is not material, a bachelor as an unmarried man, a stool as a seat for one without a back to it. But it must not be thought that because a term is negative in form it is necessary to define it negatively. Intemperance, e.g., can be defined affirmatively as the excessive indulgence in strong drink.

(6) Dr. Mellone adds a sixth rule to the five given above, and he calls it a 'counsel of perfection.' The rule is: *the definition should contain nothing superfluous.* This, in fact, sums up the essentials of definition, in so far as the aim of this logical process is to 'mark off' the object defined from the others by exhibiting neither more nor less than its essence.

iii. The Limits of Definition

The traditional type of definition is achieved, as was shown above, by mentioning the proximate genus of the thing to be defined and the characteristic differences which distinguish it from other species. In certain cases these conditions of a logical definition cannot be fulfilled. It is not possible to define the *summum genus* which has no genus above it. And again, though species like 'man' or 'metal' may be defined, *individual* characteristics cannot be stated by means of a definition. Hence it is said sometimes that the *summum*

genus and individuals are indefinable. Again there are other terms like 'space,' 'time,' 'life,' 'thought,' which are also indefinable, since they denote objects that are *sui generis* or of their own class.

iv. Systematic Definition

These difficulties arise only if we adhere to the traditional method of defining terms. Definition *per genus et differentiam* assumes the existence in the world of fixed classes or kinds arranged in an unchanging, hierarchical order. Only then is it possible to subsume a species under its immediate genus and differentiate it from its co-ordinate species which in effect is definition as understood in traditional logic. But modern science recognises no fixed species or real kinds. There is no immutable trait by virtue of which a particular class may be distinguished from the rest. The theory of evolution has revealed to us that all things are subject to a process of change. Man, we are told, is descended from the ape. Newer and newer species are evolved from the older ones. From the unicellular amoeba to the multicelled man there runs a single chain of evolution. And so in the realm of science it is no longer possible to define terms in the old traditional way. Instead of defining by genus and differentia, science adopts what may be called the *systematic* method, *i.e.*, it tries to show the position of the thing defined as an element in a system. The whole universe is a system; and in it there are ever so many smaller systems. Nothing hangs in the air as it were, unrelated to others. The supreme task of science is to organise apparently isolated entities into system; or rather it is to discover the system to which a thing belongs. And when this is done, a thing is said to be defined. The scientific or systematic definition, then, consists in exhibiting the relation of the object defined to other elements, in the system and to the system as a whole. The definition of a thing is also possible by tracing it to its origin or showing the mode of its genesis, as when a circle is *genetically* defined as 'the plane figure generated by revolving a straight line about one of its extremities which remains fixed.' In the view of the scientist, then, there is nothing which cannot be defined. Unlike traditional logic, it admits of no indefinables.

CHAPTER V

DIVISION

i. Division and Definition

Division is the complement of definition. If definition is the process of making distinct the connotation of a term, division is the method of exhibiting the denotation of a term. In defining a term, we state its essential nature; in dividing it, we analyse its extension. Both aim at the same end, *viz.*, of making clear the meaning of terms. In fact, division is extensive definition. It is a process of analysing a genus into its various sub-classes. It supplements logical definition which we studied in the last chapter by showing in detail the denotative aspect of the significance of a term. And by its very nature, it has to follow definition, since it is not possible to divide things, without some knowledge about their essential nature. In order to divide conscious beings into men and beasts, for instance, we should know first the differentia of men and the differentia of beasts. When we divide a genus, we think of an attribute which characterises some of its members and thus serves to distinguish them from the others. It is this which suggests the basis of division. It will be evident, then, that in order to analyse the denotation of a term, we must also have a knowledge of its connotation, which again shows what we urged in an earlier chapter, *viz.*, that every term has a double significance.

ii. Division and Classification

Natural science has familiarised us with a process known as *classification* which is substantially the same as division. The only distinction between the two is this. In division, we start with a genus and distinguish the species within it: in classification, we start with the particulars of a genus and classify them into groups, according to their likenesses and differences. When one analyses the genus 'man' into the species, African, American, Asiatic, Australian and European, it is division. When one collects stamps and groups them according to value, or country or date, it is classification. In division, we move downwards from the more general to the more special; in classification, we move upwards from the more

special to the more general. The distinction between the two processes, however, is only theoretical; for in actual thinking we use them together. As Joseph says, "In actual practice our thought may move in both directions at once; and the process of dividing a genus is at the same time one of classifying the things in a genus. If, for example, one were asked to divide the genus *novel*, he might suggest a division into the novel of adventure, of character, and of plot; but he would at the same time run over in thought the novels that he had read, and ask himself if they could be classed satisfactorily under these three heads."

A distinction is usually made between artificial or diagnostic classification and natural or scientific classification. When a classification is made on the basis of some superficial quality to serve some practical purpose, it is artificial. In a catalogue the books are arranged according to the initial letter of their names. Here, there is no vital principle involved. The arrangement is only for the sake of convenience and easy detection of the book one wants, "A farmer does not divide plants, like a botanist, into dicotyledonous and monocotyledonous, but into useful plants and weeds."* This again is an artificial classification intended to serve an ulterior end. A natural classification, on the contrary, is made purely from a theoretic interest. It is guided by the essential properties of things. All the classifications in science are of this nature. The distinction between the two kinds of classification, however, should not be regarded as an absolute one. Even the artificial classification is based on some natural property; only that property may not be an essential one. And very often it is the artificial classification that leads to the natural classification.

iii. The Principle of Division

The purpose of division is to distinguish the various species subsumed under a genus. This is achieved by analysing the denotative implications of the genus on the basis of a principle. This principle is called the *fundamentum divisionis*. If we divide men into white, black, yellow, etc., the principle of division is the colour of their skins. Thus every division must be made on the ground of a principle.

* Mill: *System of Logic*, bk. iv, ch. vii, sec. 2.

The same genus may be divided according to different principles, not in the same but in different divisions. Thus, on the basis of the relative lengths of the sides of a triangle, triangles may be divided into equilateral, isosceles and scalene; and on the basis of the size of the angles, they may be divided into right-angled, obtuse-angled and acute-angled. When the same genus is thus divided in different ways, the process is called *co-division*.

A division may be carried through several stages; the species into which a genus is divided may themselves be subdivided into simpler species. This process is called *sub-division*; and it has to come to a stop when we reach the *infima species* which cannot be subdivided into smaller classes e.g., men may be first divided into Asiatic, European, etc.; Asiatic into Indian, Chinese etc., Indian into Hindu, Muslim, etc.; Hindu into Saiva, Vaishnava, etc.

iv. Rules of Division

As we discussed certain rules for definition in the last chapter, so we have rules also for division. These rules may be directly derived from the nature of the process. They are as follows:

(1) *Every division is made on the basis of a unity which reveals itself in its differences or varieties.*

This requires little comment, as it merely states the nature of division already explained. Logical division is always of a genus into its constituent species. The genus stands for the common nature or unity, and the species for the differences within it. If there is a class which has no distinctions within it, then no division is possible. Nor is division possible in the case of a heterogeneous collection of entities bearing no mutual resemblance whatever.

(2) *Every act of division must be based on a single principle or fundamentum divisionis.* Breach of this rule is called *cross-division*. We stated above that division proceeds on a principle or *fundamentum*. While it is legitimate to divide a genus on different bases in different acts of division (co-division), it is illogical to use more than one principle in a single process of division. For instance, it is wrong to divide men into fair, dark, uncivilised, cultured, and Mongol; for the division involves three principles, *viz.*, varieties of colour, of

culture and of race. The aim of division is to set forth clearly the denotation of a term. In the example of a division given above, the denotation of man is anything but clear. A dark man may be cultured; even so a Mongol. By changing the principle of division we make for confusion and not for clarity. A cross-division goes against the very purpose of division. It is worse than useless, says Joseph, for instead of assisting to an orderly arrangement of things in thought, it introduces confusion.

(3) *The constituent species or dividing members must not overlap, but must be mutually exclusive.* This is implied even in the second rule. If the principle of division is changed, then the groups into which the genus is divided will not be mutually exclusive. In the example of cross-division given in the last paragraph the classes into which man is divided overlap. Fair men and cultured men are not divisions of men, for among fair men some may be cultured and others not. The proverb which distinguishes 'fish, flesh, fowl and good red herring' and Dr. Johnson's classification of the inhabitants of Britain north of the Tweed into Scotchmen and Damned Scotchmen are also cases of overlapping division. In these cases, observes Joseph, the logical error points a sarcasm; but in itself it is comparable to the procedure of the philosopher, who cut two holes in his door, a large one for the cat and a small one for the kitten.

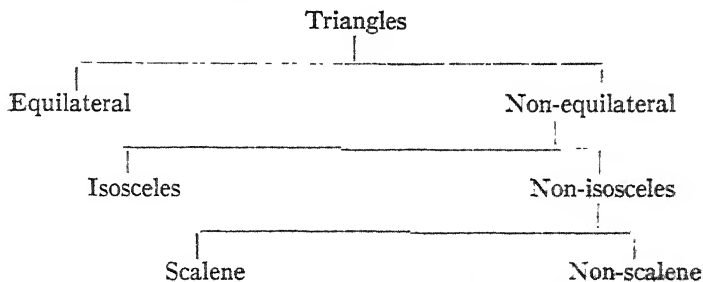
(4) *The division must be exhaustive, i.e., the dividing members must be together coextensive with the divided whole.* In a process of division all the species of a genus must be stated. Nothing should be omitted; and nothing else should be included. The division of triangles into equilateral and isosceles is incomplete, because there is the third variety, scalene. The division of triangles into equilateral, isosceles, scalene and square, is again faulty, because square is not a species of triangle.

(5) *If the division be carried through several stages, each stage must as far as possible be proximate.* That is to say, each species must emerge from its proximate genus so that nothing in the middle may be omitted. The old logicians expressed this in the formula, *divisio ne fiat per saltum*, division must not make a leap. If this rule is violated, some

members of the whole we start to divide will be left out. A breach of this rule is made when, for instance, religions are divided into Polytheism, Judaism, Christianity, and Christianity into Catholicism and Protestantism. The proper division would be: religion into Monotheism, Polytheism, etc., Monotheism into Judaism, Christianity, etc., and Christianity into Catholicism, Protestantism, etc.

v. Division by Dichotomy

A purely formal method of division known as *dichotomy* was in vogue in the Middle Ages. It consists in the division of a term into two parts, one positive and the other its corresponding negative. The principle underlying this method is the Law of Excluded Middle* which asserts that between logical contradictories there is no middle ground. Terms like *a* and not-*a* exhaust the universe. Anything must be either *a* or not-*a*. One or the other of these contradictories must belong to every possible subject. Applying this logical principle, A may be divided into B and not-B, not-B into C and not-C, not-C into D and not-D, and so on. This is division by dichotomy. According to this method, the term 'triangle' would be divided thus:



Such dichotomic divisions are formally perfect but they are not of much value and are open to objections.

(1) In the first place, at each step of the division, one of the sub-classes is left vague. The term not-B does not denote any definite class. On the basis of dichotomy even such absurd divisions like 'virtue is either square or not-square' will be valid.

* See the Chapter on the Laws of Thought.

(2) Secondly, in so far as the method is formal, it is entirely hypothetical. The division does not guarantee the reality of any of the sub-classes. Non-scalene is a sub-class in the illustration given above. But there are no non-scalene triangles which are also non-equilateral and non-isosceles.

(3) Thirdly, the method is excessively cumbersome. It represents co-ordinate species as if they were subordinate to one another. Equilateral, isosceles and scalene are co-ordinate species of triangle. But in the division they appear as if they were subordinate to one another.

vi. Other Kinds of Division

There are kinds of division which are not logical in character and which must be distinguished from logical division.

(1) *Physical partition* is the division of an individual thing or aggregate into the parts of which it is composed, e.g., the division of a chair into seat, back and legs; of Madras Presidency into the various districts.

(2) *Verbal division* is the process by which different senses of an ambiguous name are distinguished, e.g., 'vice' into moral fault and a mechanical tool.

(3) *Metaphysical division* is conceptual analysis whereby the attributes of a thing or class of things are distinguished, e.g., silver into something white, malleable, etc. This is a division which can be accomplished only in thought, and hence is conceptual.

All these are different from logical division. In logical division the whole, which is divided, can be predicated of its parts—e.g., animal is divided into man, lion, etc.; and the divided whole 'animal' may be predicated of each of the dividing members; we may say 'man is animal,' 'lion is animal' and so on. In metaphysical analysis the parts can be predicated of the whole—e.g., we may say, 'silver is white,' 'silver is malleable,' etc. In physical partition we can neither predicate the parts of the whole nor the whole of the parts—e.g., we can neither say that the chair is back or legs, nor that the back or legs are the chair.

CHAPTER VI

PROPOSITIONS

i. Introduction

Having learnt the implications of terms let us now turn to a study of propositions. A proposition, according to Aristotle, is a statement in which something is said regarding something else, either affirmatively or negatively. That about which something is asserted is the *subject* of the proposition; that which is asserted of the subject is the *predicate*; and the assertion is the *copula* which is usually signified by the words 'is' or 'is not,' 'are' or 'are not.' Thus when analysed, a proposition seems to consist of three parts. But really speaking, the copula is not a part at all. It is not a coupling link connecting the subject term and the predicate term. It simply expresses the mental *act* of judgment. When we judge 'man is mortal' there is nothing in our thought corresponding to the word 'is'; we are merely thinking of the mortality of man, and the copula represents our *judging*. In common speech we do not always employ the copula. The sentence 'What can't be cured must be endured' is certainly the verbal expression of a judgment. But it does not contain the words 'is' or 'is not.' It is only for the sake of convenience that such sentences are reduced to logical form with a copula in each. If it is to be stated as a strictly logical proposition, the sentence given above will take the form 'All things which cannot be cured are things which must be endured' where the word 'are' is the *expression* of the copula. But it should be remembered that the analysis of a proposition into subject, predicate and copula is an artificial arrangement whose purpose is to avoid ambiguity in reasoning.

ii. Logical Proposition and Grammatical Sentence

The logical proposition must be distinguished from the grammatical sentence. The logical proposition is the expression of a judgment. It embodies thought and hence conveys information which may be either true or false. The scope of a grammatical sentence is not so restricted. Not only thoughts, but wishes, commands, feelings, etc. may be expressed in sentences. Hence Whately refers to the proposition as 'an indicative sentence.' What is not indicative of a

thought is not a proposition. Questions like 'Breathes there the man with soul so dead?' commands like 'Do thy duty without caring for the reward,' and exclamations like 'Horrible!' are not logical propositions as they stand. But it will be noted that in each of these cases there is a proposition implied. The question carries with it its own answer. It means in effect that no man with soul so dead lives. The command implies that duty is to be done without a thought as to its reward. And the exclamation refers to the judgment that something (given in the context) is horrible. So in such cases the proposition must be extracted and stated in the logical form.

Even indicative sentences which are media of information are not usually found in the propositional form. The logical form of a proposition is 'S is (or is not) P'. But sentences need not be stated in this form. The order of words may be changed for the sake of effectiveness, style etc. Thus the sentence 'Blessed are the merciful' is not a strict logical proposition. In order to assume the proper form it must be rewritten thus: 'All merciful people are blessed.' Very often the grammatical subject and predicate are not the same as the logical subject and predicate. In the sentence 'we know the soul through introspection' the grammatical subject is *we*, the principal noun in the nominative case, and the predicate is *know*, the main verb. But the logical subject is not 'we' but 'our knowledge of the soul'; the logical predicate is contained in the rest of the sentence; and the copula is included in the grammatical predicate. Reduced to logical form, the proposition will be 'our knowledge of the soul is knowledge which comes through introspection.' Thus, it must be clear that all grammatical sentences—even indicative sentences—are not propositions, though it is true that all propositions are grammatical sentences. When a sentence is given for logical treatment, the first task that a student of logic has to do is to recast it into the correct propositional form with a subject and a predicate connected by a copula which should be in the present tense of the verb 'to be.' In a subsequent section of this chapter will be found hints for reducing sentences to logical propositions.

iii. Classification of Propositions

(a) According to Relation

There are several ways of classifying propositions. The most important of them is the classification of propositions into the *categorical* and the *conditional*. In a categorical proposition the predicate is either affirmed or denied of the subject without any condition. A categorical proposition is an unconditional statement. It is of the form 'S is P' or 'S is not P,' e.g., 'Socrates is a man,' 'The weather is not pleasant.' Conditional propositions are assertions under conditions. Of these there are two kinds, the *hypothetical* and the *disjunctive*. The hypothetical proposition is expressed in one of two forms: (a) If A is B, C is D; (b) If A is B, A is C. 'A is B' expresses the condition and is called the *antecedent*; 'C is D' or 'A is C' is the *consequent* or what follows from the condition; e.g., 'If wishes were horses, beggars would ride: If he is brave, he would defy unjust authority.' The disjunctive propositions can also be expressed in one of two forms: (a) Either A is B or C is D; (b) Either A is B or A is C; e.g., 'Either war should be banished or humanity will perish'; 'The signal lights are either red or green.' We shall confine ourselves for the present to a study of categorical propositions, postponing a consideration of conditional propositions to a later stage.

(b) According to Quality

We have seen that every proposition either affirms or denies, i.e., it states that a thing is such and such or is not such and such. This is called the *quality* of the proposition. In respect of quality, then, propositions are either *affirmative* or *negative*. In an affirmative proposition the predicate is asserted as belonging to the subject. In a negative proposition the predicate is denied of the subject. The affirmative proposition is of the form 'S is P'. The negative proposition is of the form 'S is not P'. In the assertion 'Roses are red,' the predicate 'red' is affirmed of the subject 'roses.' In the statement 'Man is not perfect,' the predicate 'perfect' is denied of the subject 'man.'

(c) According to Quantity

According to quantity, propositions are divided into *universal* and *particular*. The *quantity* of a proposition is deter-

mined by the extension or denotation of the subject term. When the predicate is affirmed or denied of the whole of the subject, the proposition is universal. Thus in the proposition 'All men are mortal,' the predicate 'mortal' is affirmed of the whole subject 'men,' and in the proposition 'No men are perfect,' the predicate 'perfect' is denied of the entire subject 'men'. When the predicate is affirmed or denied only of a part of the denotation of the subject, the proposition is particular. Thus in 'Some men are wise' wisdom is predicated affirmatively of a portion of the subject, and in 'Some men are not wise' it is predicated negatively of a portion of the subject 'men.'

There are some propositions with a singular term as the subject, *e.g.*, 'The highest peak in the world is Mount Everest,' 'Harischandra is the classic example of a votary of truth', 'The League of Nations is no more.' These are called singular propositions. Their subjects are indivisible and must ever be used in their whole extent. Hence, for the sake of convenience, these propositions are also called universal.

Another class of propositions called 'indefinite' or 'indesignate' has been recognised by some logicians. This class is considered to include propositions which do not give us any clue as to the quantity of the subject. We cannot say by looking at the form of those propositions whether they are universal or particular, *e.g.*, 'Man is frail,' 'Crime is punished'. Such propositions, given apart from their context, are difficult to be classified either as universal or as particular. In fact, they are not propositions at all, but sentences. In the words of Mellone, "Indefinite propositions have no place in logic, unless they are merely abbreviations, and their real quantity is obvious, as in the following: 'Triangles have three interior angles together equal to two right angles' or 'Men are rational.' etc." Thus some propositions may be indefinite in form but definite in meaning. While characterising a proposition as universal or particular, we must consider the meaning and not the form. If the predicate is an invariable and necessary attribute of the subject as in 'Glass is fragile' the proposition is universal. If on the other hand, the predicate is only an accident as in 'Glass is rough', the proposition is particular.

The words 'all' and 'some' are the only signs of quantity sanctioned in deductive logic. When other words signifying quantity are found in sentences, they must be converted into one of these words—'all' if the predicate speaks of the whole of the subject, and 'some' if it asserts only of a part of the subject. Thus, for example, 'most' and 'many' must be represented merely as 'some.' 'Most persons in India are poor,' expressed in logical form, becomes 'Some persons in India are poor.' In logic, it must be borne in mind, the word 'some' means any indefinite quantity, and not as in popular usage, a small fraction of the whole. Moreover, it does not exclude the possibility of 'all.' It signifies *some at least* and not *some only*.*

We have now seen that all propositions have quality and quantity. They are either affirmative or negative, and universal or particular. Combining these distinctions, we have four types of propositions. In order to symbolise these types the vowels A, E, I and O are employed. A and I are the first two vowels in the Latin word *affirmo* (I affirm) and they stand respectively for universal affirmative and particular affirmative propositions. E and O are the vowels in the Latin word *nego* (I deny) and they signify respectively universal negative and particular negative propositions. Thus a universal affirmative proposition is called A and is of the form 'All S is P' (SAP); a universal negative proposition is called E and is of the form 'No S is P' (SEP); a particular affirmative proposition is called I and is of the form 'Some S is P' (SIP); and a particular negative proposition is called O and is of the form 'Some S is not P' (SOP).

The E proposition (No S is P) requires some comment. If we look at the copula 'is,' it will seem as though the proposition is affirmative. But, in truth, it is not so. The subject of the E proposition is 'All S' and the copula is 'is not.' Yet the proposition is not given in the form 'All S is not P' because in the English language, as in many other languages, 'All S is not P' really means 'Some S is not P'. The meaning of the statement 'All is not gold that glitters' is that some things that glitter are not gold. To avoid the ambiguity in the form 'All S is not P', 'No S is P' is used to signify the uni-

* See Latta and Macbeath: **The Elements of Logic**, p. 56.

versal negative. Where, however the E proposition is a singular negative proposition, it is expressed not in the form 'No S is P' but in the form 'S is not P' or 'The S is not P' because in this case there is no possibility of ambiguity. The subject is used as indivisible and therefore 'Some S is not P' would be impossible. *e.g.*, 'The Ganges is not a South Indian river.'

(d) *According to Modality*

Propositions are also distinguished on the basis of modality. The modality of a proposition consists in the degree of certainty or probability with which the predicate is either affirmed or negated of the subject. There are three kinds of propositions under this head: *assertoric*, *problematic* and *apodeictic*. An assertoric proposition is that in which a simple unqualified statement is made: S is P; 'Crows are black.' A problematic proposition states a possible relation between the subject and the predicate: S may be P; 'Mankind may become absolutely non-violent.' An Apodeictic proposition expresses that the relation between the subject and the predicate is necessarily grounded in their nature: S must be P; 'An equilateral triangle must be equiangular.' These three varieties of propositions indicate the possibility, the actuality or the necessity, not of the subject or the predicate, but of the relation between the two. If we are merely aware that S and P are related without knowing its grounds, our judgment is assertoric. If we do not know the grounds of the relation and if we have not even observed the relationship, our judgment is problematic. If we know the grounds of the relation between S and P definitely, our judgment is apodeictic.

(e) *According to Import*

Propositions have been divided by some into *analytic* (or explicative) and *synthetic* (or ampliative). An analytic proposition is one in which the predicate merely states what is already implied in the subject, *e.g.*, 'A triangle is a three-sided figure.' A synthetic proposition is one in which the predicate adds something new to what is said in the subject and asserts what is not already implied therein, *e.g.*, 'The rose is red.' Kant makes the distinction thus: "There are two ways in which the predicate of an affirmative judgment may be related to the subject. Either the predicate B is

already tacitly contained in the subject A, or B lies entirely outside of A, although it is in some way connected with it. In the one case I call the judgment *analytic*, in the other case *synthetic*. Analytic judgments are those in which the predicate is related to the subject in the way of identity, while in synthetic judgments the predicate is not thought as identical with the subject."

This distinction, however, is not absolute and cannot be justified. It is one's degree of knowledge that determines whether the predicate adds or does not add something new to what is said in the subject. A proposition may be analytic to one and synthetic to another. That the interior angles of a triangle are together equal to two right angles is analytic knowledge to the geometrician but synthetic to the layman. Again, knowledge is a growth, and it progresses from less to more. Hence a proposition which is synthetic at one time may become analytic at another.

iv. Reduction of Sentences to Logical Form

We said that the beginner in logic must acquire the skill to reduce given sentences to logical form. We give below certain hints which will help him accomplish this task:

(1) A, E, I and O are the four types of categorical propositions recognised in logic. If sentences which are not in the strict logical form are given, they should be reduced to one of these four types. Poetical or rhetorical assertions are to be transformed into logical propositions with the least possible sacrifice of meaning.

(2) In reducing a sentence to its logical form, we must be guided by the meaning of the sentence. As Hamilton says, "Before dealing with a judgment or reasoning expressed in language, the import of its terms should be fully understood."

(3) We meet frequently with sentences which combine two or more propositions. Such compound propositions were called by the mediæval logicians *exponible*. When reducing them to logical form they must be split up into simple propositions. e.g., 'Gold and silver are precious metals' = 'Gold is a precious metal' and 'Silver is a precious metal.'

(4) There are some propositions which are called *exclusive*. In them the subject is limited by words like 'alone,' 'only,' 'none but,' 'none except,' 'none who is not,'

e.g., 'Graduates alone are eligible.' There are two ways of reducing such propositions to logical form: (a) By inverting the subject and the predicate of the given proposition, an A proposition may be formed. The example given above may be reduced to 'All those who are eligible are graduates.' (b) By taking the contradictory of the given subject as the subject while the predicate remains the same, an E proposition can be constructed. Thus the above proposition will take the form 'No non-graduates are eligible.'

(5) In what are called *exceptive* propositions the predicate is asserted of the whole subject with the exception of certain cases. The application of the predicate is cut off from a portion of the subject by a word like 'unless,' 'except,' 'but,' etc. If the exceptions are definitely known, such propositions are to be regarded as universal, *e.g.*, 'All metals except mercury are solid.' If the exception is indefinite, the proposition is to be treated as particular, *e.g.*, 'All metals except one are solid' = 'Some metals are solid.'

(6) In reducing a sentence to logical form, one should first discover the true subject. The real subject can be got at by asking the question 'What is being spoken about?'

(7) Then one must ask, what is stated about the subject? The answer to this will reveal the logical predicate, and show whether it is affirmed or denied of the subject.

(8) The copula must be clearly set forth, and, as we know, it must consist of the present tense of the verb *to be* with or without the negative particle *not*.

(9) The next task is to determine the quantity of the subject. This is done by getting the answer to the question: Is the predicate intended to apply to the whole of the subject, or does the proposition intend to commit itself to an assertion about 'some only' or 'some at least'? If the former, the proposition is universal; if the latter, it is particular.

(10) Words like 'all,' 'every,' 'each,' 'any,' when joined to the subject, signify an A proposition, *e.g.*, 'Every soldier fought valiantly' = 'All soldiers are persons who fought valiantly'; 'Each and every one of the students should study hard for the examination' = 'All students are those who should study hard for the examination'; 'Any house is a port in the storm' = 'All houses are ports in the storm.'

(11) Propositions with words like 'all,' 'every,' 'each,' 'any,' containing the sign of negation 'not' are generally to be regarded as particular negative, *e.g.*, 'All is not gold that glitters' = 'Some things that glitter are not gold'; 'Every disease is not fatal' = 'Some diseases are not fatal'; 'Any excuse will not suffice' = 'Some excuses are not those which will suffice.'

(12) Words like 'no,' 'none' added to the subject signify an E proposition.

(13) The absence of any sign of quantity usually signifies a universal proposition, *e.g.*, 'Blessed are the pure in heart' = 'All those who are pure in heart are blessed'; 'Murder will out' = 'All murders are discovered sooner or later.'

(14) Propositions with words such as 'most,' 'a few,' 'certain,' 'many,' 'almost all,' 'all but one,' 'several' are to be treated as particular, *e.g.*, 'Most of the legislators did not attend the meeting' = 'Some of the legislators are not those who attended the meeting'; 'A few students have prepared their lessons' = 'Some students are those who have prepared their lessons'; 'Certain interested persons are at the bottom of this mischief' = 'Some interested persons are those who are at the bottom of this mischief.'

(15) Propositions containing words like 'mostly,' 'generally,' 'frequently,' 'often,' 'perhaps,' 'nearly always,' 'sometimes' are to be regarded as particular, *e.g.*, 'White cats with blue eyes are generally deaf' = 'Some white cats with blue eyes are deaf'; 'Students sometimes indulge in cat-calls' = 'Some students are those who indulge in cat-calls.'

(16) The word 'few' means 'almost none' and its logical equivalent is 'some not.' Thus a sentence beginning with the word 'few' and not containing any sign of negation is to be treated as an O proposition; and if it contains a sign of negation, it should be regarded as an I proposition. since two negatives have the force of an affirmative, *e.g.*, 'Few books on logic are easy to read' = 'Some books on logic are not easy to read'; 'Few persons are not selfish' = 'Some persons are selfish.'

(17) Words like 'seldom,' 'hardly,' 'scarcely' have a negative meaning. When they occur in sentences not con-

taining any negative signs, those sentences should be reduced to O. But when they are found in sentences containing a sign of negation, the sentences should be reduced to I. *E.g.*, 'Unasked advice is seldom accepted' = 'Some pieces of unasked advice are not accepted'; 'Prosperous lawyers are not seldom honest' = 'Some prosperous lawyers are honest.'

(18) A proposition whose subject is a singular term is to be treated as universal, *e.g.*, 'Krishna is the teacher of the *Gita*,' 'The oldest Scripture in the world is the Veda.'

(19) Sometimes, interrogative sentences do not leave us in doubt about their answers. In such cases, they can be reduced to logical form, *e.g.*, 'Can the leopard change his spots?' = 'The essential nature of things is such as will not leave them.'

(20) It is not possible to give all the hints that are required for reducing sentences to logical form. The basic rule is that the student should transform sentences into logical propositions without altering their meaning.

v. Euler's Circles

The relation between the subject and the predicate of a categorical proposition is explained by formal logic in terms of extension. There exists a certain relation between the class of things denoted by the subject and that denoted by the predicate. This relation may be one of inclusion or of exclusion, *e.g.*, the proposition 'All men are mortal' signifies that 'men' are included in the class of 'mortal beings.' On the other hand, 'No man is a perfect being' means that the two classes 'men' and 'perfect beings' are mutually exclusive.

Euler represented the relation between S and P in categorical judgments diagrammatically by means of circles. Fig. 1 stands for the A proposition. In an A proposition the thing or class of things denoted by the subject is included in and forms part of the class denoted by the predicate. When we say 'All men are mortal beings' the class of men is meant to fall entirely within the class of mortal beings.

The bigger circle stands for the denotation of the predicate term 'mortal beings' and the smaller circle for the denotation of the subject term 'men.'

The proposition A does not generally assert anything about the whole of its predicate. In the proposition 'All men

are mortal beings' a reference is made only to those mortal beings who are identical with men. There are mortals other

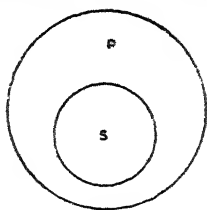


Fig. 1

than men, and about them nothing is said. Thus ordinarily the predicate of a universal affirmative proposition is understood to denote only a part of its extent. In the case of some A propositions however, the predicate term may not be wider than the subject term but equal to it in extent, *e.g.*, 'All equilateral triangles are equiangular.' If such a proposition were to be represented geometrically, then the circle signifying S must be made to coincide with that representing P. The S and P of any logical definition stand in this relation of equality. But unless the P is known to be of the same extent as S, it must be regarded as wider in its scope.

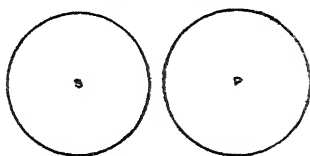


Fig. 2

In the E proposition the circles representing S and P fall outside each other (see Fig. 2). The proposition E states that the class denoted by the subject is entirely outside the class denoted by the predicate. When it is said 'No men are perfect beings,' it is meant that the class of men is completely outside the class of perfect beings. The circles never meet.

Fig. 3 represents the proposition I. The two circles intersect or overlap each other. When we assert 'Some men are wise' we mean that a portion of mankind is identical with

इति तद्रव्यविप्रकर्त्राद्यभावे वेदितव्यम् । अत एव जाबालिः—

जन्मनोऽनन्तरं कार्यं जातकर्म यथाविधि ।

दैवादतीतकालं चेदतीते सूतके भवेत् ॥

इति । श्राद्धमेतदामद्रव्येण कार्यम् । यदाह प्रचेताः—

स्त्री श्रूद्रस्वपचश्चैव जातकर्मणि वाऽप्यथ ।

आमश्राद्धं तथा कुर्याद्विधिना पार्वणेन तु ॥

इति । स्वपचस्वयंपच इत्यर्थः । सत्यव्रतोऽपि—

‘पुत्रजन्मनि नाभिकर्तनात्पूर्वं’ कृतजातकर्मा श्राद्धं^२ कुर्यात्’

इति । यत्पुनर्व्यासेनोक्तं—

द्रव्याभावे द्विजाभावे प्रवासे पुत्रजन्मनि ।

हेमश्राद्धं प्रकुर्वीत यस्य भार्या रजस्वला ॥

इति । तदामद्रव्याभावे द्रष्टव्यम् । अतोऽस्य हीनत्वाद्व्यमत्र भोज्यं विवक्ष्यते । द्विजः भोक्ता । आदिपुराणे तु जातश्राद्धे पक्वान्ननिषेधो दर्शितः—

जातश्राद्धे न दद्याद्वै पक्वान्नं ब्राह्मणेष्वपि ।

यस्माच्चान्द्रयणाल्लुद्धिस्तेषां भवति नान्यथा ॥

इति । कुमारजन्मदिनं प्रकृत्वाह हारीतः—

‘जाते कुमारे पितृणामामोदात्पुण्यं तदाहः’

are plane figures' the entire class of triangles is included in the class of plane figures. The subject of an A proposition is universal in quantity, and so it is distributed. The predicate, however, is undistributed because its entire extension is not considered, and only a part thereof is spoken about. The statement 'All triangles are plane figures' does not refer to all plane figures, but only to a portion of the class of plane figures which is identical with the class of triangles.

The universal negative proposition (E) distributes both subject and predicate, for in it the entire extent of the subject is excluded from the whole extension of the predicate, *e.g.*, the proposition 'No birds are mammals' excludes the entire subject 'birds' from the whole class of mammals.

In the particular affirmative proposition (I), such as 'some birds are web-footed,' the subject is not distributed, because the assertion is made only of a part of its extent, and the predicate too is not distributed because it is used in a limited extent. Some birds are not all the web-footed beings, but only some of them.

The particular negative proposition (O) excludes a portion of the extent of the subject from the entire extent of the predicate, *e.g.*, in 'Some men are not wicked,' a part of mankind is excluded from the whole class of wicked beings. Hence the subject of O is undistributed, while its predicate is distributed.

To sum up the results: universals (A and E) distribute their subjects; particulars (I and O) do not; negatives (E and O) distribute their predicates; affirmatives (A and I) do not.

Proposition A, subject distributed, predicate undistributed.

Proposition E, subject distributed, predicate distributed.

Proposition I, subject undistributed, predicate undistributed.

Proposition O, subject undistributed, predicate distributed.

These considerations may be summarised in the mnemonic *Asebinop*, which means A distributes subject only, E both, I neither, and O predicate only.

CHAPTER VII

THE IMPORT OF PROPOSITIONS

i. Introduction

While explaining Euler's circles we said that formal logic interprets the relation between subject and predicate in terms of extension. But this is by no means the only possible way of interpreting the import of propositions. Since there are two terms in a proposition and since each term has a double significance, denotative and connotative, there are four possible modes of understanding the import. Both subject and predicate may be read in extension. This is the denotative-denotative view, or, as it is otherwise called, class view. The subject may be taken in extension and the predicate in intension. This is known as the denotative-connotative view or predicative view. Both subject and predicate may be used in intension. This is connotative-connotative view or attributive view. Lastly, the subject may be understood in intension and the predicate in extension. This is connotative-denotative view. We shall now study these four views regarding the import of propositions.

ii. Class View

The class view, according to which both subject and predicate are read in extension, is very important and popular in formal logic. The diagrammatic representation of the categorical propositions by means of Euler's circles and the doctrine of the distribution of terms are founded on this view. It is called class view because it understands the subject and predicate terms as primarily referring to classes. The proposition 'All men are mortal' means that the class of men falls under or is included in the class of mortal beings. The proposition 'No dogs are bipeds' implies that the race of dogs falls outside or is excluded from the class of bipeds. Thus the relation between subject and predicate is regarded as one of inclusion or of exclusion.

In the usual types of categorical propositions A, E, I and O, the quantity mark is found only before the subject and not before the predicate. But on the class view the predicate also must be quantified. And so Sir William Hamilton proposed to supply the quantity mark to the predi-

cate. According to him, "We ought to take into account the *quantity*, always understood in thought, but usually, and for manifest reasons elided in its expression, not only of the *subject*, but also of the *predicate* of a judgment." Since the predicate, like the subject, may be either universal or particular, there should be four additional types of categorical propositions. Hamilton names them U, N, Y and W. The eight forms are:

A	All S is some P	U	All S is all P
E	No S is any P	N	No S is some P
I	Some S is some P	Y	Some S is all P
O	Some S is not any P	W	Some S is not some P

An examination of the four new forms suggested by Hamilton would show that they are quite unnecessary. The U proposition 'All S is all P' is a combination of two A propositions, 'All S is P' and 'All P is S.' The N proposition 'No S is some P' is the same as O 'Some P is not S'. The Y proposition 'Some S is all P' is in no way different from the A proposition 'All P is S'. The W proposition 'Some S is not some P' has no value. The meaning of it is 'Some (but not all) S is not some (but not all) P'. 'Some men are not some rational animals'. That is to say, a certain portion of the class 'men' is not equal to a certain portion of the class 'rational animals' but is equal to the remainder of the rational animals; and the remaining portion of the class 'men' is equal to the first mentioned portion of the class 'rational animals.' As Latta and Macbeath rightly observe, this is merely a round-about way of saying that all men are rational animals and all rational animals are men, which two statements when combined constitute the U proposition. Thus it is seen that the new forms suggested by Hamilton serve no purpose and that the quantification of the predicate leads us nowhere.

The class view which is responsible for the quantification of the predicate has been developed by some modern logicians into what is known as the equational theory of the proposition. Hamilton himself said that "a proposition is always an equation of its subject and predicate." If the predicate, like the subject, is a quantity, then the proposition signifies a comparison or identification of quantities, and the sign of equation may be substituted for the copula 'is.' The propo-

sitions may then be stated in the form $All\ S = Some\ P$, $All\ S = All\ P$, etc.

Let us understand the significance of this equation. Hamilton quantified the predicate, because he thought that an unquantified predicate is indefinite. 'All roses are flowers' may mean either that all the roses are the flowers or that all roses are some flowers. To preclude the possibility of drawing the first implication which is obviously wrong, the predicate is to be particularised and the proposition is to be stated in the form 'All roses are some flowers,' 'All roses = some flowers.' But even this is indefinite, since 'some' is not a definite quantity. Definiteness may be achieved by assessing the proportion between roses and flowers, we may modify the form and say ' $All\ S = 1/500\ P$.' But then we will be entering into the realm of mathematics leaving that of logic; and logic would become merely a branch of mathematics, interested in striking out algebraic equations, and not in the general laws of thought. To obviate this difficulty, Jevons suggested another way of making definite the relation between subject and predicate. He thought that instead of quantifying the predicate, we may qualify it. We saw that the proposition 'All roses are flowers' are to be stated, according to Hamilton, in the form 'All roses are some flowers.' But the difficulty, we found, was to determine *what* flowers are roses. The answer evidently is, rose flowers. And so, the predicate is to be qualified by the word 'rose' and the proposition should take the form 'All roses are rose-flowers.' ' $S = SP$.' Now, this form cannot be agreeable to a mathematical mind. Is P an addition to S or an analysis out of the content of S ? If it is an addition to S , there cannot be an equation because the predicate would be more than the subject. If it is an analysis out of S , then to avoid ambiguity P must be added to S in the subject side also. The form of the proposition would then be ' $SP = SP$ ' which is a meaningless tautology.

We have worked out the implications of the equational theory merely to show the absurdity of the class view of the proposition. If the proposition is regarded as expressing a relation between two classes, then we will inevitably be led into the untenable position of the equational theorist. As we:

have already shown, it is impossible to think of extension to the exclusion of intension. When we judge 'roses are red', we do not intend to bring roses under the class of red things, but think of the red colour of roses. The class view may be useful in many logical exercises, but it is not the natural way of interpreting most of our judgments.

iii. Predicative View

The theory which takes the subject in extension and the predicate in intension is known as the denotative-connotative view or predicative view. Those who uphold this theory maintain that the subject stands for a thing or group of things and the predicate for an attribute or set of attributes. The proposition either affirms or denies that the quality connoted by the predicate belongs to the things or class of things denoted by the subject. Aristotle seems to have favoured this view, and his classification of categorical propositions into A, E, I and O where the subject alone is quantified is based on it. This theory appears to be the natural one for interpreting most propositions, e.g., 'All men are mortal,' 'the weather is awful,' 'the moon is bright', etc.

iv. Attributive View

The attributive view advocated by J. S. Mill reads both subject and predicate in intension, and is also called connotative-connotative view. It regards the subject not as primarily referring to things but to attributes in virtue of possessing which a class of beings is known by that name. The proposition, then, is to be understood as showing a relation between two sets of attributes. It asserts "that the attributes which the predicate connotes are possessed by each and every individual possessing certain other attributes; that whatever has the attributes connoted by the subject has also those connoted by the predicate; that the latter set of attributes *constantly accompany* the former set. Whatever has the attributes of man has the attribute of mortality; mortality constantly accompanies the attributes of man."* Thus, according to the attributive view, the implication of a proposition is that certain attributes do or do not go with certain other attributes.

This theory fits in with Mill's general metaphysical posi-

* Mill: *System of Logic*, bk. I, ch. v, sec. 4.

tion which regards things as collocations of attributes or phenomena. But it is untrue to the facts of logic. The class view erred by doing away with differences and asserting an absolute identity or equation between subject and predicate. The attributive view is guilty of the opposite mistake of ignoring identity and stressing the difference. It says that there is no unity between subject and predicate; the attributes signified by the predicate just happen to accompany the attributes connected by the subject. But, for predication we require both unity and difference. And further, we do not think of the subject as signifying a quality, but as primarily a thing or class of things of which a quality connoted by the predicate is asserted.

v. Connotative-Denotative View

This theory which reads the subject in intension and the predicate in extension is so unnatural that no one has sponsored it so far. Such judgments like 'Nothing gaseous is a metal' may seem to justify this theory. But even in such a case the judgment may ordinarily take the form 'No metal is gaseous.' The defect of this, as of the previous view, is that it interprets the subjects to mean a quality.

vi. Conclusion

Of the four views we have considered, the second, *viz.*, the predicative theory, seems to be the most natural one, since it interprets the judgment as expressing a unity in difference. But even this theory would have to be declared unsatisfactory, if we mean by it that the subject is to be taken purely in extension and the predicate purely in intension. As we had occasion to observe in an earlier chapter, extension and intension are inseparable aspects of the significance of terms. It is impossible to have the one without the other. In so far as the four views emphasise the one *or* the other of the two aspects of terms, they are all defective.

A judgment (or proposition), as we stated above, expresses identity in difference. When we say 'Man is mortal,' we mean that mortality as well as other human attributes belong to the individuals of the class 'man.' Mortality is different from the other human attributes like rationality, etc. That is, there is a difference in connotation. And yet, there is an identity in denotation, since all these attributes refer to

the same individual or individuals. Even a negative judgment is no exception to the rule that judgment expresses a unity in difference. But only in the case of a negative judgment there is an underlying identity of connotation in spite of a difference in denotation. A negation, to be logical, must be significant. There is no meaning, for instance, in the denial 'Virtue is not square.' The proposition 'Red is not blue,' on the other hand, is significant because we find here both identity and difference. 'Red' and 'blue' belong to the universe of colour, and hence there is identity of connotation. But their denotations are different since red things cannot at the same time be blue. This view of judgment is satisfactory precisely because it neglects neither of the two aspects of the significance of terms, *viz.*, denotation and connotation.

CHAPTER VIII

THE LAWS OF THOUGHT

i. Introduction

We have so far studied the nature of the elements of reasoning, *viz.*, terms and propositions. And before proceeding to understand the characteristics of reasoning itself, we shall have to grasp the implications of the grounds on which reasoning is based. These grounds are usually expressed in what are known as the Laws of Thought—principles of reasoning which govern all knowledge.

The term 'law' is rather ambiguous as it is used in different contexts. (i) There are, for instance, Laws of Nature which it is the endeavour of science to discover. (ii) There are civil or positive laws enforced by the State on its subjects. (iii) And there are laws which are more subtle and operate in the realm of morals. It will be useful to learn in what sense the term 'law' is employed in each of these contexts and to compare these different kinds of law with the laws of thought.

(i) The Laws of Nature are *descriptive* in character. They state *how* changes in Nature occur, how phenomena are caused. The law of gravitation, *e.g.*, does not order stones and apples to fall, but only describes how and why they fall. Science is interested in the discovery of such laws—laws which are but the principle that govern natural phenomena. The

the seventeenth century, added a fourth and he called it the Principle of Sufficient Reason. These Laws of Thought cannot be proved, since they form the ground of all proof. They are self-evident and axiomatic. Without them there can be no reasoning. They are postulates which are pre-supposed in all thinking.

ii. The Law of Identity

The old formula in which this law is generally expressed is 'Whatever is, is' or 'A is A.' This is, however, misleading, and has led to a gross misrepresentation of the meaning of thought and reality. The formula 'A is A' may be taken to mean that A is A and no more. If this be the case, then it is 'aggressively untrue,' as Professor Bosanquet says, 'for it denies the synthesis of differences which alone can make a judgment.' Bare identity is significant neither of thought nor of reality. To say 'A is A' 'Socrates is Socrates' is tautologous and meaningless. What a judgment aims to convey is that in and through differences there is a pervading unity or identity characterising the logical subject. The Law of Identity is not a denial of difference. It asserts rather that there is identity in difference. As the German philosopher Hegel urges in his *Logic*, "The main thing in connection with thought, is not to confuse the true identity with an abstract identity, identity of barren form." True identity does not exclude difference. Socrates was not the same all through. 'Socrates changes, or is different from day to day and from year to year. But he also remains identical with himself; he is in his old age the same Socrates who talked with Parmenides in his youth and fought at Potidaea in middle life.'* Concrete identity means identity in difference. Judgment expresses the unity that persists in diversity. A is B and yet does not cease to be A. The law thus means that, "in spite of or in virtue of the differences expressed in a judgment, the content of judgment is a real identity, that is to say, has a pervading unity. It says that there is such a thing as identity in difference."† It states that Reality is a system of inter-related elements, that there is unity of different properties or parts in the whole.

* Creighton and Smart: *An Introductory Logic*, p. 398.

† Bernard of Bosanquet: *Logic*, vol. ii, p. 210.

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iii. The Law of Non-contradiction

The familiar name by which this is known is 'the Law of Contradiction.' But the name is misleading, because, the purpose of the law is not to enjoin but to warn against contradiction.

The usual form in which the law is stated is: it is not possible for the same thing both to be A and not to be A, or, A is not not-A. It is evident that this law expresses in a negative way what is affirmed by the Law of Identity and is complementary to it. Just as we found a danger in interpreting the Law of Identity, there is a similar danger here also. The formula 'A is not not-A' may be understood as a bare negation. Then, the law would be either meaningless or 'aggressively false'—meaningless because it would be a tautology to say 'A is not not-A,' if 'not not-A' means A, and false because it would not square with reality to say that A is not other than *mere* A.

Aristotle states the principle in a form which is at once true and acceptable. He says, "It is impossible that the same predicate can both belong and not belong to the same subject at the same time and in the same sense." The qualifications 'to the same subject,' 'at the same time' and 'in the same sense' are essential and should not be overlooked. It is not a contradiction to say 'this paper is white,' and 'that paper is not white,' because the two judgments do not have the *same subject*. Nor is it a contradiction to say 'Arjuna knows archery' and 'Arjuna does not know archery,' if the two judgments refer to the state of Arjuna at different times, say, youth and infancy. Knowledge of archery and non-knowledge of archery would become contradictories only if they are predicated of Arjuna at the *same time*. Similarly, one may say without contradiction that a man is both fast and not fast—fast with reference to another man who walks slow and not fast with reference to a third man who runs. In this case the word 'fast' is not used in the *same sense*, and so there is no contradiction.

The obvious truth of the Law of Non-contradiction is this, *viz.*, that a statement and its denial cannot both be true. It is absurd to say 'This paper is white and not-white' at the same time, but not so to assert 'This paper is white and

smooth.' 'White' and 'not white' are contradictories, and they cannot be predicated of an identical thing, *viz.*, 'this paper.' But 'white' and 'smooth' are differences, and they may logically co-exist in a subject. Thus the Law of Non-contradiction "does not forbid the union of *differences* in one judgment, but of *contradictories*, or of what would destroy the integrity of the judgment and render it unmeaning."

iv. The Law of Excluded Middle

The symbolic form of this law is 'A is either B or not B. This is only the disjunctive way of stating the same principle as appears in the Laws of Identity and Non-contradiction. It asserts that between contradictories there is no middle ground. In the words of Aristotle, 'Between the assertions of a logical contradiction there is no middle.' Of two contradictories one or the other must be true. A given person is either Socrates or not-Socrates. If an orange is not sweet, it must be not-sweet. As Jevons says, "The very name of the law expresses the fact that there is no third or middle ground; the answer must be Yes or No."

Here, again, the formula 'A is either B or not-B' must be interpreted with care. It means that if a subject belongs to a universe of discourse, and if the universe of discourse admits of only two possibilities, the denial of one implies the affirmation of the other. We cannot say, for instance, that ghosts are either white or not-white, for we have no method by means of which we can ascertain that ghosts belong to the universe of colour. We can say, on the other hand, that if the signal light is red it is not green, or if it is green, it is not red, for colour is relevant to light and the system of signalling consists only of two alternatives, red and green. Thus the Law of Excluded Middle also stands witness to the systematic nature of Reality. It affirms that Reality is 'a system of reciprocally determinate parts.' Only then is it possible for every negation to be significant. Bare affirmation and bare denial have no place in the thought system. The 'not-B' in the formula 'A is either B or not-B' has a positive implication, say, 'C.'

v. The Law of Sufficient Reason

The law is usually stated thus: everything must have a

sufficient reason why it is so and not otherwise. Leibniz who formulated this principle said that there must always be some sufficient reason why a thing is what it is. If Reality is a system of inter-connected parts, then each part must depend on some other and ultimately on the whole. This is the truth given expression to by the principle of sufficient reason. A specific form of this principle is known as the law of causation which regards every event as the effect of a cause. This we shall study in greater detail when we come to the inductive postulates. But what concerns us here is another aspect of the principle of Sufficient Reason. And that is, that every proposition or judgment which is held to be true has its own sufficient grounds; it presupposes premises or grounds and therefore is an element in a reasoning. Thought is a system, and not a mechanical mosaic of isolated judgments. Each judgment is an implicit inference with its grounds unexpressed. If it is to be true, it must be capable of being shown as the conclusion from certain premises.

All these Laws of Thought are but different ways of representing the demand of intelligence for understanding Reality. They express the one great principle that Reality is systematic, coherent and intelligible. Consistency and necessity characterise the nature of Reality. The Law of Identity says that in spite of differences a thing remains identical with itself. The Law of Non-contradiction states the same truth in a negative way, *viz.*, that a thing cannot be both itself and not-itself. These two laws give expression to the fact that Reality is consistent or non-contradictory. The Law of Excluded Middle represents the Real as a system of inter-connected parts governed by necessity. The Law of Sufficient Reason too tells us that the whole is the necessary ground on which all its parts depend. These laws are principles of our rational experience, principles without which thought cannot function.

CHAPTER IX

THE OPPOSITION OF PROPOSITIONS

i. Introduction

Categorical propositions, we have seen, are of four kinds, A, E, I and O. The classification is based on considerations

of difference in quantity and in quality. The difference between any two of these four types, the subject and predicate being the same, is technically called 'opposition.' The ordinary meaning of opposition, however, is not this. By opposition we mean incompatibility or repugnance. But in the technical sense in which we use it here, 'opposition' includes also cases of propositions where there is no real conflict. Propositions are to be regarded as 'opposed' when, having the same terms as subject and predicate, they differ in quantity, or in quality, or in both; *e.g.*, the propositions 'All men are wise' and 'No men are wise' are opposed to each other, because they differ in quality, while having the same subject and predicate.

Opposition is of four kinds. If the two opposing propositions differ both in quantity and in quality, the opposition is called *contradiction*. If the difference is only in quality and if the two propositions are universal, the opposition is called *contrariety*. Again, differing only in quality, if the two propositions are particular in quantity, the opposition is known as *subcontrariety*. Lastly, if the two propositions agree in quality but differ in quantity, the opposition—which really is no opposition—is termed *subalternation*.

ii. Contradiction.

Contradiction is complete disparity. Two propositions which differ both in quantity and in quality are in contradictory opposition. A and O, and E and I are the two pairs of contradictories. 'All men are wise' is the contradictory of 'Some men are not wise'; 'No men are infallible' is the contradictory of 'Some men are infallible.' The relation between the contradictories is governed by the Law of Excluded Middle. Of contradictory opposites, one must be true and the other false; they cannot both be true, nor can they both be false. If A is rejected, O must be accepted and *vice versa*. Similarly of E and I. Of the two propositions 'All adults have a vote' and 'Some adults have no vote' one must be true and the other false. So also, of the contradictories 'No tyrants are just' and 'Some tyrants are just,' one must be true and the other false.

iii. Contrariety

If two propositions differ in quality and if they are both

universal, there is contrary opposition between them. A and E are contraries. 'All men are perfect' and 'No men are perfect' are contrarily opposed. Contrary propositions cannot both be true, but may both be false. If one is true, the other must be false; but if one is false, the other is doubtful. Of the two contraries, both cannot be accepted, but both may be rejected.

iv. Subcontrariety

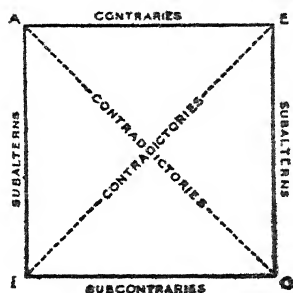
Subcontrary opposition obtains between the two particular propositions I and O which differ in quality alone. They may both be true, but cannot both be false. If 'Some men are virtuous' is true, 'Some men are not virtuous' may or may not be true. But if the former is false, the latter must be true. Of the subcontraries, both may be accepted, but both cannot be rejected.

v. Subalternation

Two propositions which differ in quantity only are in subaltern relation. The universal and the corresponding particular are related by way of subalternation. A and I, and E and O are the two pairs of subalterns. The universal is called the subalternant and the particular subalternate. A is the subalternant of I and I the subalternate of A; E is the subalternant of O and O the subalternate of E. In a subaltern relation, if the universal is true, the particular must be true; but if it is false, the particular may or may not be false. If the particular is false, the universal must be false; but if it is true, the universal may or may not be true. If 'All men are mortal,' then it follows that 'Some men are mortal.' But from the truth of the statement 'Some men are wicked' we cannot say anything about 'All men.' If the universal is accepted, the particular must be accepted; if the universal is rejected, the particular may or may not be rejected. If the particular is rejected, the universal must be rejected; if the particular is accepted, the universal may or may not be accepted. If A is true, I is true; if A is false, I is doubtful. If I is false, A is false; if I is true, A is doubtful. Similarly of E and O.

vi. The Square

The four types of relation we have explained above are usually represented in a diagram which is called the *square of opposition*.



- (1) Contradictories: A and O; E and I.
- (2) Contraries: A and E.
- (3) Subcontraries: I and O.
- (4) Subalterns: A and I; E and O.

The results we have obtained may be summed up in the following table:—

	A	E	I	O
If A is true ..	true	false	true	false
If A is false ..	false	doubtful	doubtful	true
If E is true ..	false	true	false	true
If E is false ..	doubtful	false	true	doubtful
If I is true ..	doubtful	false	true	doubtful
If I is false ..	false	true	false	true
If O is true ..	false	doubtful	doubtful	true
If O is false ..	true	false	true	false

vii. Singular propositions

A note must be added on the place of singular propositions in the scheme of opposition. The singular proposition is classed as universal for purposes of formal logic, since its subject which is a singular term can be used only in its entire extent. The proposition 'Sivaji was a heroic ruler' has no corresponding particular form. It is an A proposition and can be opposed only to E. But the contrary opposition here between A and E is indistinguishable from contradictory opposition. Of the two statements 'Sivaji was a heroic ruler'

and 'Sivaji was not a heroic ruler,' one must be true and the other false. It is therefore said that in the case of the singular proposition the contradictory and the contrary coincide.

CHAPTER X

IMMEDIATE INFERENCE

i. Introduction

There are conflicting theories regarding the meaning of inference. But formal inference, with which alone we are concerned here, may be defined as the intellectual process by which we derive a proposition from one or more given propositions. It is "a process of thought which, starting with one or more judgments, ends in another judgment made necessary by the former".* Inference, in short, is *reasoning* from what is given to something else that is not given. If what is given consists of one proposition and if a conclusion is drawn therefrom, the inference is called *immediate*. If the given consists of two or more propositions which lead to a conclusion, the inference is called *mediate*. If I say, 'P is S because S is P,' it is a case of immediate inference. What I mean is that 'P is S' is implied in the statement 'S is P'. If I argue, 'S is P because M is P and S is M,' it is a case of mediate inference. Here the relation between S and P is known through their relation to a third term, *viz.*, 'M' which serves as the mediating link. When we pass directly from the proposition 'No men are infallible' to the statement 'No infallible beings are men', we have an instance of immediate inference. When it is argued 'All men are mortal; Socrates is a man; therefore, Socrates is mortal', we have an instance of mediate inference. Immediate inference will form the subject of study in this chapter, and mediate inference in the next and succeeding ones.

In the last chapter we saw that when two propositions are logically opposed to each other, we can, in some cases, infer the truth or falsity of one of them from the falsity or truth of the other. From the truth of the statement 'No birds are mammals' we can infer the falsity of the assertion 'Some birds are mammals'. The process by means of which this is done has been described by some logicians as a form of imme-

* Joseph: *Introduction to Logic*, p. 209.

mediate inference. But it is usual to apply the name of immediate inference to those reasonings in which we infer the truth of one proposition from the truth of another. These processes are also called *eductions*, since they seek to draw out or explicate the meaning or implication of propositions. Whether they can legitimately be named processes of immediate inference is a question to which we shall return at a later stage.

The processes of eduction or immediate inference are based on the 'class' view of the import of propositions. That is, the subject and predicate of a proposition are to be taken in their extent and the relation between the two is to be regarded as one of inclusion or exclusion. According to the 'class' view, 'S is P' means S is included in P as a part of it. This implies, then, that a part of P is identical with S. The denotative interpretation of 'S is not P' is that S is excluded from P. From this it can be inferred that P is excluded from S.

There are two main kinds of immediate inference, *viz.*, conversion and obversion. The other educts (*i.e.*, the results of eduction) are got by the repeated alternate application of these two methods.

ii. Conversion

In the inferential process known as *conversion* the subject and predicate terms change their places. We are said to convert when from a given proposition we infer another having the subject of the original proposition for its predicate and the predicate of the original proposition for its subject. From a proposition of the form S-P we infer an equivalent one of the form PS. The given proposition is called the *convertend* and the inferred proposition is called the *converse*. In conversion the quality of the proposition is not changed. Only the terms S and P are transposed. But in so transposing we should take care to see that no term which is undistributed in the original is distributed in the converse, for it is illogical to infer the more from the less. It is no fallacy, however, when a term which is distributed in the convertend is not distributed in the converse, for it is quite legitimate to infer the less from the more.

The rules of conversion are:—

- (1) *The quality (affirmative or negative) of the original proposition is unchanged in the converse.*

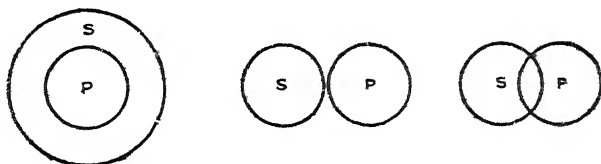
- (2) *No term must be distributed in the converse which is not known to be distributed in the convertend.*

The meaning of the first rule is evident. If the quality is changed, the meaning of the proposition will be changed, and the converse will be no longer an inference from the convertend. The need for the second rule also will be clear if we try, for example, the conversion of the proposition 'All apes are animals', without paying heed to the rule, into 'All animals are apes'. The converse, on the face of it, is absurd because apes are not the only animals; and the absurdity is due to the violation of the second rule. 'Animals' which is undistributed in the convertend is wrongly distributed in the converse.

The conversion is *simple* when the quantity of the converse is the same as that of the convertend. E and I are to be converted thus. The converse of 'No S is P' is 'No P is S', and of 'Some S is P' is 'Some P is S.' But this method of simply changing the places of S and P and leaving the quantity undisturbed is not possible in the case of the A proposition. The conversion of A is by limitation or *per accidens*, because its converse is I, and the two differ in quantity. The converse of 'All S is P' is 'Some P is S.' P in the convertend is undistributed as the predicate of an affirmative proposition and it should not be distributed in the converse where it is the subject. This is possible only if the converse is a particular proposition, as particular propositions alone do not distribute their subject. S in the convertend A proposition is distributed, but in the converse it is undistributed. But this is no fallacy because, as we saw above, a term which is distributed in the original may be undistributed in the converse. The O proposition is not amenable to conversion because its subject which is undistributed would become distributed in the converse as the predicate of a negative proposition; and this would be a transgressing of the second rule. From 'Some Indians are not Andhras' we cannot logically infer 'Some Andhras are not Indians.' As Minto points out, "Some S is not P does not enable you to make any converse assertion about P. All P may be S, or No P may be S, or some P may be not S."* And he gives three diagrams illustrative of the three propositions to show how they are all compatible with

* William Minto : *Logic, Inductive and Deductive*, p. 152.

“Some S being excluded from P” which is the meaning of the O proposition.



We may now apply the rules of conversion to the four forms of categorical propositions A, E, I and O and educe their respective converses.

- A. *Convertend*: All S is P; All men are mortal.
Converse: Some P is S; Some mortals are men.
 - E. *Convertend*: No S is P; No men are perfect.
Converse: No P is S; No perfect beings are men.
 - I. *Convertend*: Some S is P; Some men are wise.
Converse: Some P is S; Some wise beings are men.
- O has no converse.

We stand above that the A proposition should be converted by limitation. The converse of ‘All S is P’ is not ‘All P is S’ but only ‘Some P is S’. There is, however, an exception to this rule. In the case of formal definitions simple conversion may be adopted; e.g., the converse of ‘All equilateral triangles are equiangular’ is ‘All equiangular triangles are equilateral.’ This, of course, is obtained not from the form of the original proposition but from our knowledge of Geometry. The strict *form* of the A proposition justifies a conversion only to I.

Singular propositions are, as we know, to be regarded as universals. If in an affirmative proposition both the terms are singular, the conversion may be made without limitation. Thus ‘Mount Everest is the highest peak in the world’ is converted into ‘The highest peak in the world is Mount Everest.’ If in an affirmative proposition the subject term is singular and the predicate term is general, the conversion should be made by limitation. Thus the converse of ‘Nalanda is an ancient university of India’ is ‘Some one of the ancient universities of India is Nalanda.’

iii. Obversion

In the form of immediate inference known as *obversion* there is no transposition of terms, but the quality of the proposition is changed and the predicate is replaced by its contradictory. By means of this process we are able to pass from an affirmative proposition to a negative statement of the same truth, and *vice versa*. From a proposition of the form 'S-P' we infer a proposition of the form 'S not-P.' This process is based on the principle that every proposition can be expressed either affirmatively or negatively. It is all the same whether we say 'All men are fallible' or 'No men are infallible.' These two are only different ways of expressing the same truth, *viz.*, the fallibility of mankind. It is evident that the process of obversion is governed by the Law of Excluded Middle. If S is P, then it can be inferred that S is not not-P. The original proposition is called the *obvertend* and the inferred proposition is called the *obverse*. The rule of obversion is: *change the quality of the proposition, keep the subject as it is, and substitute for the predicate its logical contradictory.*

Applying this rule to the four types of categorical propositions, we get the following results:

- A. *Obvertend*: All S is P; All men are fallible.
Obverse: No S is not-P; No men are non-fallible.
- E. *Obvertend*: No S is P; No men are perfect.
Obverse: All S is not-P; All men are non-perfect.
- I. *Obvertend*: Some S is P; Some men are virtuous.
Obverse: Some S is not not-P; Some men are not non-virtuous.
- O. *Obvertend*: Some S is not P; Some men are not wise.
Obverse: Some S is not-P; Some men are non-wise.

The logical contradictory of a term is obtained by prefixing 'not-' or 'non-' to the term. But we may use instead such phrases as 'other than.' The obverse of 'Some mistakes are not proofs of ignorance' may be given as 'Some mistakes are other than proofs of ignorance'. The use of terms of the form 'non-P' may be avoided as far as possible by means of such prefixes as 'in-', 'un-' etc. Instead of obverting 'All men are fallible' into 'No men are non-fallible,' we may obvert it into 'No men are infallible.' But these prefixes sometimes indicate contrary, not contradictory terms; *e.g.*, 'unhappy' is the

contrary of 'happy.' In such cases the contradictory must be formed by prefixing 'non-' or 'not,' or an equivalent phrase such as 'other than' should be used. And it should also be remembered that just as not-P is the contradictory of P, P is the contradictory of not-P. If the obvertend is of the form 'S is not-P,' the obverse will be of the form 'S is not P.'

iv. Other Educts

The processes of immediate inference consist in drawing from given propositions other equivalent propositions differing in form. By conversion we obtain from a proposition of the form S-P another proposition of the form P-S; and by obversion we get a proposition of the form S-not-P. These are not all the possible forms. There are five others, *viz.*, (1) P not-S, (2) not-P S, (3) not-P not-S, (4) not-S P; and (5) not-S not-P. These educts are got by applying alternately the processes of conversion and obversion to the given proposition of the form S P. The first of these five 'P not-S' is called *obverted converse*; the second 'not-P S' is *partial contrapositive*; the third 'not-P not-S' is *full contrapositive*; the fourth 'not-S P' is *partial inverse*; and the fifth 'not-S not-P' is *full inverse*.

(1) *Obverted Converse*. By first converting the given proposition S P and then obverting the converse we get the obverted converse P not-S.

A. *Original proposition*: All S is P.

Converse: Some P is S.

Obverted converse: Some P is not not-S.

E. *Original proposition*: No S is P.

Converse: No P is S.

Obverted converse: All P is not-S.

I. *Original proposition*: Some S is P.

Converse: Some P is S.

Obverted converse: Some P is not not-S.

O. There is no converse: hence no obverted converse.

(2) and (3) *Contrapositive, partial and full*. Contraposition is a process by which from a given proposition we infer another proposition having the contradictory of the original predicate (not-P) for its subject. When the predicate of the contrapositive is the original subject (S), the contrapositive is *partial* (not-P S); and when its predicate is the

contradictory of the original subject (not-S), the contrapositive is *full* (not-P not-S).

The partial contrapositive is got by first obverting the given proposition and then converting its obverse.

A. *Original proposition*: All S is P.

Obverse: No S is not-P

Partial contrapositive: No not-P is S.

E. *Original proposition*: No S is P.

Obverse: All S is not-P.

Partial contrapositive: Some not-P is S.

I. *Original proposition*: Some S is P.

Obverse: Some S is not not-P.

There is no partial contrapositive, since the obverse of I which is an O proposition cannot be converted.

O. *Original proposition*: Some S is not P.

Obverse: Some S is not-P.

Partial contrapositive: Some not-P is S.

The full contrapositive is got by obverting the partial contrapositive of the given proposition.

A. *Original proposition*: All S is P.

Partial contrapositive: No not-P is S.

Full contrapositive: All not-P is not-S.

E. *Original proposition*: No S is P.

Partial contrapositive: Some not-P is S.

Full contrapositive: Some not-P is not not-S.

I. There is no partial contrapositive; and so no full contrapositive either.

O. *Original proposition*: Some S is not P.

Partial contrapositive: Some not-P is S.

Full contrapositive: Some not-P is not not-S.

(4) and (5) *Inverse, partial and full*. Inversion is the name given to the process by which from a given proposition we infer another proposition having for its subject the contradictory of the original subject (not-S). The inverse is *partial* when its subject is the contradictory of the original subject (not-S) and its predicate is the same as the original predicate (P). It is *full* when its predicate also is the contradictory of the original predicate (not-P).

By alternate process of obversion and conversion we get from the given proposition (S-P) the partial inverse (not-S P)

and full inverse (not-S not-P). The particular propositions I and O have no inverses because in the process of applying obversion and conversion alternately to them we get at a particular negative in each case which cannot be converted. It is only the universal propositions A and E that yield inverses. In the case of A we have to begin with obversion; and in the case of E, with conversion.

- A. All S is P (*original proposition*)
 No S is not-P (*obversion*)
 No not-P is S (*conversion*)
 All not-P is not-S (*obversion*)
 Some not-S is not-P (*full inverse*)
 Some not-S is not P (*partial inverse*)
- E. No S is P (*original proposition*)
 No P is S (*conversion*)
 All P is not-S (*obversion*)
 Some not-S is P (*partial inverse*)
 Some not-S is not-P (*full inverse*).

v. Eduction Schemes

All the forms of immediate inference have been considered above. They may be presented schematically in the following diagrams where the kind of proposition is represented by the small letters a, e, i or o, and the contradictory of a term by adding a dash to the letter signifying the term (S and P).

S a p

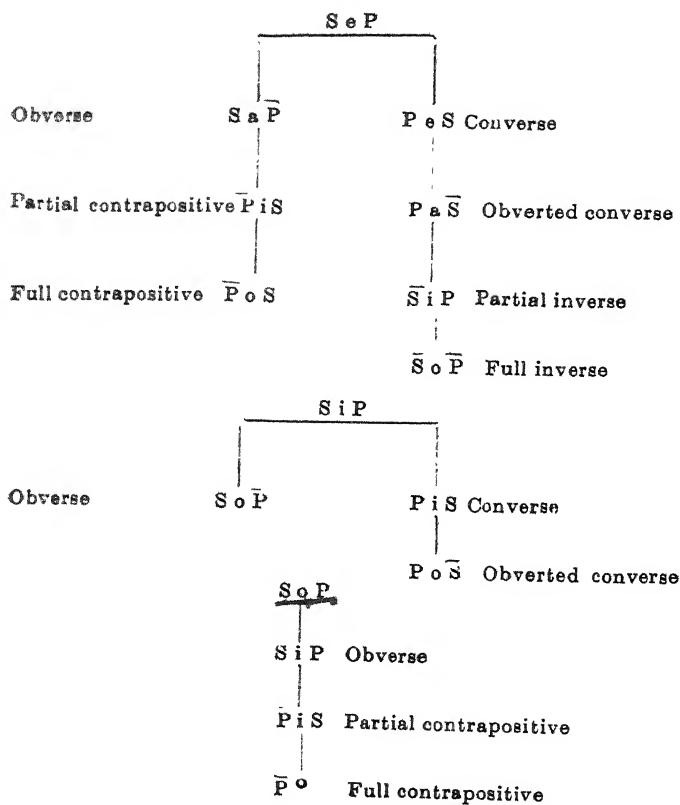
Obverse S e \bar{P} P i S Converse

Partial contrapositive P e S P o \bar{S} Obverted converse

Full contrapositive \bar{P} a \bar{S}

Full inverse \bar{S} i \bar{P}

Partial inverse \bar{S} o P



The results may be summarised in the following table. Here again \bar{S} and \bar{P} are employed to denote not-S and not-P respectively.

			A	I	E	O
1	Original proposition	...	SaP	SiP	SeP	So
2	Obverse	...	Se \bar{P}	So \bar{P}	Sa \bar{P}	Si \bar{P}
3	Converse	...	PiS	PiS	PeS	
4	Obverted converse	...	Po \bar{S}	PoS	Pa \bar{S}	
5	Partial contrapositive	...	\bar{P} eS		\bar{P} iS	PiS
6	Full contrapositive	...	\bar{P} a \bar{S}		PoS	\bar{P} o \bar{S}
7	Partial inverse	...	\bar{S} oP		SiP	
8	Full inverse	...	Si \bar{P}		\bar{S} o \bar{P}	

vi. Other Kinds of Immediate Inference

The forms of immediate inference we have explained so far in this chapter are based on the relation of inclusion and exclusion. There are other forms which are dependent on more complex kinds of relations. Jevons describes two of them, 'immediate inference by added determinants' and 'immediate inference by complex conception.' The former of these is obtained by adding an adjective or other determinant to the subject of the given proposition and adding the same determinant to the predicate as well; *e.g.*, 'All Todas are fellow creatures', therefore 'all Todas in suffering are fellow creatures in suffering' or 'A comet is a material body,' therefore 'a visible comet is a visible material body.' This method is based on the principle that if an individual or a species by itself has a certain characteristic, the same characteristic will inhere in it as a member of a wider class, and that since the subject and predicate of a proposition are equal (equational view), the addition of a qualification to the subject will necessitate the addition of the same qualification to the predicate. But this is really not so. A qualification

added to a term has not the same value when it is added to another term. And a future which characterises a thing or species taken by itself may not characterise it when viewed as a member of a wider class. For instance, an artist is a man; but it does not follow that a good artist is a good man. A tortoise may be fast in comparison with other tortoises; but it is not therefore fast as an animal.

In immediate inference by complex conception both subject and predicate become modifications of a third term; by the addition of the same words conceptions which are more complex than the original subject and predicate are formed; e.g., 'A horse is a quadruped,' therefore 'the head of a horse is the head of a quadruped' or 'Physics is a science,' therefore 'physical treatises are scientific treatises.' The defects which attend on immediate inference by added determinants are present here also. For instance, from 'All Vaishnavas are Hindus' we cannot infer that 'a majority of Vaishnavas are a majority of Hindus.' 'A cow is not a horse'; but from this we cannot argue that 'the owner of a cow is not the owner of a horse.'

Keynes gives an account of a more important form of immediate inference. He calls it 'immediate inference by converse relation.' This is a process by which from a statement of the relation in which P stands to Q we pass to a statement of the relation in which Q consequently stands to P. This implies that the two terms are reciprocally related and are members of or in a system like the family, time, space, etc. Thus from 'P is the grand-parent of Q' we infer immediately that 'Q is the grand-child of P.' From 'A is a day before B' we draw the inference 'B is a day after A.' From 'M is wider than N' we infer that 'N is narrower than M.' From 'X is to the left of Y' we conclude 'Y is to the right of X.' In this kind of inference the two terms of the original proposition are transposed and the word by which their relation is expressed is replaced by its correlative. The inference to be valid must be based on a knowledge of the relations governing the terms concerned. In the absence of such knowledge, Formal Logic cannot help us in deciding whether a given inference is valid or not.

vii. Is Eduction Immediate Inference?

Opinion is divided on the question whether the processes we have studied in this chapter may properly be called modes of inference. Some logicians like Mill and Bain who emphasise the element of novelty as a characteristic of inference maintain that the so-called kinds of immediate inference are not inferential processes. Mill calls them 'inferences improperly so-called' and goes on to say, "In these cases, there is not really an inference, there is in the conclusion no new truth—nothing but what was already asserted in the premise . . . The fact, asserted in the conclusion, is either the very same fact or part of the fact, asserted in the original proposition." Bain declares, "In none of these cases is there inference properly so called, that is to say, the transition from a fact to some different fact. There is merely *the transition from one wording to another wording of the same fact.*"

Logicians like Bosanquet, however, do not insist on novelty. Novelty, according to them, is an accidental feature of inference. What is essential for an inference is that it should be necessary. Necessity is the characteristic of system; and system is the basis of inference. When we pass in inference from one fact to another, we do so because the two are known to be necessarily related as parts of a system. Necessity then is the heart of inference. If this be so, the processes considered above may be rightly called inferences.

Mill and Bain are wrong in saying that immediate inferences are mere verbal changes and that they simply express the original proposition in different words. The valuable service rendered by these processes is to bring out clearly the implications of a given proposition by making explicit what is implicitly contained therein. And this is a task pertinent to inference. Even if the eductions are not regarded as inferences, they are not useless. A Creighton and Smart observe, "Whether or not they may properly be called *inferences*, they render important service by helping us to understand all that is really implied, both in the way of affirmation and denial, in the propositions we use."*

* **An Introductory Logic**, p. 119.

CHAPTER XI

THE CATEGORICAL SYLLOGISM

i. Introduction

In the last chapter we referred to mediate inference as a process by which we arrive at a conclusion on the ground of two or more given propositions. The typical form of mediate inference consists of three propositions, the first two being the grounds for inferring the third; and it is of such a nature that the third proposition is not derivable from either of the other two taken alone. The two given propositions are the *premises*, and the third is the *conclusion*. Any and every pair of propositions will not lead to a conclusion. From the statements 'Dogs are animals' and 'Man is rational' no conclusion can be drawn, because they have nothing in common. There must be some identity of nature if two propositions are to lead to a third proposition. In the following argument—

All men are mortal

Socrates is a man

∴ Socrates is mortal,

there is a transition from the first two propositions which are the premises to the third which is the conclusion. This is made possible because the two premises have a common term which is called the *middle term*. It is through the *mediation* of the middle term that the inference is drawn. Hence this process of inference is regarded as *mediate*.

Aristotle calls this inferential process *sylogism* (a thinking 'together,' i.e., thinking two propositions together). In the words of Jevons, syllogism may be defined as "the act of thought by which from two given propositions we proceed to a third proposition, the truth of which necessarily follows from the truth of these given propositions." In a syllogism the truth of the given propositions is to be taken for granted. Assuming the truth of the premises, we have to see what conclusion can be drawn from them. A syllogism is thus a formal statement of the way in which two propositions which have a common term lead to a third proposition.

The propositions that compose a syllogism may be categorical, hypothetical, or disjunctive. And a syllogism may be pure or mixed. A pure syllogism consists of propositions of the same kind. A mixed syllogism is composed of

propositions of different kinds. We shall for the present study the pure categorical syllogism.

ii. Structure of the Syllogism

In a syllogism, we repeat, there are two propositions taken as true, and another inferred or following from them. The former are called premises, and the latter is known as conclusion. The term which is common to the two premises is called the *middle term*. The terms that constitute the subject and predicate of the conclusion are described as the *extremes*. Of these, the subject is known as the *minor term*, and the predicate as the *major term*. The premise in which the major term appears is called the *major premise*. The premise which contains the minor term is called the *minor premise*. When the syllogism is stated in the proper order, the major premise stands first and the minor premise second. Thus in the syllogism—

All organisms are mortal

Man is an organism

∴ Man is mortal

the major term is 'mortal,' and the major premise 'All organisms are mortal'; the minor term is 'man,' and the minor premise 'Man is an organism'; and the middle term is 'organism.' The reason for calling the term which is common to both the premises *middle* is evident. The middle term is that through which the conclusion is *mediated*. The term which has the greatest extension is the predicate of the conclusion, and that which has the least extension is the subject term. Hence they are called respectively *major* and *minor* terms. And if the three terms are arranged in the order of their extension, it will be found that the middle term comes in between the two extremes, which is another reason for calling it middle term. It will be convenient to use symbols for these terms. We shall use the letters P, S and M. S (= subject of the conclusion) will indicate the minor term. P (= predicate of the conclusion) the major term, and M the middle term. Substituting the symbols for the terms of the syllogism given above, we have—

M — P

iii. The Rules of the Syllogism

The syllogism depends for its formal validity on certain conditions which are commonly known as the rules or canons of the syllogism. They are as follows:

(a) *Relating to the structure of the syllogism:*

(1) A syllogism must contain three and only three, terms.

(2) A syllogism must contain three, and only three, propositions.

(b) *Relating to quantity:*

(3) The middle term must be distributed in one, at least, of the premises.

(4) No term must be distributed in the conclusion which is not distributed in the premise.

(c) *Relating to quality:*

(5) From two negative premises there can be no conclusion, *i.e.*, one, at least, of the premises must be affirmative.

(6) If one premise is negative, the conclusion must be negative, and if the conclusion is negative, one premise must be negative.

(d) *Corollaries:*

(7) From two particular premises, there can be no conclusion.

(8) If one premise is particular, the conclusion must be particular.

The first two rules tell us what a syllogism is. A syllogism is an argument in which from two given propositions we infer a third proposition. Hence (1) there must be only three propositions. If there be more than three we have more than one syllogism; if less than three, we have no syllogism. And (2) there must be only three terms, for the premises have a common term. It is on the basis of the middle term that the major term is predicated of the minor term in the conclusion. If there be more than three terms, the object of mediation would be defeated. A syllogism containing four terms commits the fallacy of *Quarternio Terminorum* or four terms. In certain cases a term may be used twice but in different senses. For example,—

The *end* of a thing is its perfection

Death is the *end* of life

∴ Death is the perfection of life.

Here the word 'end' is used first in the sense of 'aim' and then in the sense of 'termination.' Therefore it is not to be considered a middle term. There are really four terms in the argument which as a consequence is guilty of the fallacy of *ambiguous middle*. The ambiguity may be as regards the major term or minor term as well. Even then there will be four terms. For example,—

No courageous creature *flies*

The eagle is a courageous creature

∴ The eagle does not *fly*.

No man is made of paper

All *pages* are men

∴ No *pages* are made of paper.

The third rule guards us against the fallacy of *undistributed middle*. It demands that the whole extent of the middle term must be referred to at least in one premise. If this is not done, the major term may refer to one part of the extent of the middle term and the minor term to another part, and there will be no basis for comparison and consequent assertion of a relation between the major and minor terms in the conclusion. This may be illustrated by means of three diagrams, any of which may be found to be consistent with an argument where the middle term is undistributed.

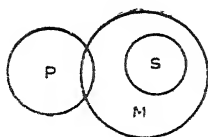


Fig. 1

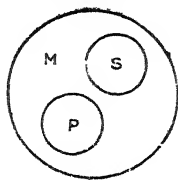


Fig. 2

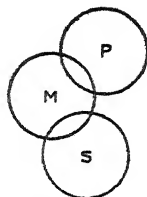


Fig. 3

In all these figures neither S nor P is related to the whole extent of M: and so no relation can be asserted between S and P. For example,—

All men are mortal

All monkeys are mortal

∴ All monkeys are men.

The conclusion is absurd because it is not warranted by the premises. 'Men' constitute a portion of mortal beings and 'monkeys' another portion of mortal beings with the result that no identity can be established between 'men' and 'monkeys.'

The fourth is a double rule. (a) The minor term should not be distributed in the conclusion unless it is distributed in its premise. A breach of this rule is called an 'illicit process of the minor term' or shortly *illicit minor*. (b) The major term should not be distributed in the conclusion unless it is distributed in its premise. A violation of this rule is called an 'illicit process of the major term' or shortly *illicit major*. Thus the fourth rule asks us not to assert more in the conclusion than is contained in the premises. If an assertion is not made about the whole extent of a term in the premise, it cannot logically be made about the whole extent of that term in the conclusion.

The argument—

All men are mortal

All men are *rational*.

∴ All *rational beings* are mortal

commits the fallacy of illicit minor.

And the argument—

All cruel men are *cowards*

No college men are cruel

∴ No college men are *cowards*

commits the fallacy of illicit major.

The present rule prevents us from taking *more* of a term in the conclusion than is referred to in the premise; but it does not prohibit us from taking *less*. There is no illicit process when a term is distributed in the premise and undistributed in the conclusion, as in the following:

All M is P

All S is M

∴ Some S is P.

The fifth rule states that from two negative premises no conclusion can be reached. From the fact that S and P are excluded from M, we can conclude nothing as regards their relation to each other. The negative statements give us no ground for inference. At least one premise must be

affirmative. Sometimes the premises may appear to be negative, while in reality they are not. In such cases valid conclusion may be drawn. For example,—

Whatever is not a compound is an element
 Gold is not a compound
 ∴ Gold is an element.

In this argument both the premises seem to be negative. But a closer attention will reveal that neither of them is negative. A slight change in the wording of each of the premises will exhibit this truth.

Whatever substance is not a compound is an element
 Gold is a substance that is not a compound
 ∴ Gold is an element.

The sixth rule says that if one premise be negative, the conclusion must be negative, and *vice versa*. In a syllogism where one premise is negative and the other affirmative, it is asserted that of the major and minor terms, one agrees and the other does not agree with the middle term. The inference that has necessarily to be drawn from these premises is that the major and minor terms do not agree with each other. Hence the conclusion must be negative. For example—

No men are perfect
 X is a man
 ∴ X is not perfect.

The negative premise asserts that one term is excluded from the middle term. The affirmative premise states that the other term is connected with the middle term. From this it follows that the two terms, S and P, are not related. Conversely, it is easy to understand how, if the conclusion be negative, one of the premises must be negative.

The seventh rule is that from two particular premises no conclusion is possible. This may be deduced as a corollary from the preceding rules. There are two particular propositions I and O. As either of them may be major or minor premise, there are four possible combinations, II, IO, OI and OO. (a) Of these, OO is not possible, because from two negative premises no inference can be drawn (rule 5). (b)

The combination II is to be ruled out, as it distributes no term and thus violates rule 3 according to which the middle term must be distributed at least once. (c) In IO and OI only one term is distributed, *viz.*, the predicate of O. If this term be not the middle term, rule 3 is broken. If it be the middle term, neither the minor nor the major term can be distributed. But in the conclusion which must be negative (rule 6), the predicate, *viz.*, the major term is distributed. And as the major term is not distributed in its premise, rule 4 is violated, *i.e.*, there is an illicit process of the major term. Hence, from two particular premises no conclusion can be had.

The eighth rule, that if one premise be particular, the conclusion must be particular, may be proved by examining the eight possible combinations of premises: AI and IA, AO and OA, EI and IE, EO and OE. (a) EO and OE are excluded by rule 5. (b) In AI and IA only one term is distributed, *viz.*, the subject of A. This distributed term must be the middle term (rule 3). As there is no other distributed term, the minor term is not distributed in its premise, and, to avoid the fallacy of illicit minor, it should not be distributed in the conclusion. This can be achieved only if the conclusion is particular, for particular propositions alone do not distribute their subject. (c) In AO and OA two terms are distributed, *viz.*, the subject of A and the predicate of O. One of the distributed terms must be the middle term (rule 3). There is only one other term, either the major or the minor, that can be distributed. As one of the premises is negative, the conclusion is negative (rule 6), and it distributes the predicate, *viz.*, the major term. As only one term can be distributed in the conclusion (for one term alone except the middle term is distributed in the premises), and as this distributed term is the major, the minor term cannot be distributed. If the minor term is not distributed, the conclusion must be particular. (d) In EI and IE* two terms are distributed, and so the proof is as in the case of AO and OA. The conclusion, therefore, must be particular, if one premise be particular.

* No valid conclusion is possible from IE (See page 85).

iv. The Axiom of the Syllogism

The governing principle of syllogistic reasoning is called the *dictum de omni et nullo* (the principle of all and none). It means that if anything be true of all the members of a class, it must be true of each separately, and that if anything be true of none, it can be denied of each separately. Whatever is affirmed or denied of a whole may be affirmed or denied of a part thereof. For example, 'mortal' is true of all members of the class 'man'; 'Socrates' is a member of the class 'man'; therefore 'mortal' is true of 'Socrates'.

It has been urged by some that the Dictum is redundant if not useless, as it is undistinguishable from the principle of subalternation. 'If the universal be true, the particular is true', and that it is based on the 'class' view which is mechanical and meaningless. It has also been said that the syllogism is but an elaborate process of begging the question. The conclusion is already implied in and known along with the premises. If I know that all men are mortal then it goes without saying that Socrates who is a man is mortal. We are only re-stating in the conclusion either what we explicitly know or have assumed without question in accepting the premises.

It is true that the Dictum states a self-evident axiomatic truth. As for the charge that the syllogism is a process of begging the question we shall consider it in a later chapter. But for the present we may observe that the Dictum is descriptive of the *modus operandi* of the syllogism, and that it is directly dependent on the Laws of Thought. These laws, we have seen, are fundamental to reasoning. They express the truth that Reality or the whole is a system and that it is the systematic nature of reality which makes reasoning possible. The Dictum endorses the same principle. The universal in the form of the middle term is the heart of syllogistic reasoning. It is in virtue of this identity in difference that there is a transition from one fact to another. The German philosopher, Hegel, declared the keynote of his philosophy in the epigram 'The truth is the whole'. Every syllogism brings out a phase of and is based on the systematic nature of the whole. The Laws of Thought as well as the Dictum postulate the same principle.

CHAPTER XII THE FIGURES AND MOODS

i. The Figures

In the classical example of a syllogism given in the last chapter—

All men are mortal
Socrates is a man
∴ Socrates is mortal

the middle term is subject in the major premise and predicate in the minor premise. But this is not the only order in which the middle term may be found in a syllogistic argument. The middle term may be predicate in both premises, or subject in both, or predicate in the major and subject in the minor premise. As there are two premises in a syllogism and as the middle term occurs in both, there are four possible ways of arranging the terms in an argument. These four forms of syllogistic reasoning are called the figures of the syllogism. It is the position of the middle term in the premises that determines the figure of a syllogism. The four figures may be symbolically represented as follows:—

Fig. I	Fig. II	Fig. III	Fig. IV
M — P	P — M	M — P	P — M
S — M	S — M	M — S	M — S
— — — —	— — — —	— — — —	— — — —
∴ S — P	∴ S — P	∴ S — P	∴ S — P
— — — —	— — — —	— — — —	— — — —

In the first figure the middle term is subject in the major premise and predicate in the minor premise.

In the second figure the middle term is predicate in both the premises.

In the third figure the middle term is subject in both the premises.

In the fourth figure the middle term is predicate in the major premise and subject in the minor premise.

Aristotle recognised only the first three figures, and of these two, the first is the only perfect figure, as it proceeds directly from the Dictum. He regarded the fourth figure as inverted form of the first. But later logicians gave it an independent status and recognised it as a separate figure.

ii. The Moods

There are three propositions in a syllogism; and each of the propositions may be of any one of the four forms A, E, I, or O. Syllogisms thus vary in respect of the quantity and quality of the propositions which constitute them. This character of the propositions which go to form a syllogism is called the *Mood* of the syllogism. If all the three propositions of a syllogism be universal affirmative, we say, the argument is of the mood AAA; and if the two premises be A and E, and the conclusion E, we say, the mood is AEE; and so on.

As there are three propositions in each syllogism, and as each of the propositions may assume any one of the four forms A, E, I and O, the total number of possible moods is 4^3 or 64. Thus for each figure we have $4 \times 4 \times 4$ or 64 possible varieties. We may write out these 64 combinations and determine which of them are valid. But it is rather a cumbersome process. It is much easier to determine the valid combinations of premises first and then draw the conclusion in each case for ourselves. In each syllogism there are two premises, and each premise may take any one of the four forms A, E, I or O. Thus we get sixteen combinations in all.

AA	EA	IA	OA
AE	EE	IE	OE
AI	EI	II	OI
AO	EO	IO	OO

Some of these combinations will have to be eliminated on the basis of the general rules of the syllogism. We know that from two negative premises no conclusion is possible; hence, the combinations EE, EO, OE and OO are to be ruled out. From two particular premises no inference can be drawn; hence the combinations II, IO and OI are invalid. The combination IE does not lead to a valid conclusion. If there be any conclusion from IE, it must be a negative proposition. The major term as predicate in the conclusion will then be distributed. But in its premise which is I it is not distributed either as subject or as predicate. This is an illicit process of the major. Therefore the combination IE too is impossible. Eliminating these invalid combinations, there are eight valid ones left over:

AA	EA	IA	OA
AE	—	—	—
AI	EI	—	—
AO	—	—	—

Now, we have to see which of these combinations of premises will yield valid conclusions in the first, second, third and fourth figures respectively. In order to determine the valid moods in each of the figures, we shall have to find out, first, the special rules of each figure. After this is accomplished, we shall be able to determine which of the eight combinations are in accordance with the special canons.

iii. The Special Canons of the Four Figures

(a) *First Figure:*

- (1) *The minor premise must be affirmative.*
- (2) *The major premise must be universal.*

M — P

S — M

∴ S — P

To show that the minor premise must be affirmative, we employ the indirect method of proof. Let us suppose that the minor premise is negative. Then, since one premise is negative, the conclusion will be negative. If the conclusion is negative, then, the major term as its predicate is distributed. And to avoid an illicit process it must be distributed in the major premise. Since it is the predicate of the major premise, and since negative propositions alone distribute the predicate, the major premise must be a negative proposition. But we have assumed by hypothesis that the minor premise is negative. If the major premise also be negative, both the premises will then be negative; but from two negative premises no conclusion is possible. This absurdity is due to our supposition that the minor premise in the first figure is negative. Hence the minor premise must be affirmative.

Since we have proved that the minor premise must be affirmative, it is easy to establish that the major premise must be universal. In the minor premise, the middle term is the predicate. Since the minor premise is affirmative, and since affirmative propositions do not distribute the predicate, the middle term is not distributed in it. But the middle term

must be distributed in at least one of the premises. It must, therefore, be distributed in the major premise. In the major premise the middle term is the subject. Since universal propositions alone distribute the subject, and since the middle term is the subject of the major premise, the major premise must be universal.

(b) *Second Figure:*

(1) *One premise must be negative and the conclusion is therefore negative.*

(2) *The major premise must be universal.*

P — M

S — M

\therefore S — P

The reason for the first rule is this. The middle term is predicate in both the premises and it must be distributed at least once. Since a negative proposition alone can distribute the predicate, one of the premises must be negative, and if one premise is negative, the conclusion must be negative.

The need for the second rule follows almost immediately. Since the conclusion is negative, the major term, its predicate, is distributed, and it must be distributed in the major premise. In the major premise P is the subject; and in order that it may be distributed the major premise must be universal.

(c) *Third Figure:*

(1) *The minor premise must be affirmative.*

(2) *The conclusion must be particular.*

M — P

M — S

\therefore S — P

That the minor premise must be affirmative may be proved by the indirect method. Let us suppose that the minor premise is negative. Then, since one premise is negative, the conclusion must be negative. The major term in the conclusion, as the predicate of a negative proposition, will be distributed and to avoid an illicit process of the major, it must be distributed in its premise. In the major premise P is the predicate, and in order that it may be distributed, the major premise must be negative. We have assumed that the minor

premise is negative. Thus we are left with two negative premises from which no conclusion is possible. Therefore our assumption that the minor premise is negative is wrong; that is, the minor premise must be affirmative.

Having proved that the minor premise is affirmative, it is easy to show that the conclusion must be particular. The minor premise, being affirmative, does not distribute its predicate S. To avoid an illicit process of the minor it must remain undistributed in the conclusion. S is subject in the conclusion; and as particulars alone do not distribute the subject, the conclusion must be particular.

(d) *Fourth Figure:*

(1) *If either premise is negative, the major premise must be universal.*

(2) *If the major premise is affirmative, the minor premise must be universal.*

(3) *If the minor premise is affirmative, the conclusion must be particular.*

P — M

M — S

∴ S — P

If either premise is negative the conclusion will be negative and P will be distributed in the conclusion. In order that it may be distributed in its premise in which it is the subject, the major premise must be universal.

If the major premise is affirmative, it does not distribute its predicate, the middle term. As the middle term must be distributed at least once, it must be distributed in the minor premise. M is the subject in the minor premise. Since universals alone distribute the subject, the minor premise must be universal.

If the minor premise is affirmative, it does not distribute its predicate S. This term must remain undistributed in the conclusion of which it is the subject. Since particulars alone do not distribute the subject, the conclusion must be particular.

iv. Determination of the Valid Moods

Applying these special canons we have now to see what moods are valid in each of the four figures. We have already

determined the possible combinations of premises which do not violate the general rules. They are

AA	EA	IA	OA
AE	—	—	—
AI	EI	—	—
AO	—	—	—

Now, we have proved that in the first figure the major premise must be universal, and the minor affirmative. The combinations that conform to these rules are AA, EA, AI and EI. Drawing the proper conclusions from these four pairs, we have the following four valid moods of the first figure:

AAA, EAE, AII, EIO.

From this it is clear that the first figure yields a valid conclusion in any of the four kinds of propositions A, E, I and O.

The special rules of the second figure state that the major premise must be universal, and one premise negative. The combinations that satisfy these conditions are EA, AE, EI and AO. When the conclusions are drawn, we have the moods:

EAE, AEE, EIO, AOO.

It is evident that the second figure yields only negative conclusions.

In the third figure the minor premise must be affirmative, and the conclusion particular. Taking all the combinations in which the minor premise is affirmative, we have AA, AI, IA, EA, OA and EI. It must be remembered that the conclusion in the third figure is always particular, even when both the premises are universal. The valid moods in this figure are:

AAI, IAI, AII, EAO, OAO, EIO

The combinations which stand the test of the special rules of the fourth figure are: AA, AE, IA, EA and EI. In drawing conclusions from these premises it is necessary to keep in mind the third canon of this figure, which states that where the minor premise is affirmative, the conclusion must be particular. Hence, the valid moods are:

AAI, AEE, IAI, EAO, EIO.

In this figure the conclusion may be either E, or I or O. It is only in the first figure that the conclusion can be of the form A.

We have thus nineteen valid moods in all—four in the first figure, four in the second, six in the third, and five in the fourth figure.

v. Mnemonic Lines

The Schoolmen* invented a set of mnemonic lines to facilitate easy remembrance of the nineteen moods:

Barbara, Celarent, Darii, Ferioque, prioris;

Cesare, Camcstres, Festino, Baroco, secundae;

Tertia, Darapti, Disamis, Datisi, Felapton, Bocardo,

Ferison, habet; quarta insuper addit,

Bramantip, Camenes, Dimaris, Fesapo, Fresison.

These lines mean that *Barbara, Celarent, Darii* and *Ferio* belong to the first figure; *Cesare, Camcstres, Festino* and *Baroco* to the second; *Darapti, Disamis, Datisi, Felapton, Bocardo* and *Ferison* to the third; and *Bramantip, Camenes, Dimaris, Fesapo* and *Fresison* to the fourth. Each of these words stands for a mood; and the vowels contained in the word signify the quality and quantity of the three propositions that compose a mood. Thus *Barbara* is a mood of the first figure whose component propositions are all of them universal affirmative AAA, and *Cesare* is a mood of the second figure whose constituent propositions are EAE. The significance of the other letters contained in each of the mnemonic words will be explained later.

vi. Strengthened and Weakened Moods

Of the nineteen valid moods mentioned above, *Darapti* and *Felapton*, in the third figure, and *Bramantip* and *Fesapo* in the fourth figure are called *strengthened* moods. In each of these moods a particular conclusion is drawn from two universal premises. Even if one of the premises were particular, the same conclusion can be had. In *Darapti* the two premises are AA and the conclusion is I.

M a P

M a S

∴ S i P

If instead of the given major premise M a P, we have

*The teachers of the Middle Ages in Europe (476-1453 A.D.) who examined the doctrines of the Church in the light of philosophic ideas.

M i P as in *Disamis*, the same conclusion will follow. Since in the moods like *Darapti* one of the premises is unnecessarily stronger than what is required to prove the conclusion, those moods are called *strengthened*.

In five of the nineteen moods the conclusion is universal, viz., *Barbara* and *Celarent* in Fig. I, *Cesare* and *Camestres* in Fig. II, and *Camenes* in Fig. IV. As the universal includes the particular, in each of these cases a particular conclusion may be drawn. There is nothing to prevent one from being satisfied with less when more is offered. We have seen in an earlier chapter that if the universal is true, the particular is necessarily true. If in a syllogism a particular conclusion is inferred while a universal is possible, the syllogism is called *weakened*. A weakened syllogism or a subaltern mood is one in which a particular conclusion is drawn, though a universal conclusion is justified by the premises. The weakened moods are: *Barbari* (AAI) and *Celarent* (EAO) in the first figure; *Cesaro* (EAO) and *Camestros* (AEO) in the second figure; and *Camenos* (AEO) in the fourth figure. There are no weakened syllogisms in the third figure, because in the moods of the figure the conclusions are all particular.

CHAPTER XIII

REDUCTION

i. Introduction

Aristotle regarded the first figure as the perfect figure, and the second and the third as imperfect figures. He did not recognise the fourth figure as an independent argument-form, because he thought it was only an inversion of the first figure. The reason for considering the first figure to be perfect is that it is the most natural form thought takes. In this figure the subject of the conclusion is subject in the minor premise and the predicate of the conclusion is predicate in the major premise. This shows that the transition from premises to conclusion is direct, easy and natural. Further, it is only in the first figure that the conclusion can be a universal affirmative (A) proposition. In fact, all the four forms A, E, I and O are possible as conclusions in this figure. And above all, the first figure exemplifies best the principle

of syllogistic reasoning, the *dictum de omni et nullo*—a fact which is very important for Aristotle. All these considerations made him think that the first figure is the perfect figure, and that syllogisms in the imperfect figures are to be tested by showing them to be equivalent to syllogisms in figure one. This process of testing the validity of the moods of the imperfect figures is called *Reduction*. It consists in *reducing* the arguments in the imperfect figures (including the fourth added by Gallen) to those in the first figure and seeing if they are equal. Reduction, then, is the process of changing or transforming the moods of the second, third and fourth figures into moods of the first figure. If an argument given in an imperfect figure is reducible to a mood in the perfect figure, it is valid, according to Aristotle; if not, it is invalid.

ii. Kinds of Reduction

There are two kinds of reduction—direct and indirect. *Direct* or *ostensive* reduction is the process by which a mood in the imperfect figures is *directly* transformed into a mood of the first or perfect figure. The means employed for this purpose are conversion, obversion, contraposition and transformation of premises. Indirect reduction or *reductio per impossibile* is the method of *reductio ad absurdum*. That is, the conclusion of the given argument is proved by first assuming its contradictory to be true and then showing that such an assumption is absurd, since it contradicts the truth of one of the given premises. Aristotle had recourse to this method for reducing *Baroco* and *Bocardo*, because he thought that these two cannot be reduced directly. But as we shall see below, every one of the imperfect moods can be reduced both directly and indirectly.

iii. Direct Reduction

The mnemonic lines contain clues for reducing directly syllogisms of the imperfect figures to arguments in the first figure. We give below the significance of the letters that compose the mnemonic words.

(1) The vowels, we have observed already, stand for the mood. In *Barbara*, e.g., the vowels AAA indicate the qualitative and quantitative nature of the three propositions constituting the syllogism.

(2) The initial letters of the mnemonic names of the

moods of the first figure are the first four consonants of the alphabet, *B, C, D, F*. The initial consonant of each of the moods of the imperfect figures, except *Baroco* and *Bocardo*, indicates that the mood is to be reduced to a mood of the first figure having the same initial consonant. Thus, the letter *B* in *Bramantip* shows that the mood must be reduced to *Barbara*; *C* in *Cesare* indicates that it is to be reduced to *Celarent*; *D* in *Darapti* suggests that it is to be reduced to *Darii*; *F* in *Festino* signifies that it should be reduced to *Ferio*, and so on.

(3) The letter *s* appearing in the mnemonic lines of the imperfect figures means that the proposition signified by the vowel preceding it is to be converted *simply*. In *Cesare*, *s* shows that the major premise *E* is to be converted simply.

(4) The letter *p* in the middle of a mnemonic word means conversion *per accidens* of the preceding proposition. Thus in *Darapti*, *p* indicates that the minor premise *A* is to be converted *per accidens*.

(5) Where *s* or *p* occurs after the third vowel, the conclusion of the new syllogism in figure one is to be converted either simply or *per accidens* as the case may be, and not the conclusion of the given syllogism.

(6) The letter *m* means *mutare* or *metathesis*, i.e., transposition of premises. By this process the major premise of the given syllogism becomes the minor premise of the new syllogism in the first figure, and the minor premise of the given syllogism becomes the major premise of the new syllogism in the first figure.

(7) The letter *k* stands for obversion of the preceding proposition. *Ks* means first obversion and then simple conversion, i.e., contraposition; and *sk* means first simple conversion and then obversion.

(8) The letter *c* appearing in the middle of a word shows that the syllogism is to be reduced *per contradictionem*, i.e., by the indirect method. The letter *c* occurs only in *Baroco* and *Bocardo*. Aristotle thought, as we remarked above, that these two moods cannot be reduced directly. But he was wrong. It is possible to reduce them directly; and for this purpose *Baroco* and *Bocardo* are renamed as *Faksoko* and *Doksamosk* respectively.

(9) The other letters occurring in the mnemonic lines, r, t, l, b, d and n have no meaning. They are only helps to easy pronunciation.

With the help of the clues given above let us reduce the imperfect moods to moods in the first figure.

I. *Moods of the Second Figure:*

(1) <i>Cesare</i>		<i>Celarent</i>
PeM	s	MeP
SaM		SaM
<hr/>		<hr/>
∴ SeP		∴ SeP
(2) <i>Camestres</i>		<i>Celarent</i>
PaM	s	MeS
SeM		PaM } m
<hr/>		<hr/>
∴ SeP		∴ PeS s
		∴ SeP
(3) <i>Festino</i>		<i>Ferio</i>
PeM	s	MeP
SiM		SiM
<hr/>		<hr/>
∴ SoP		∴ SoP
(4) <i>Baroco = Faksoko</i>		<i>Ferio</i>
PaM	ks	MeP
SoM	k	SiM̄
<hr/>		<hr/>
∴ SoP		∴ SoP

II. *Moods of the Third Figure:*

(1) <i>Darapti</i>		<i>Darii</i>
MaP		MaP
MaS	p	SiM
<hr/>		<hr/>
∴ SiP		∴ SiP
(2) <i>Disamis</i>		<i>Darii</i>
MiP		MaS
MaS	s	PiM } m
<hr/>		<hr/>
∴ SiP		∴ PiS s
		∴ SiP

(3) <i>Datissi</i> MaP MiS	s	<i>Darii</i> MaP SiM
∴ SiP		∴ SiP
(4) <i>Felapton</i> MeP MaS	p	<i>Ferio</i> MeP SiM
∴ SoP		∴ SoP
(5) <i>Bocardo</i> = <i>Doksamosk</i> MoP MaS		<i>Darii</i> MaS \overline{P} iM } m
∴ SoP		∴ \overline{P} iS s ∴ Si \overline{P} k ∴ SoP
(6) <i>Ferison</i> MeP MiS	s	<i>Ferio</i> MeP SiM
∴ SoP		∴ SoP

III. *Moods of the Fourth Figure:*

(1) <i>Bramantip</i> PaM MaS		<i>Barbara</i> MaS PaM } m
∴ SiP		∴ PaS p ∴ SiP
(2) <i>Camenes</i> PaM MeS		<i>Celarent</i> MeS PaM } m
∴ SeP		∴ PeS s ∴ SeP
(3) <i>Dimaris</i> PiM MaS		<i>Darii</i> MaS PiM } m
∴ SiP		∴ PiS s ∴ SiP

(4) <i>Fesapo</i>		<i>Ferio</i>
P e M	s	M e P
M a S	p	S i M
<hr/>		<hr/>
∴ S o P		∴ S o P
(5) <i>Fresison</i>		<i>Ferio</i>
P e M	s	M e P
M i S	s	S i M
<hr/>		<hr/>
∴ S o P		∴ S o P

Following the directions contained in the mnemonic words we have reduced above the moods of the imperfect figures to moods of the first figure. We shall explain here in detail the direct reduction of two of the moods, *Baroco* and *Bocardo*, since they are likely to present some difficulty to the beginner in logic.

Baroco is a mood of the second figure with AOO as its component propositions; and for the purposes of direct reduction it receives a new name, *Faksoko*. The initial letter *F* stands for *Ferio* the mood of the first figure into which it is to be transformed. *k* and *s* after A indicate that the major premise P a M is to be first obverted and then converted; the obverse of P a M is P e \bar{M} , and the converse of the latter is M e P. The letter *k* after the second vowel (*o*) in *Faksoko* implies that the minor premise S o M is to be obverted. The obverse of S o M is S i \bar{M} . The two premises \bar{M} e P and S i \bar{M} yield the conclusion S o P. Thus *Baroco* of the form—

$$\begin{array}{c} P a M \\ S o M \\ \hline \therefore S o P \end{array}$$

has been transformed into *Ferio* of the form—

$$\begin{array}{c} \bar{M} e P \\ S i \bar{M} \\ \hline \therefore S o P \end{array}$$

Bocardo is a mood of the third figure with OAO as its constituent propositions. It gets the name *Doksamosk* for

giving us clues for direct reduction. *D* stands for *Darii* of the first figure, *k* and *s* after the first vowel *o* indicate that the proposition signified by the vowel is to be first obverted and then converted. The obverse of $M o P$ is $M i \bar{P}$; and the converse of $M i \bar{P}$ is $\bar{P} i M$. The letter *m* appearing in the mnemonic word means that the premises are to be transposed. That is, $M a S$ becomes the major premise and $\bar{P} i M$ the minor. These two premises yield the conclusion $\bar{P} i S$. The last two letters *s* and *p* in *Doksamosk* imply that the new conclusion $\bar{P} i S$ is to be first converted and then obverted in order to get the original conclusion. The converse of $\bar{P} i S$ is $S i \bar{P}$; and the obverse of the latter is $S o P$. Thus *Bocardo* of the form—

$$\begin{array}{c} M o P \\ M a S \end{array}$$

$$\therefore S o P$$

has been reduced to *Ferio* of the form—

$$\begin{array}{c} M a S \\ \bar{P} i M \end{array}$$

$$\therefore P i S$$

$$\therefore S i \bar{P}$$

$$\therefore S o P$$

iv. Indirect Reduction

Indirect reduction is the method of demonstrating the validity of a mood in the imperfect figures by showing that the assumption of the truth of the contradictory of the given conclusion leads to absurdity. This is done by substituting for one of the premises the contradictory of the given conclusion so as to form an argument in the first figure. The resulting conclusion would contradict the premise for which the substitution has been made. But this is absurd, as the truth of the given premises cannot be called in question. Thus it may be proved that the original argument is valid.

Aristotle applied this method to the reduction of two moods, *Baroco* and *Bocardo*, because he thought they are irreducible by the direct method. The initial consonant *B*

indicates that the two moods are to be reduced to *Barbara*; and the letter *c* means that they are to be reduced *per contradictionem*, i.e., indirectly. Though the indirect method was originally employed only in the case of *Baroco* and *Bocardo*, it need not be restricted. It can be applied to any of the imperfect moods. Here we shall reduce, *Baroco* and *Bocardo* alone, leaving it to the student to apply the same method to the other cases.

(1) *Baroco*:

$$\begin{array}{c} P a M \\ S o M \\ \hline \therefore S o P \end{array}$$

The indirect method of reducing *Baroco* consists in showing, by means of a syllogism in *Barbara*, that the conclusion must be true. Let us suppose that the conclusion is false. Then its contradictory *S a P* must be true. With this as the *minor* premise, and the original major as the major premise, let us form a syllogism in the first figure.

$$\begin{array}{c} P a M \\ S a P \\ \hline \therefore S a M \end{array}$$

This is *Barbara* with *P* as the middle term. The new conclusion *S a M* is the contradictory of the original minor premise *S o M*. The original minor, however, is given as true and hence it cannot be false. It is only the new conclusion *S a M* that must be false. To what is its falsity due? Not due to the major premise *P a M*, because it is the same as the given. The false conclusion must be the result of the new minor premise *S a P* which, therefore, must be false. If it is false, then its contradictory, the original conclusion *S o P* must be true.

(2) *Bocardo*:

$$\begin{array}{c} M o P \\ M a S \\ \hline \therefore S o P \end{array}$$

If the conclusion *S o P* be false, its contradictory *S a P* must be true. With this as the *major* premise and the given minor as the *minor* premise, let us constitute a fresh syllogism in the first figure.

$$\begin{array}{c}
 S \ a \ P \\
 M \ a \ S \\
 \hline
 M \ a \ P
 \end{array}$$

This is *Barbara* with S as the middle term. The conclusion that is thus reached, *viz.*, M a P is the contradictory of the original major premise M o P, and so must be false. Its falsity is due to the new major premise S a P which, therefore, must be false. If this be false, then, its contradictory, the original conclusion S a P, must be true.

v. An Easier Method

The processes of reduction were taught by Aristotle as the only methods of determining the validity of an argument in the imperfect figures. But the moods of the imperfect figures do not need to be reduced to corresponding moods of the first figure for exhibiting their validity. Any concrete syllogistic reasoning may be tested by casting its propositions into logical form, discovering its figure and mood, and ascertaining whether or not it belongs to one or other of the valid forms.

Professor Robert Latta gives a simple method of testing the validity of arguments, following up a suggestion made by Professor Leonard J. Russel. We should first put the argument whose validity we want to determine into logical form. In arranging the form of the argument we should take care to see that the major premise stands first, then the minor premise, and finally the conclusion, that both the premises are not negative and that if one premise be negative the conclusion is negative. Having done this, we should see if the argument satisfies the rules of distribution. Let us examine a concrete case 'All trained logicians can detect fallacies; but few electors are trained logicians, so that few electors can detect fallacies.' Expressed in logical form, the argument will appear thus:

- All trained logicians are those who can detect fallacies
- Some electors are not trained logicians
- ∴ Some electors are not those who can detect fallacies.

The distribution of the terms may be indicated in the following diagram:

<i>m</i>	<i>d</i>			<i>u</i>
	<i>u</i>		<i>m</i>	<i>d</i>
<hr/>			<hr/>	
	<i>u</i>			<i>d</i>

In this diagram *d* stands for 'distributed,' *u* for 'undistributed' and *m* for 'the middle term.' The above argument satisfies the rule regarding the distribution of the middle term, but the major term in the conclusion is distributed without being distributed in its premise. Therefore the reasoning is invalid, the fallacy being an illicit process of the major.

vi. Value of the Second and Third Figures

It is true that the first figure offers the best form for the expression of our highest thoughts. But it does not follow that the other figures—specially the second and the third—have no value.

As regards the fourth figure, it must be confessed that it is an unnatural form for expressing any of our thoughts. The syllogism—

All nitrogenous foodstuffs are flesh-forming

All grains are nitrogenous

∴ Some flesh-forming foodstuffs are grains

is awkward in form and involved in meaning. The same argument may be better expressed in the first figure thus:

All nitrogenous foodstuffs are flesh-forming

All grains are nitrogenous

∴ All grains are flesh-forming.

The difference between the two figures will be evident if we compare the two conclusions which, however, are based on the same premises. The conclusion in the first figure is 'All grains are flesh-forming'; and the conclusion in the fourth figure is 'Some flesh-forming foodstuffs are grains' which, if converted, would be 'Some grains are flesh-forming.' There is some loss in expressing the argument in the fourth figure. Thus the fourth figure is not of much value. But, nevertheless, it is valid, and does not stand in need of reduction to the first figure for its validity being demonstrated, though the

argument may gain in directness and clarity by such reduction.

Even this charge of unnaturalness and lack of clarity cannot be levelled against the second and the third figures which are not merely different from the first figure in form but represent different levels of thought.

It is no doubt true, as Aristotle says, that "the scientific knowledge of the nature of a thing (literally, what a thing is) can be ascertained through the first figure alone." But this scientific knowledge which consists in defining a thing or species in terms of universals is not reached at a jump. There are intervening stages; and to these the second and third figures are appropriate.

The second figure represents the stage when a thing is sought to be defined by showing what it is not. In this type of reasoning, if M is affirmed of P, it is denied of S, or if M is denied of P, it is affirmed of S, with the result that the conclusion states a relation of exclusion as between S and P. But as every significant negation has a positive basis, the negative conclusion of the second figure paves the way for the determination of the character of S. Suppose we are interested in knowing what true patriotism is. Before arriving at a final definition, it will be useful if we eliminate concepts with which patriotism is likely to be confused, *e.g.*,

War-mongering is bad

Patriotism is not bad

∴ Patriotism is not war-mongering.

The third figure marks another incomplete stage in the process of defining a subject. The conclusions in this figure are all particular. Hence at this stage, there is a suggestion of a possible connection or disparity between the minor and the major terms, which may serve as a clue for further investigation and analysis, *e.g.*,

Sankara, Ramanuja and Madhva are eminent philosophers

Sankara, Ramanuja and Madhva are religious reformers

∴ Some religious reformers are eminent philosophers.

Thus the second and the third figures represent stages in the development of thought; and it would be a mutilation of

their content and nature to squeeze them into the first figure-form. The so-called reduction serves no more useful purpose than that of providing an interesting logical exercise.

CHAPTER XIV

CONDITIONAL ARGUMENTS

i. Introduction

We have so far studied the categorical syllogism. In arguments of this type the constituent propositions are categorical in nature. But there may be reasonings with either hypothetical or disjunctive propositions. In this chapter we shall consider these. But before proceeding to a study of the structure and kinds of conditional arguments, let us learn about the nature and place of conditional propositions.

ii. Conditional Propositions

In chapter VI* we classified propositions according to relation into the categorical and the conditional, and sub-divided the conditional into the hypothetical and the disjunctive. We have elaborately dealt with the categorical propositions which is an unconditional assertion of a relation (either affirmative or negative) between the subject and the predicate. Here we shall explain the import of the other two kinds, *viz.*, the hypothetical and the disjunctive.

There are two forms of the hypothetical proposition: (a) If A is B, C is D; (b) If A is B, A is C; *e.g.*, 'If the weather is foggy, the train is late'; 'If a metal is heated, it expands.' The first part which begins with an 'If' expresses the condition and is called the *antecedent*; and the second part states what is asserted under the condition and is called the *consequent*. Of the two forms of the hypothetical proposition given above, in the first the antecedent and the consequent have different subjects; and in the second they have the same subject. In these forms the antecedent and the consequent are both affirmative. But they need not be so in all hypothetical propositions. Either the antecedent or the consequent may be negative, or both of them may be negative. For instance, the first form need not always be 'If A is B, C is D'. The other alternatives are: If A is not B, C is D; If A is B, C is not D; If A is not B, C is not D.

* See p. 37.

In the case of the categorical proposition, the quality is indicated by the copula. But how are we to determine the quality of the hypothetical proposition! Latta and Macbeath believe that the quality of the proposition is the same as the quality of the consequent. "A hypothetical proposition is affirmative, when its consequent is affirmative, whether or not its antecedent is also affirmative; and it is negative when its consequent is negative, whether or not its antecedent is negative."* Joseph, however, is of a different view† He thinks that the distinction of affirmative and negative does not apply to the hypothetical judgment as a whole. Even where either antecedent or consequent or both are negative, the judgment affirms the dependence of a consequent on a condition. The antecedent or consequent severally may be either affirmative or negative. The hypothetical judgment as such is neither affirmative nor negative. Its function is to connect a consequent with a condition; and so the hypothetical cannot express a negative relation as between the antecedent and the consequent.

Distinctions of quantity too do not apply to the hypothetical proposition. In the categorical the reference is primarily to individuals or denotation. Since the function of the categorical is to assert a relation between an attribute and its subject, it is pertinent to ask whether the attribute belongs (or does not belong) to the whole of the subject or to a part of it? If the predication is made of the whole of the subject, the proposition is universal; if it is made of a part of the subject, the proposition is particular. But in the case of the hypothetical such a distinction would be meaningless. The hypothetical expresses the dependence of a consequent on an antecedent. Since this dependence is a necessary relation, the proposition which expresses it must be universal—universal, not in the quantitative sense of an exhaustive collection of all the particular objects, but in the sense that the attributes, signified by the antecedent and the consequent are necessarily related. The hypothetical refers to individuals only indirectly. When we say 'If man, then mortal', what we mean is that the characteristic of being mortal is necessarily related to the

* *The Elements of Logic*, p. 89.

† *An Introduction to Logic*, p. 164.

characteristics of being man. Thus the hypothetical is mainly connotative in its significance, and so distinctions of quantity are inapplicable to it.

Controversy has been raging between rival groups of logicians over the relation between the categorical and hypothetical. Some writers regard the distinction between the two as merely formal; and according to them the same content may be expressed either categorically or hypothetically. For instance, the hypothetical proposition "If commodities are scarce, they are dear" may be stated in the form of a categorical proposition thus: "Scarce commodities are dear". There are other logicians who think that the categorical and the hypothetical are altogether different, the one being an unconditional statement of a fact and the other a statement of dependence as between a condition and a consequent.

Both these views seem to be erroneous. All categorical propositions are not unconditional statements. In most cases the conditions are not expressed because they are very evident. Pondering over the carnage of war and the blood thirst of nations, if a person says 'the world is too bad', it is not an unconditional statement that he makes. He does not specify the conditions because, in his view, they are too evident. Many of the universal or individual categorical propositions are of this sort. Propositions like 'Man is mortal' and 'Heat expands bodies' are really in each case the assertion of a relation between a condition and a consequent; they may be stated with greater propriety in the hypothetical form. 'If man, then mortal', 'If a body is heated, it expands.' These are called generic universals as contrasted with collective universals which are the result of an enumeration of particular cases constituting a class. Suppose I make the statement 'All modern nations maintain Air Forces'. After studying the military dispositions of the various nations, it is a case of the collective universal, and is categorical in nature, for I cannot say 'If a modern nation, then it must necessarily have an Air Force.' But if I make the same statement on a knowledge of the deplorable state of the modern world with its warring nations, and of the importance of the Air Arm, then my judgment would be hypothetical. Thus the difference between the categorical and the hypothetical is a difference in degree. The

conditions are implicit in the categorical, while they are developed and made explicit in the hypothetical. Though the reduction of the hypothetical proposition to the categorical form may serve as a useful logical exercise, it must be borne in mind that the two types of propositions represent different levels in the evolution of thought. We start our intellectual career by identifying things and discovering certain characteristics in them. At this level the categorical is adequate. But when we probe deeper into the connections of things, and are interested not so much in individuals as in types, we are at a higher level of scientific thought which is best expressed in the hypothetical form.

The disjunctive proposition sets forth alternative possibilities. It may take either of two forms: (1) A is either B or C, (2) Either A is B or C is D: Number is either odd or even; either slavery is justifiable or Aristotle is wrong. Whatever be the form, the principle of disjunction is the same. It is to exhibit the alternatives of a system, and is expressed by the words 'either . . . or.' The logical process of division which we have studied is based on the same principle. When a genus is divided into its various species, it means that it is realised in those alternatives. For instance, from a knowledge of the dividing members of the genus 'triangle' we may say 'Triangles are either equilateral, isosceles or scalene.' It should be noted here that the alternatives need not be only two in number but may be more.

Distinctions of quality and quantity do not apply to the disjunctive. There cannot be any negative disjunction. 'Neither A is B nor C is D' is a conjunction of negations, not a disjunction. It means merely that 'A is not B' and 'C is not D.' Some logicians speak of particular disjunctives.* But there is no justification for this. 'Some A is B or C' has not the character of a knowledge of system, and hence is not disjunctive.

The true disjunctive, as has been stated, sets forth the alternatives of a system, as for example, the constituent species of a genus. The alternatives of the disjunction must be exhaustive; and they must also be mutually exclusive. To this latter characteristic some writers raise an objection. 'He

* See G. H. Joyce: *Principles of Logic*, p. 66.

is either a knave or a fool' precludes his being neither; but is there not a possibility that 'he is both a fool and a knave'? Surely, in this case the alternatives are not exclusive, but they are not exhaustive either, because there is the third alternative, 'knave and fool.' 'He is either a knave or a fool,' properly speaking, is not a true disjunction. It is a disjunction of ignorance. Propositions which are of the form 'either...or' are not all of them disjunctive. True disjunction is the result of a thorough knowledge of the universe of discourse. 'A triangle is either equilateral, isosceles or scalene.' 'The signal light is either red or green.' In these cases of genuine disjunction, the alternatives are exclusive as well as exhaustive.

There is a view which holds that the disjunctive is a union of hypotheticals. The disjunctive 'Either A is B or C is D', may be resolved, it is said, into four hypothetical propositions: if A is B, C is not D; if A is not B, C is D; if C is D, A is not B; if C is not D, A is B. It is true that the implications of a disjunctive statement may be shown by means of hypothetical propositions. But still, the hypothetical and the disjunctive are not the same. The latter represents a higher stage in the development of knowledge than the former. The knowledge expressed in the hypothetical proposition is not systematic. The antecedent is only one of the conditions of the consequent, and so the relation between the two is not reciprocal. 'If the rains fail, there will be famine.' Here, failure of rain is but one of the conditions that cause famine. If the rains fail, then, we may be sure, famine will follow. But if there is famine, we cannot say that it must have been caused by failure of rains alone. Similarly, if there is no famine, we may conclude that the rains did not fail. But from the non-failure of rains, we cannot infer that there will be no famine. We shall say more about this when we come to hypothetical arguments. What we are interested here in pointing out is that the relation between the antecedent and the consequent in the hypothetical proposition is not reciprocal. And to that extent our knowledge is imperfect. In the disjunctive proposition this defect is got over. The alternatives set forth in the disjunctive are so systematically related that from the affirmation of either alternative (assuming there are two alternatives) the other may be denied, and from the

denial of either the other may be affirmed. This shows that the disjunctive proposition goes beyond the hypothetical while including it, even as the hypothetical proposition transcends the categorical, while preserving its essentials. Knowledge grows from the lower levels to the higher. And in the evolution, the categorical, the hypothetical and the disjunctive represent successive stages. In the realms of sense perception and history, our knowledge is for the most part categorical; in science it is hypothetical; and in philosophy which aims at a systematic understanding of reality, our knowledge tends to be disjunctive.

Having examined at some length the nature and import of the hypothetical and the disjunctive propositions, we shall now proceed to a study of conditional reasonings. Under this head we shall consider three forms: (1) the hypothetical syllogism, (2) the disjunctive syllogism, and (3) the dilemma.

iii. The Hypothetical Syllogism

The hypothetical syllogism is of two kinds: (1) pure and (2) mixed. The pure hypothetical syllogism is an argument in which all the constituent propositions are hypothetical in character.

If A is B, C is D
If E is F, A is B
∴ If E is F, C is D.

e.g., If a country is prosperous, civilisation flourishes.
 If the people are diligent, a country is prosperous.
 ∴ If the people are diligent, civilisation flourishes.

This type of argument, however, is not common. What is ordinarily meant by the hypothetical syllogism is the mixed variety which consists of a hypothetical major premise, a categorical minor premise and a categorical conclusion. This is also known as the hypothetico-categorical syllogism.

e.g., If the strike had been called off, the men would be
 back at work.

The strike has been called off.

∴ The men are back at work.

The rule of the hypothetical syllogism is: *either affirm the antecedent or deny the consequent*. That is, the categorical minor premise must either affirm the antecedent or deny the consequent of the hypothetical major premise. If it

affirms the antecedent the conclusion will affirm the consequent. If it denies the consequent, the conclusion will deny the antecedent. The reason for this rule will be evident, since we have already explained the nature of the hypothetical proposition. The antecedent is only one of the conditions and not the *sole* condition of the consequent. And so, if the antecedent is present, we may say the consequent is present, or if the consequent is absent, we may infer, the antecedent is absent. But from the absence of the antecedent we cannot argue that the consequent is absent, for the consequent may be present as a result of some other condition. Similarly we cannot conclude that the antecedent is present because of the presence of the consequent, for the latter might be due to other conditions. If a person takes a certain quantity of arsenic, he dies. But if he is dead, we cannot assert that it must have been due to arsenic, for his death might have been the result of other causes like gun-shot, disease or age. Similarly, from a knowledge that the person has not taken arsenic, we cannot say that he is not dead, for the same reason as given above. Hence the rule of the hypothetical syllogism is that either the antecedent should be affirmed or the consequent should be denied.

Here the words 'affirm' and 'deny' should be understood with care. They do not refer to the quality of the minor premise. If the antecedent of the major premise is negative in form, its affirmation in the minor would take the form of a negative proposition. Similarly, if the consequent of the major premise is negative, its denial in the minor would assume the form of an affirmative proposition. Hence 'affirming' means stating what is given in the major premise, and 'denying' means contradicting what is given in the major. And it should also be remembered that when the consequent is denied in the minor premise, the conclusion will be the contradictory, and not the contrary, of the antecedent of the major premise. We shall illustrate these two points by means of an example.

If all men were actuated by the highest motives, the courts of law would not be necessary.

The courts of law are necessary.

∴ Some men are not actuated by the highest motives.

Here the minor premise denies the consequent of the major; and the denial takes the form of an affirmative proposition because the consequent is a negative. Since the minor premise denies the consequent, the conclusion denies the antecedent. But the denial, it should be noted, is the contradictory, and not the contrary, of the antecedent.

There are two moods of the hypothetical syllogism one in which the minor premise affirms the antecedent, and the other in which the minor premise denies the consequent. The first is called the constructive syllogism or the *modus ponens*, and the second is called the destructive syllogism or the *modus tollens*.

Modus ponens:

If A is B, C is D

A is B

∴ C is D

E.g., If a nation is ease-loving, the enemy can subdue it easily

This nation is ease-loving

∴ The enemy can subdue it easily.

Modus tollens:

If A is B, C is D

C is not D

∴ A is not B.*

E.g., If an author is obscure in his writings, people ignore him

People do not ignore this author

∴ This author is not obscure in his writings.

There are two formal fallacies of hypothetical reasoning, one the form in which the minor premise denies the antecedent, and the other the form in which the minor premise affirms the consequent. These arise, it will be seen, as a result of disobeying the rule of the hypothetical syllogism.

Denying the antecedent:

If A is B, C is D

A is not B

∴ C is not D.

*The principle is the same for the other form of the hypothetical proposition: if A is B, it is C.

Affirming the consequent:

If A is B, C is D

C is D

∴ A is B.

It is possible to reduce the hypothetical syllogism to categorical form. But it should be remembered that this reduction is not very helpful, though it may serve as a useful exercise in formal logic. The categorical proposition and the hypothetical, as we have said, belong to two different levels of thought, and by reducing the one to the form of the other, at least a part of the meaning is lost. Bearing this in mind, let us reduce the two moods of the hypothetical syllogism to their corresponding categorical types.

By this process the *modus ponens* will become *Barbara*.

All cases of A being B are cases of C being D

This is a case of A being B

∴ This is a case of C being D

The *modus tollens*, when reduced to categorical form, will become *Camestres*:

All cases of A being B are cases of C being D.

This is not a case of C being D

∴ This is not a case of A being B

Similarly, the two fallacious types, if reduced to categorical form, will be found guilty respectively of illicit major and undistributed middle. That is, a hypothetical syllogism in which the antecedent is denied corresponds to a categorical syllogism in which there is an illicit process of the major thus:

All cases of A being B are cases of C being D

This is not a case of A being B

∴ This is not a case of C being D.

And a hypothetical syllogism in which the consequent is affirmed corresponds to a categorical syllogism which commits the fallacy of undistributed middle thus:

All cases of A being B are cases of C being D

This is a case of C being D

∴ This is a case of A being B.

There is one other point we should like to advert to. Affirming the consequent and denying the antecedent become fallacies only because in the ordinary hypothetical proposition the antecedent is but one of the conditions of the consequent.

But, if it is known in a hypothetical proposition that the antecedent is the only condition of the consequent, then it is no fallacy to infer the absence of the consequent from the absence of the antecedent or the presence of the antecedent from the presence of the consequent.

E.g., If a triangle is equilateral, it is equiangular

This triangle is not equilateral

\therefore This triangle is not equiangular.

If a triangle is equilateral, it is equiangular

This triangle is equiangular

\therefore This triangle is equilateral.

iv. The Disjunctive Syllogism

The disjunctive syllogism is a reasoning in which the major premise is a disjunctive proposition and the minor premise a categorical which either affirms or denies an alternative of the disjunction. If the minor premise affirms one alternative, the conclusion will deny the other alternative (assuming there are only two alternatives in the disjunction), and if the minor premise denies one alternative, the conclusion will affirm the other. An argument in which the minor affirms and consequently the conclusion denies is in the *modus ponendo tollens*. And an argument in which the minor denies and as a consequence the conclusion affirms is in the *modus tollendo ponens*. Thus there are two moods of the disjunctive syllogism.

Nidus ponendo tollens

Either A is B or C is D Either A is B or C is D

A is B

or

C is D

C is not D

$\therefore A$ is not B

Modus tollendo ponens:

Either A is B or C is D Either A is B or C

A is not B

or

C is not D

$\therefore C$ is D

$\therefore A$ is B

The *modus ponendo tollens* is not admitted by some logicians, according to whom the alternatives of a disjunction need not be mutually exclusive. If the alternatives are not exclusive, then it is evident that by affirming one of them we cannot deny the other, for both of them may be true. Let us illustrate, taking the old example: 'He is either a knave or a fool.' If he is not a knave, then he a fool; but if he

is a knave, then it cannot be said that he is not a fool, for he may be both. Thus, where the alternatives of a disjunction are not mutually exclusive, the *modus ponendo tollens* will not be valid. But as we have already stated, it is not a proper disjunctive which sets forth alternatives that are not mutually exclusive. If by the affirmation of one alternative we cannot deny the other, then we are on no higher level than the hypothetical. The true disjunctive is based on a systematic knowledge of the various alternatives in their interconnections. And when this is achieved, it is a matter of indifference whether affirmation is made through denial or denial is made through affirmation.

In the types of disjunctive syllogism given above the major premise sets forth only two alternatives. But there may be more, e.g., Either A is B or C is D or E is F. In such cases if the minor premise affirms one member, the conclusion will deny the others, and if the minor premise denies one member, the conclusion will affirm the others disjunctively. It is also possible that the minor premise denies all the members but one, in which case the conclusion will affirm that one. The forms of these types are as follows:

Either A is B or C is D or E is F

A is B

∴ C is not D and E is not F.

Either A is B or C is D or E is F

A is not B

∴ Either C is D or E is F.

Either A is B or C is D or E is F

Neither A is B nor C is D

∴ E is F*

v. The Dilemma

The dilemma is a syllogism with a compound hypothetical proposition as its major premise and usually a disjunctive proposition as its minor premise. That is, the major premise consists of two hypothetical propositions joined together. The disjunctive minor either affirms the two antecedents or denies the two consequents of the compound hypothetical major

*In the examples given in this section we have taken one form of the disjunctive. The principle is the same as regards the other form: A is either B or C.

premise. Consequently, following the rule of the hypothetical syllogism, the conclusion either affirms the consequents or denies the antecedents. The dilemma in which the disjunctive minor premise affirms the antecedents is called *constructive*. And the dilemma in which the disjunctive minor premise denies the consequents is called *destructive*. The compound hypothetical major premise of a constructive dilemma may contain two antecedents and a single consequent, in which case the conclusion will be a categorical proposition affirming the consequent; and the dilemma is called *simple*. If however, there are two consequents, the conclusion will be disjunctive and the dilemma is called *complex*. Similarly, in a destructive dilemma, if there is a single antecedent, the conclusion will be categorical and the dilemma is *simple*, and if there are two antecedents, the conclusion will be disjunctive, and the dilemma is *complex*. Thus there are four types of dilemma, the simple constructive, the complex constructive, the simple destructive and the complex destructive.

(1) *Simple Constructive*:

If A is B, E is F; and if C is D, E is F

Either A is B or C is D

∴ E is F.

E.g., If a man follows the dictates of his conscience, he is criticised; and if he follows the way of others, he is criticised.

A man must follow either the dictates of his conscience or the way of others.

∴ In any case he is criticised.

(2) *Complex Constructive*:

If A is B, E is F; and if C is D, G is H

Either A is B or C is D

∴ Either E is F or G is H.

E.g., Caliph Omar is said to have given the following argument in support of his action ordering the destruction of the Alexandrian library:

If the books are in conformity with the Koran, they are superfluous; and if they are not in conformity with the Koran, they are pernicious.

Either the books are in conformity with the Koran or they are not.

∴ Either the books are superfluous or they are pernicious.

(3) *Simple Destructive:*

If A is B, E is F; and if A is B, G is H

Either E is not F or G is not H

∴ A is not B.

E.g., A politician may find himself sometimes torn between loyalty to his convictions and loyalty to his party. He may argue thus:

If I am to continue in politics, I must feel able to support both my convictions and my party.

But now I must either act against my convictions or oppose my party.

∴ I cannot continue in politics.*

In the form and example of the simple destructive dilemma given above the minor premise is a disjunctive proposition. But very often in this type, it should be noted, the minor premise is a conjunction of two negatives.

If A is B, either C is D or E is F

But neither C is D, nor E is F

∴ A is not B.

E.g., Zeno, the Greek philosopher, used the following argument to prove that motion is not intelligible.

If a body moves, it must move either in the place where it is, or in the place where it is not.

But it can neither move in the place where it is, nor in the place where it is not.

∴ It cannot move.

It is true that in this case the minor premise is not disjunctive; and yet the argument is a dilemma because the minor premise denies a disjunctive proposition.

(4) *Complex Destructive:*

If A is B, E is F; and if C is D, G is H

Either E is not F or G is not H

∴ Either A is not B or C is not D.

E.g., If your education is broad you have read everything and if you are virtuous you have not read immoral literature.

* See H. W. B. Joseph: *An Introduction to Logic*, p. 332.

But either you have not read everything or you have read immoral literature.

∴ Either your education is not broad or you are not virtuous.

The dilemma is a dialectical device to confound the opponent; and so it does not aim at truth. It is a weapon of effective argumentation, and is employed to compel the adversary to choose between two alternatives either of which will be unacceptable to him. While it is, in the words of Schiller, "the prettiest and dialectically the most effective form of conditional reasoning," it is mostly fallacious. If it is to be valid, the disjunction in the minor premise must be exclusive and exhaustive. But generally this condition is not satisfied in the dilemma. And so, if the adversary is resourceful, he can find a way out of the intellectual difficulty presented to him.

There are three ways of meeting a dilemma:

(1) A person to whom a dilemma is offered may point out that the alternatives or horns as they are called (evidently a metaphor borrowed from bull-fights) are not exhaustive, and that therefore the conclusion does not follow. That is, he may say that, as the disjunction is not complete, there is another alternative and he is not bound to choose either. This method is described as *escaping between the horns of the dilemma*. We shall illustrate this by means of an example. One who does not favour institution of prizes may argue thus: 'If boys are bright, they do not require prizes; and if they are dull, prizes will have no effect on them. Boys are either bright or dull. Therefore prizes are useless.' The dilemma may be met by showing that the minor premise does not exhaust all the possibilities. It classifies boys into two groups, bright and dull. But there are mediocre boys for whom prizes hold out a good inducement. In the case of these boys, it may be said, prizes are not useless.

(2) The second way of meeting the dilemma is by showing that the consequents proposed in the major premise do not follow from the antecedents. This course is described as *taking the dilemma by the horns*. The following example illustrates this method. 'If a man remains single, he will be unhappy because he has no one to take care of him, and if a

‘नाप्सु श्लाघमानस्स्नायात्’

इति । अत्र ये त्वसाधारणधर्मा अग्नीन्धनादयस्ते यावत्समावर्तनं तावदेव कार्या इत्याह मनुः—

अग्नीन्धनं भैक्षचर्यमधश्शयां गुरोर्हितम् ।

आसमावर्तनं कुर्यात्कृतोपनयनो द्विजः ॥

एतच्चान्येषामप्येवंजातीयानां¹मुपलक्षणार्थम् । अत एव पुराणं—

मेखलामजिनं दण्डमुपवीतं च नित्यशः ।

कौपीनं कटिसूत्रं च ब्रह्मचारी तु धारयेत् ॥

यमोऽपि—

दण्डं कमण्डलुं वेदं मौर्झीं च रशनां तथा ।

धारयेद्ब्रह्मचर्यं च भिक्षान्नाशी गुरौ वसन् ॥

वेदो दर्भमुष्टिः । गुरौ गुरुगृह इत्यर्थः । अत्र वसतो ये धर्मास्ता-

नाह व्यासः—

जघन्यशायी पूर्वं स्यादुत्थाय गुरुवेश्मनि ।

यच्च शिष्येण कर्तव्यं कार्यं दासेन वा पुनः ॥

कृतमित्येव तत्सर्वं कृत्वा तिष्ठेत्तु पार्श्वतः ।

किङ्करस्सर्वकार्येषु सर्वकर्मसु कोविदः ॥

नाभुक्तवति नाश्नीयादपीतवति नो पिबेत् ।

न तिष्ठति तथाऽसीत नासुप्ते प्रस्वपेत्तथा ॥

in the counter-dilemma; 'If A is B, G is not H; and if E is F, C is not D.' In the major premise the consequents are transposed and negated. This, however, is not the only way of rebutting. If the alternative conditions are such that they cannot be combined with the contradictory of each other's consequents, the method detailed above will be of no use. But the same object of rebutting may be achieved with the help of altogether different premises. The essential of this process is that a dilemmatic argument with contradictory conclusion must be produced. Some dilemmas may not admit of rebuttal. Then the other methods 'escaping through the horns' and 'taking the dilemma by the horns' must be tried.

In the dilemma the major premise contains two hypothetical propositions. But instead of two, we may have three or four or more, in which case the argument will be called a trilemma, tetralemma or polylemma. These arguments proceed on the same principle as that of the dilemma, and call for no separate treatment.

CHAPTER XV

ABRIDGED AND CONJOINED SYLLOGISMS

i. Introduction

We were concerned in the last few chapters with the principles and types of syllogistic reasoning. Whenever we set forth a form of syllogism or a syllogistic argument, we took care to state the major premise first, then the minor premise and lastly the conclusion. And also, we have been dealing with isolated syllogisms. *i.e.*, one syllogistic argument at a time. These may be all right in a textbook on logic. But when we think within ourselves or express our thoughts to others, we do not make use of full-fledged syllogisms in their isolation. We omit to express parts of the syllogism which are obvious, and we present a series of connected arguments. The principles of reasoning, however, are the same as those we have already studied; only the forms of expression vary. These we shall discuss in the present chapter.

ii. Enthymeme

When a syllogistic argument is not completely stated, it is called an enthymeme. There are three propositions in a syllogism. But it is not necessary to state all the three.

Sometimes it may even seem pedantic and unnatural to set forth the major premise, the minor premise and the conclusion in succession. It is more natural to say 'Socrates is mortal because he is a man' than to say 'All men are mortal; Socrates is a man; therefore Socrates is mortal.' It is enough if two propositions of a syllogism are given, for it is quite easy to construct the third. Such an abridged syllogism where one of the propositions is omitted is an enthymeme. It obeys all the rules of syllogism except the one which states that there should be three propositions. And this, as we have shown above, is no defect at all. On the contrary, it may contribute to the effectiveness of an argument.

In an enthymeme any one of the three propositions may be suppressed. If the major premise is left out, the enthymeme is of the first order. If the minor premise is omitted, the enthymeme is of the second order. If the conclusion is not expressed, the enthymeme is of the third order. We may illustrate the three orders by expressing the following argument enthymematically:—

Complete argument:

All prophets are wise
Buddha is a prophet
∴ Buddha is wise.

First order:

Buddha is a prophet
∴ Buddha is wise.

Second order:

All prophets are wise
∴ Buddha is wise.

Third order:

All prophets are wise and
Buddha is a prophet.

We have expressed above a categorical syllogism in the form of enthymemes. Other kinds of syllogism, the hypothetical etc. may also be abridged in a similar way. *E.g.*, I said, "If I am well, I would go with you", and now I am well. This is an abridged hypothetical syllogism of the third order.

iii. Poly-syllogism

Seldom do we think or express our thoughts in single syllogisms. Reasoning is mostly in the form of a chain in which

several syllogisms are conjoined in such a way that they lead to a single conclusion. A chain of reasoning so constituted is called a poly-syllogism. In a poly-syllogism the arguments form a related series. Any syllogism either supports or is supported by the syllogism adjacent to it. The syllogism that supports is called a *pro-syllogism*, and the syllogism that is supported is called an *epi-syllogism*. That is, a pro-syllogism is one whose conclusion becomes a premise in another syllogism; an epi-syllogism is one which takes for a premise the conclusion of another syllogism. Let us consider the following example:

- | | |
|-----|---------------|
| (1) | All D is E |
| | All C is D |
| | ∴ All C is E. |
| (2) | All C is E |
| | All B is C |
| | ∴ All B is E |
| (3) | All B is E |
| | All A is B |
| | ∴ All A is E |

Here, the first syllogism is a pro-syllogism in relation to the second because its conclusion becomes the major premise in the latter. The second is an epi-syllogism in relation to the first and a pro-syllogism in relation to the third. The third syllogism is an epi-syllogism in relation to the second. The terms 'pro-syllogism' and 'epi-syllogism,' then, are relative, any argument proving one of the premises of another syllogism being a pro-syllogism, and any argument using as a premise the conclusion of another syllogism being an epi-syllogism.

In the example of a poly-syllogism given above, we saw that the first argument is a pro-syllogism in relation to the second, and the second a pro-syllogism in relation to the third. The chain of reasoning here proceeds forward from a pro-syllogism to an epi-syllogism; the advance of thought is from the supporting argument to the argument which it supports. Such a poly-syllogism is said to be *progressive* or *synthetic*.

The train of reasoning may go backward too, *i.e.*, from the argument which is supported to the argument which supports it, from the epi-syllogism to the pro-syllogism. In such

a case, the poly-syllogism is said to be *regressive* or *analytic*. The order of the example we gave above will have to be reversed in order to get the regressive poly-syllogism.

- (1) All A is E
 \therefore All B is E and
 All A is B
- (2) All B is E
 \therefore All C is E and
 All B is C
- (3) All C is E
 \therefore All D is E and
 All C is D

Here, the first argument is an epi-syllogism in relation to the second, because a premise of the first is the conclusion of the second and the second argument is an epi-syllogism in relation to the third, because a premise of the second is the conclusion of the third. The chain of reasoning is regressive, since it goes from an epi-syllogism to a pro-syllogism.

The following examples are given to illustrate the two types of poly-syllogism:

Progressive poly-syllogism:

- (1) All sentinels of the spirit are honoured
 All inspired persons are sentinels of the spirit
 \therefore All inspired persons are honoured.
- (2) All inspired persons are honoured
 All poets are inspired persons
 \therefore All poets are honoured.
- (3) All poets are honoured
 Rabindranath is a poet
 \therefore Rabindranath is honoured.

Regressive poly-syllogism:

- (1) Socrates is mortal
 because all men are mortal
 and Socrates is a man.
- (2) All men are mortal
 because all organisms are mortal
 and all men are organisms.
- (3) All organisms are mortal
 because all created beings are mortal
 and all organisms are created beings.

In the foregoing examples of poly-syllogism the conjoined arguments are stated in full, none of the propositions being omitted. But it may be effective sometimes to state the conjoined arguments in enthymematic form. Such abbreviations may be made both in the progressive and the regressive chains of reasoning. When a progressive poly-syllogism is abridged, it is called a *sorites*. When a regressive poly-syllogism is abridged, it is called an *Epicheirema*.

iv. Sorites

A sorites, as has been indicated above, is an abridged progressive chain of reasoning. It is a series of pro-syllogisms and epi-syllogisms in which all the conclusions except the last are suppressed. Since it is a progressive chain of reasoning, the progress of thought is from a pro-syllogism to an epi-syllogism. A sorites may be described as a series of enthymemes progressively arranged.

E.g., All A is B
 All B is C
 All C is D
 All D is E
 ∴ All A is E

This sorites, when expanded, will be seen to consist of three syllogisms:

- (1) All B is C
 All A is B
 ∴ All A is C
- (2) All C is D
 All A is C
 ∴ All A is D
- (3) All D is E
 All A is D
 ∴ All A is E

Two forms of sorites are recognised by logicians, *viz.*, the Aristotelian and the Goalenian:

(a) *Aristotelian Sorites*

The example given above is of the Aristotelian sorites.*

* The name 'Aristotelian' is not apt. Though this form of reasoning is found in Aristotle's writing, he does not discuss the sorites anywhere.

In it the suppressed conclusion of each pro-syllogism forms the minor premise of its corresponding epi-syllogism.

The following is adapted from the *Bhagavad-Gita* (Chapter II, verses 62 & 63) as an illustration of the Aristotelian sorites:—

- A man who is attached to the objects of sense is one in whom desire arises.
- A man in whom desire arises is one who gets angry.
- A man who gets angry is one who is deluded.
- A man who is deluded is one in whom there is loss of recollection.
- A man in whom there is loss of recollection is one who ruins his understanding.
- A man who ruins his understanding is one who perishes.
- ∴ A man who is attached to the objects of sense is one who perishes.

If this long sorites is expanded, it will be found to consist of five syllogisms, each succeeding one taking for its minor premise the conclusion of the preceding syllogism.

(b) *Goclenian Sorites*

Rudolf Goclenius (1547-1628), a professor at Marburg, was the first to call attention to another form of sorites which is just the reverse of the Aristotelian form. In the Goclenian sorites the suppressed conclusion of each pro-syllogism forms the major premise of the corresponding epi-syllogism.

- E.g., All D is E
 All C is D
 All B is C
 All A is B
 ∴ All A is E

This sorites can be expanded into three syllogisms thus:

- (1) All D is E
 All C is D
 ∴ All C is E
- (2) All C is E
 All B is C
 ∴ All B is E
- (3) All B is E
 All A is B
 ∴ All A is E

The two kinds of sorites, it will be seen, differ only in form. The order of premises in the Goclenian sorites is exactly the reverse of that of the Aristotelian sorites. The following points of structural contrast between the two may be noted:—

(1) In the Aristotelian sorites the predicate of each premise is the subject of the next. In the Goclenian sorites the subject of each premise is the predicate of the next.

(2) In the Aristotelian sorites the predicate of the last premise is the major term and subject of the first premise is the minor term. In the Goclenian sorites the predicate of the first premise is the major term and the subject of the last premise is the minor term.

(3) In the Aristotelian sorites the suppressed conclusion of each pro-syllogism forms the minor premise of the corresponding epi-syllogism. In the Goclenian sorites the suppressed conclusion of each pro-syllogism forms the major premise of the corresponding epi-syllogism.

(4) In the Aristotelian sorites the first premise is a minor premise and all other premises are major premises. In the Goclenian sorites the first premise is a major premise and the others are minor premises.*

Rules of sorites:

The following rules hold good when the component syllogisms are in the first figure:

(1) *Only one premise can be negative. If a premise be negative, it must be the last in the Aristotelian form and the first in the Goclenian form.*

Proof: (a) Only one premise can be negative because it is not possible to have more than one negative premise. If a premise be negative, the conclusion of the syllogism of which it is a premise will be negative. This negative conclusion becomes a premise in the next syllogism. If another premise also be negative, then ultimately one of the component syllogisms will contain two negative premises from which no conclusion can be drawn.

(b) If any premise be negative then the last conclusion will be negative. If the last conclusion be negative, its pre-

*The Aristotelian sorites has been called progressive, and the Goclenian sorites regressive.

dicate, the major term will be distributed. To avoid the fallacy of illicit major, the major term must be distributed in its premise where it is the predicate. Hence that premise must be negative.

Now, the premise in which the predicate of the last conclusion appears as the predicate is the last premise in the Aristotelian sorites and the first in the Goclenian form. Therefore only the last premise in the Aristotelian and the first in the Goclenian sorites can be negative.

(2) *Only one premise can be particular. If a premise be particular, it must be the first in the Aristotelian sorites and the last in the Goclenian sorites.*

Proof: (a) It is not possible to have more than one particular premise in a sorites. If more than one premise be particular, the fallacy of two particular premises will result. With one universal premise and another particular premise we get a particular conclusion. If this conclusion be combined with another particular premise, no further conclusion can be reached. Therefore in a sorites there cannot be more than one particular premise.

(b) If any premise be particular, it must be the first in the Aristotelian sorites and the last in the Goclenian sorites.

We stated above that these rules apply only when all the component syllogisms of a sorites are in the first figure. In the Aristotelian sorites all the premises except the first are major premises. One of the special canons of the first figure is that the major premise must be universal. Since in the Aristotelian sorites the first premise alone is the minor premise, it can be particular, and not the others which are all major premises. If any other premise be taken as particular, the fallacy of undistributed middle will result.

In the Goclenian sorites, if any premise be particular, it must be the last premise. If any premise except the last were particular, then the conclusion of the syllogism in which it occurs will be particular. In the Goclenian sorites the conclusion of each syllogism becomes the major premise of the next one. But in the first figure the major premise cannot be particular. Hence in the Goclenian sorites only the last premise can be particular.

So far we have been dealing with forms of sorites whose

component propositions are categorical. Similar chains of reasoning may be had with hypothetical propositions also.

E.g., If A is B, C is D
 If C is D, E is F
 If E is F, G is H
 ∴ If A is B, G is H

The example which we adapted from the *Bhagavad-Gita* may be re-written in the form of a hypothetical sorites perhaps with better advantage, because the component propositions directly signify connections of traits rather than individuals.

If a man be attached to the objects of sense, desire arises in him.

If there be desire, anger arises.

If there be anger, there is delusion.

If there be delusion, there is loss of recollection.

If there be loss of recollection, the understanding is ruined.

If the understanding be ruined, the man perishes.

∴ If a man be attached to the objects of sense, he perishes.

v. *Epicheirema*

An *epicheirema* is a condensed regressive poly-syllogism in which the epi-syllogism is conjoined with a proof of one or both of its premises. If one premise only is proved, the *epicheirema* is called *single*; if both the premises are proved, it is called *double*. The epi-syllogism is fully stated; and the enthymeme which appears in either or both of the premises is of the first or second order.

Single Epicheirma:

Whatever is necessary to health is a duty.

Athletic games are necessary to health, because they are exercise.

∴ Athletic games are duties.

This is a single *epicheirema* because proof is offered for only one of the premises. If the *epicheirema* is expanded, there will be an epi-syllogism and a pro-syllogism, the latter proving the minor premise of the former.

(1) Whatever is necessary to health is a duty.

Athletic games are necessary to health.

Athletic games are duties.

- (2) Exercise is necessary to health.
 Athletic games are exercise.
 \therefore Athletic games are necessary to health.

Double Epicheirema:

What is not in space is indestructible because it is insusceptible of motion.

The soul is what is not in space because it is indissoluble.

\therefore The soul is indestructible.

This is a double epicheirema because both the premises of the epi-syllogism are enthymematically proved. If the epicheirema is expanded, there will be an epi-syllogism and two pro-syllogisms proving the major and minor premises of the former.

- (1) What is not in space is indestructible
 The soul is what is not in space
 \therefore The soul is indestructible.
- (2) What is insusceptible of motion is indestructible
 What is not in space is insusceptible of motion
 \therefore What is not in space is indestructible.
- (3) What is indissoluble is what is not in space
 The soul is indissoluble
 \therefore The soul is what is not in space.

CHAPTER XVI

THE LIMITS OF SYLLOGISTIC REASONING

i. Introduction

In Chapter XI we initiated a study of mediate inference in its typical form, *viz.*, the syllogism. There we defined syllogism, following Jevons, as "the act of thought by which from two given propositions we proceed to a third proposition, the truth of which necessarily follows from the truth of these given propositions." And in the light of this definition we have studied three main kinds of syllogism, *viz.*, the categorical, the hypothetical, and the disjunctive. But, strictly speaking, the hypothetical and the disjunctive arguments are not syllogistic, though to avoid confusion we have followed the traditional usage and called them so. Etymologically, syllogism covers all mediate inference. Aristotle himself defined syllogism as 'a discourse in which certain things being posited, something else than what is posited necessarily

follows on their being true.' But neither he nor his followers have used the term 'syllogism' in such a comprehensive sense. They have restricted its application to arguments "in which from the given relation of two terms, *in the way of subject and predicate*, to the same third term, there follows necessarily a relation, *in the way of subject and predicate*, between those two terms themselves."* In this sense only categorical mediate inferences can be syllogistic, for it is in the categorical proposition that the relation of subject and predicate obtains. If 'mortality' can be predicated of 'all men' and 'being man' of 'Socrates,' then it follows that 'mortality' can be predicated of 'Socrates.' The underlying principle is this: What can be predicated of a predicate can be predicated of the subject. 'Mortal' is a predicate of 'man.' 'Man' is a predicate of 'Socrates.' Therefore 'Mortal' is a predicate of 'Socrates.'

The Dictum of syllogism expresses the same principle. What is true of a whole is true of a part thereof. For instance, in the argument—

All who think lightly of their own deserts are grateful.

Modest men think lightly of their own deserts
Modest men are grateful,

we find the application of the *dictum de omni et nullo*. 'Grateful' is a predicate of 'all who think lightly of their own deserts.' 'Modest men' constitute a portion of 'those who think lightly of their own deserts.' Therefore 'grateful' may be predicated of 'modest men.' Since the categorical arguments alone admit of the application of the Dictum, the term 'syllogism' whose principle it is must refer only to those arguments.

Aristotle believed that the syllogism is the type of all inference, except the immediate. The hypothetical and the disjunctive arguments were considered syllogistic by him because they are reducible to the categorical form. Even among the categorical arguments, those in the first figure are perfect because it is they that are in strict accordance with the *dictum de omni et nullo*. Thus the Aristotelian view is that the syllogism of the first figure is the standard of all

* Joseph: *An Introduction to Logic*, p. 225.

inference. The validity of any argument can be demonstrated only if it is reducible to the syllogistic form.

As against this view, Mill urges that the syllogism is no inference at all. We shall first examine Mill's contention, and then Aristotle's claim on behalf of the syllogism.

ii. Mill's Criticism of the Syllogism

Mill's objection to the syllogism is twofold:

(1) In the first place, the syllogism is devoid of validity because it involves the fallacy of *petitio principii* or begging the question. A reasoning is guilty of this fallacy if its conclusion is assumed as one of its premises. The premises, instead of proving the conclusion, assume it, and so they are said to beg the question. Every syllogistic argument, according to Mill, is a case of begging the question. In the view of Aristotle, the first figure is the perfect figure. The major premise of a valid argument in this figure is a universal proposition. In order to establish the truth of the universal major premise the conclusion is required. But the major premise is used at the same time to prove the conclusion. And so we are begging the question, and our reasoning cannot be valid. Mill illustrates this point by analysing the classic example of syllogism.

All men are mortal

Socrates is a man

∴ Socrates is mortal.

The major premise 'All men are mortal' can be true only if the conclusion 'Socrates is mortal' is true. Hence instead of proving the conclusion, the major premise requires the conclusion for its own validation. That is, the conclusion is assumed in the major premise. Thus says Mill, "it is unanswerably urged by the adversaries of the syllogistic theory, that the proposition, 'Socrates is mortal,' is presupposed in the more general assumption, 'All men are mortal': that we cannot be assured of the mortality of all men, unless we are already sure of the mortality of every individual man: that if it be still doubtful whether Socrates, or any other individual we choose to name, be mortal or not, the same degree of uncertainty must hang over the assertion, 'All men are mortal' that the general principle, instead of being given as an evidence of the particular case, cannot itself be taken for true without

exception, until every shadow of doubt which could affect any case comprised with it, is dispelled by evidence *aliunde*; and then what remains for the syllogism to prove? That, in short, no reasoning from generals to particulars can, as such, prove anything, since from a general principle we cannot infer any particulars, but those which the principle itself assumes as known.”*

(2) Secondly, Mill contends that the syllogism is without value. Any inference to be valuable must give us new knowledge. But the conclusion of a valid syllogism cannot contain anything new. It is merely a restatement of what is already there in the premises. For, if it be otherwise, the syllogism will not be valid. But inference is the instrument of progress in knowledge. The progress of knowledge is determined by its newness or increase. If the conclusion of a syllogism add nothing to our knowledge, if it contain nothing new, of what use is it?

iii. Validity of the Syllogism

Let us now examine Mill's criticisms and see if they are sound.

(1) The charge that the syllogism involves *petitio principii* is based on a wrong view of universals. According to this view, all universals are aggregates of particulars. ‘All’ implies ‘each and every.’ The universal is reached through an examination of particular cases. Hence it is a collective or enumerative universal. Mill himself was an advocate of this view. He says, ‘All inference is from particulars to particulars: general propositions are merely registers of such inferences already made, and short formulae for making more: the major premise of a syllogism, consequently, is a formula of this description: and the conclusion is not drawn *from* the formula, but an inference drawn *according to* the formula: the real logical antecedent, or premise, being the particular facts from which the general proposition was collected by induction.”† That is, we observe several cases that resemble in some respect, and summarise our results in the form of a

* *System of Logic*, bk. ii, ch. iii, sec. 2.

† *System of Logic*, bk. ii, ch. iii, sec. 4. The method of enumerative induction will be explained and criticised in a later chapter.

universal statement. We notice, for instance, a hundred crows being black and make a universal statement 'All crows are black.' But this cannot serve as a universal major *premise* from which we can infer for certain the black colour of an unobserved crow. When we argue,

All crows are black

A is a crow

∴ A is black

A is either an observed crow or an unobserved one. If it is an observed crow, then its truth is implied or assumed in the major premise. If it is an unobserved crow, then the conclusion is doubtful, and it makes the truth of the major premise also doubtful. And so Mill concludes that there can be no genuine inference from universals to particulars. All attempts of inferring from universals are, in his view, bound to involve the fallacy of *petitio principii*. If Mill's view of universals be correct and if all universals be aggregates of particulars, then the syllogism cannot escape the charge that is levelled against it.

Unfortunately, some of the doctrines of traditional logic have themselves contributed to an enumerative view of universals. The class-inclusion view and the doctrine of the distribution of terms are based on an extensional interpretation of universals. The *dictum de omni et nullo* also suggests an enumerative view. We ourselves interpreted the Dictum in such a way,* for otherwise many of the exercises of traditional logic would be impossible.

But there is another sense in which universals may be understood and the Dictum be interpreted. Let us recall the formulation of the Dictum in an earlier chapter: whatever is affirmed or denied of a whole may be affirmed or denied of a part thereof. But what is the meaning of the term *whole*? If the *whole* be an extensional totality, then its universality will be only an aggregation of particulars, and the syllogism in which it becomes the major premise will be a begging of the question. But if the *whole* be a system of inter-related elements, a character rather than a class, then its universality will be genuine. The proposition 'All B is A' may mean either 'All the B's are A' or 'B as such is A.' In the former

* See page 82.

case the universal is collective, in the latter it is generic or genuine. If, in the syllogism,

All B is A
C is B
∴ C is A

the major premise means 'All the B's are A,' then as it could not have been arrived at if C which is a B had not been enumerated already, the conclusion is assumed in the major premise and the syllogism involves *petitio principii*. But if the major premise means 'B as such is A,' what is here asserted is an inter-connection of B and A, and that does not depend on an observation of every B. Most of the universals with which we deal are of this latter kind. To make the universal statement 'All water is H₂O' it is not necessary that we should have analysed every quantity of water. If it is to be made after an exhaustive analysis of all water, it can never be made. Similarly the universal 'All men are mortal' is not the result of an enumeration of all men being mortal. Even if it were possible to count up all the men that are dead and gone what about those who are living and are yet to be born. If 'All men are mortal' were an enumerative universal, how can any one make such a statement so long as he lives? And so, these are not collective, but generic universals. It is the word 'all' that misleads us, and makes us think that the universal is an aggregate of particulars. Properly speaking, the form of the universal should be 'Man is mortal,' 'Water is H₂O.' These universals mean respectively that mortality is an essential attribute of man, and that hydrogen and oxygen in the proportion of 2 : 1 are the constituents of water. If the major premises of syllogisms be such generic universals, then the charge of *petitio principii* cannot be levelled against them. In the classical syllogism, if, instead of having Socrates who is dead as the minor term, we have some living person, say, Mr. X, it will be easily seen that the syllogism is not a begging of the question.

All men are mortal
Mr. X is a man
∴ Mr. X is mortal

Here the conclusion is not required to prove the major premise. The universal major is not made after observing the

mortality of Mr. X. On the contrary, the mortality of Mr. X is inferred from his being a man and from the mortal nature of man as such.

Thus it will be evident that Mill's first criticism does not hold good against syllogisms which have as their major premises genuine universals.

As for the so-called collective universals, they are not universals at all. If after examining each and every book on my shelf and finding them all to deal with philosophy, I make the statement 'All the books in my shelf are on philosophy.' I am not really framing a universal proposition. In spite of the presence of the word 'all,' the proposition is only a collection of several particular propositions. It is as good as saying 'A is a book on philosophy,' 'B is a book on philosophy' and so on.* It was not in this way that Aristotle understood the major premise of the syllogism. For him the major premise is not an enumerative universal, but a generic universal. And so Mill's criticism is out of court.

(2) As for the second criticism that the syllogism is valueless because it gives nothing new in its conclusion it may be pointed out that if Mill's condition of novelty, in the sense in which he understands it, be accepted, there can be no inference at all. In all valid inference the conclusion must, in a sense, be contained in the premises. Otherwise it is not possible to base the conclusion on the premises. If the premises are to lead to a conclusion, the conclusion must be present implicitly in the premises. At the same time, it is also true that the conclusion must, in some sense, go beyond the premises. If there be nothing new in the conclusion then there would be no reasoning but only a vain reiteration of what we already know. This is what has been called 'the paradox of inference'. Inference is either useless or invalid; for if its conclusion does not contain something not given in the premises, the inference is useless; and if its conclusion does contain anything not given in the premises, the inference is invalid. Now this dilemma is fallacious as most dilemmas are. It is true that the conclusion is contained in the premises, but it is contained in them not taken separately but

* Though formal logic characterises the individual judgment as universal, it is philosophically unsound to regard the two as the same.

considered together. The conclusion is what is made explicit from out of a real union of the premises. In this sense it gives something new, and is not a mere re-assertion of either of the premises.

Thackeray's story of the priest affords a good illustration of the nature of inference.* "An old Abbe, talking among a party of intimate friends, happened to say, 'A priest has strange experiences; why, ladies, my first penitent was a murderer.' Upon this, the principal nobleman of the neighbourhood enters the room. 'Ah, Abbe, here you are; do you know, ladies. I was the Abbe's first penitent, and I promise you my confession astonished him!'" Here the inference is that the nobleman was a murderer; and this was certainly a piece of new knowledge for the Abbe's listeners. Yet it is contained in the premises put together.

Moreover, Mill overlooks the fact that in an inference the element of necessity is more important than that of novelty.† It is no doubt true that the conclusion, must be new in the sense that it must not be contained in either of the premises separately. But Mill in his enthusiasm for the 'novel' was blind to the characteristic of necessity. If the conclusion be not necessary, then any two premises may lead to any conclusion. Inference itself would be impossible, if the conclusion be not grounded in the premise. Thus Mill's criticism of the syllogism does not stand to reason; and it must be admitted that the syllogism is a valid form of mediate inference.

iv. Non-Syllogistic Reasoning

Aristotle claimed on behalf of the syllogism that it is the type of all mediate inference. Arguments which are not syllogistic in form should be reduced to a mood in the first figure in order that their validity may be demonstrated. Only then their principle which is the same as the principle of the syllogism, viz., *the dictum de omni et nullo*, will be clearly manifest. Thus, according to Aristotelian logic, the syllogism is the pattern of all mediate inference and the Dictum is the

* Quoted by Bosanquet in his *Essentials of Logic*, pp. 140 & 141.

† See Bosanquet: *Essentials of Logic*, p. 138. I ought to warn you at once that though we may have novelty in the conclusion of Inference (as in multiplication of large numbers), the necessity is more essential than the novelty.

principle of all reasoning. It believes that, only when syllogistically expressed, arguments become deductive or demonstrative.

In the last section we saw the syllogism is a valid form of a mediate inference. But should we go further and accept the Aristotelian view that there is no mediate inference which is non-syllogistic or cannot be expressed syllogistically? In other words, do all arguments proceed on the principle of the syllogism? Or, are there reasonings which employ principles other than and coordinate with the Dictum? In short, is there mediate inference which is non-syllogistic?

There are mediate inferences which are valid and yet cannot be expressed as syllogisms. They have been called relational arguments. We use them in our daily thought and discourse. They abound also in the exact science. We shall notice some of them and show how they cannot be reduced to strict syllogistic form.

$$\begin{aligned}(1) & A = B \\ & B = C \\ \therefore & A = C\end{aligned}$$

This is a valid deductive argument based on the axiom of equals. We cannot regard it as a syllogism because it contains four terms. B is not the middle term, as the predicate of the major premise is not B but 'equal to B.'

The same difficulty will be found in the following arguments which, if expressed in syllogistic form, will be guilty of the fallacy of four terms.

$$\begin{aligned}(2) & A \text{ is greater than } B \\ & B \text{ is greater than } C \\ \therefore & A \text{ is greater than } C.\end{aligned}$$

This argument is based on the principle governing relations of degrees of quantity.

$$\begin{aligned}(3) & A \text{ graduated two years before } B \\ & B \text{ graduated three years before } C \\ \therefore & A \text{ graduated five years before } C.\end{aligned}$$

This is a relational argument based on the nature of time.

$$\begin{aligned}(4) & \text{Bangalore is to the west of Madras} \\ & \text{Mysore is to the south of Bangalore} \\ \therefore & \text{Mysore is to the south-west of Madras.}\end{aligned}$$

This argument proceeds on the nature of space.

(5) Rama is the son of Dasaratha

Rama is the husband of Sita

∴ Dasaratha is the father-in-law of Sita.

This argument involves the principle governing family relationships.

(6) Most rude men repent

Most rude men act unthinkingly

∴ Some men who act unthinkingly are those who repent.

This is a valid argument; and yet if reduced to syllogistic form, its middle term 'rude men' will be undistributed since the logical value of 'most' is 'some' and the middle term is subject in both the premises.

(7) You grant that it is not right to kill animals and use them for food.

To kill and use them in sacrifices is less needful than to kill them for food.

How much more, then, should you condemn animal sacrifices?

This is an *a fortiori* argument. The Latin phrase '*a fortiori*' means 'from yet firmer ground.' It is used in introducing a statement which, provided a previous statement is accepted as true, must be still more readily accepted. If killing animals for food is admitted to be wrong, then it follows by a stronger reason (*a fortiori*) that animal sacrifice is wrong.

It is needless to multiply such instances of arguments which are non-syllogistic in character. The traditional logicians were not oblivious of these arguments. They tried to express them in syllogistic form and thought that they had proved their thesis that the syllogism is the standard of all mediate inference. But we shall show, taking the first example given above, how futile such an attempt is.

The first relational argument we gave was the one based on the axiom of equals:

$$A = B$$

$$B = C$$

$$\therefore A = C$$

The traditional logic regards this as an enthymeme of the first order with the major premise suppressed. Put into the syllogistic form, the argument would then be:

Things equal to the same thing are equal to one another

A and C are things equal to the same thing

∴ A and C are equal to one another.

As a preliminary objection it may be pointed out that B has altogether disappeared from the argument in this new shape. To remedy this defect, it may be suggested that instead of having for our major premise the axiom of equals 'Things equal to the same thing are equal to one another,' we may have one of its expressions, *viz.*, 'Things equal to B are equal to one another.' Then the argument would stand thus:

Things equal to B are equal to one another

A and C are things equal to B

∴ A and C are equal to one another.

Even thus stated, the argument is not only clumsy and confusive but also not syllogistic in character. *First*, the truth of the axiom which is given as the major premise is itself recognised only in the instances. We recognise its truth because we see that if A and C are both equal to B, they are equal to one another. Therefore the axiom cannot be one of the premises from which we reason. *Secondly*, the proposition 'A and C are things equal to B' is not a single premise but a compound of two premises, *viz.*, 'A is equal to B' and 'C is equal to B.' And so, there is no minor premise. *Thirdly*, there is no minor term either. 'A and C' cannot be the minor term, for A and C are two subjects.

All these difficulties are the result of regarding the relational argument as a syllogism. In the syllogism, a universal law or rule is applied to an instance falling within it. Hence there is the distinction of major premise and minor premise. It is not so in a relational inference. $A = B$ and $B = C$ are the two premises from which the conclusion ' $A = C$ ' is drawn. But neither of them is the major premise and neither is the minor premise.

Why the axiom cannot be the major premise we have shown already. If the axiom of equals be treated as the major premise of the argument we have been examining, then the *dictum de omni et nullo* will have to be regarded as the major premise of all syllogistic arguments. Let us take the

stock example of the syllogism:

All men are mortal

Socrates is a man

∴ Socrates is mortal.

Having the relevant particular expression of the Dictum as the major premise, we should re-write the argument as follows:

What can be predicated of man (which is a distributed term) can be predicated of

Socrates (which is a term falling under it)

Mortality can be predicated of man

∴ Mortality can be predicated of Socrates.

No votary of the traditional logic will agree to this form of the syllogism. The Dictum is the principle *according to* which the syllogism proceeds; and so it cannot be a premise *from* which the conclusion is drawn. Similarly, the axiom of equals is the principle according to which, from the premises ' $A = B$ ' and ' $B = C$ ' the conclusion ' $A = C$ ' is drawn. Thus it should be evident by now that to regard the relational arguments as syllogism contributes neither to clarity of thought nor to precision of expression. The principles that underlie the different relational arguments are coordinate with and not subordinate to the Dictum which is the principle of the syllogism. The syllogism employs the relation of predication, the relation of subject and predicate; and this relation can easily be expressed by the present tense of the verb 'to be.' In the typical syllogism the major premise predicates a characteristic of a kind or universal; the minor premise asserts that a particular individual (or species) is included (or not included) in the kind or universal; the conclusion asserts that the characteristic does (or does not) belong to the individual (or species). The principle of the syllogism, as we have already seen, is: what can be predicated of a whole can be predicated of its part. The relational inferences are based on other principles. This also we have shown. The relation is expressed not by a simple 'is' or 'are' but by such phrases as 'is equal to,' 'is greater than,' 'is the father-in-law of,' etc. Like the syllogism, each of the relational inferences also implies a system. But the nature of the system differs according to the data considered. The syllogism implies the

system of universals which are realised in particulars. The argument ' $A = B$; $B = C$; therefore $A = C$ ' implies the system of quantitative relations. Thus the relational inferences are different from the syllogism, and it is impossible to reduce them to syllogistic forms.

v. Syllogism and Deduction

The arguments which we considered in the last section are based on relations other than those of predication. And so we called them non-syllogistic reasonings. They are, no doubt, mediate inferences. But they are not syllogisms, nor deductive, in the strict sense of the term. Deductive arguments proceed from an explicit universal or system. The relational inferences do not thus proceed *from* a universal, though they proceed *according to* a universal. Hence they are not cases of deduction.

The traditional logic identified the syllogism with deduction. Even that is wrong. While it is true that all syllogism is deduction, it is not true that all deduction is syllogism. In deduction we apply an explicit universal or principle to a particular case or cases. In the syllogism also we do this, but under a particular form. The system which underlies the syllogism is an easy and simple one. If an individual is found to be a member of a kind or class, then we can infer that it has all the characteristics of the kind. But through the syllogism we cannot know the differentiating features of the individual case. Similarly, we can infer syllogistically that a particular fact comes under the operation of a universal law, but in what manner and under what conditions we cannot say. "E.g., we can syllogistically infer from the law of gravitation expressed in the form, 'All material bodies gravitate,' that a particular body which we know to be material gravitates but not the velocity with which or the direction in which it will do so."* The method by which we determine the velocity and direction of a gravitating body is deductive but not syllogistic. Such deductions are based, like the relational inferences, on the inter-relation of elements in a system; whereas the syllogism stresses the repetition of the class-nature in the individuals.

That the syllogism is not the whole of deduction will be

* Latta and Macbeath: *The Elements of Logic*, pp. 236 and 237.

evident also when we recall to our mind the nature of the hypothetical and the disjunctive arguments in relation to the categorical syllogism. We saw in Chapter XIV that, though it is possible to reduce conditional reasonings to categorical form, the meaning does not remain the same but gets altered because while the categorical syllogism represents the lower stages of thought, the hypothetical and the disjunctive inferences represent successively higher stages. The Dictum is not the principle of conditional reasonings; and so they cannot be regarded as syllogisms. In the syllogism (*i.e.*, the categorical inference), the relation between a subject and its predicate is established by means of a middle term. But in the conditional reasonings this is not the relation that is established. It is true that like the syllogisms, hypothetical and disjunctive reasonings proceed from an explicit universal or system; but the kind of system is different. As we have stated already, the syllogism emphasises the repetition of a character in the individuals that constitute a class, whereas conditional reasonings are based on the inter-connection of elements in a system.

We do admit that the syllogism is a valid form of mediate inference. Mill was wrong in condemning it. But, at the same time, we do not concede the claim of the traditional logic that the syllogism is the form of all mediate inference. Not only is it not all mediate inference; but also it is not the whole of deduction.

CHAPTER XVII

FALLACIES OF DEDUCTIVE REASONING

i. Introduction

The function of logic, we have seen, is to formulate the principles of valid thinking. Thinking proceeds in the form of inference. Inference may be immediate or mediate. There is only a difference of degree between the two. What is called judgment is also inference; it is inference *in posse*. And what is known as inference is judgment which has become conscious of its grounds. In the preceding chapters we have studied some of the forms of inference which are used for the demonstration of truths, as also the principles that govern these:

forms. Our reasoning will be correct if it is in accordance with these principles, and wrong if it violates them. Though the task of logic is to exhibit the principles of valid reasoning, this will be accomplished in an eminent way if an account of fallacies is also given. A study of the wrong forms shows up by contrast the nature of the right forms. And so we shall give, in this chapter, a descriptive account of the important types of fallacies in deductive reasoning—the mode of reasoning with which we were mainly concerned so far.

ii. Classification of Fallacies

A fallacy is a wrong or unsound inference. An inference is wrong or unsound if it ignores or transgresses the principles of reasoning. As there are innumerable ways in which the principles of valid thought may be violated, the fallacies are numberless. Joseph rightly remarks, "Truth may have its norms, but error is infinite in its aberrations, and they cannot be digested in any classification."* An argument may fail to be conclusive in many ways, in which case it would have to be classified under several heads of fallacies. There may be arguments which are so absurd that they do not fit into any type of fallacies but would have to be characterised simply as 'inconclusive.' Besides, each science has its own fallacies; and the deduction of these would require a knowledge of the subject-matter. Thus, as the ways of erroneous thinking are numerous, it is not possible to enumerate all the fallacies. What can be attempted is to give the main kinds or types of errors that commonly beset our thinking. Even in this anything like an exhaustive classification cannot be achieved.

There have been several attempts at classifying fallacies ever since the time of Aristotle. In the last book of his *Topics*, called the *Sophistici Elenchi*, Aristotle divides fallacies by dichotomy, into those which arise out of ambiguity of language and those which are not the result of such ambiguity. Under the first class which are called fallacies in *dictione*, Aristotle enumerates six. They are: Equivocation, Amphiboly, Composition, Division, Accent and Figure of Speech.

* *An Introduction to Logic*, p. 528. See also De Morgan: *Formal Logic*, p. 237. 'There is no such thing as a classification of the ways in which men may arrive at an error; it is much to be doubted whether there ever can be.'

The other class of errors are called fallacies *extra dictionem*. They have nothing in common except that they are not due to any ambiguity or looseness in the use of language. According to Aristotle, there are seven types of fallacies *extra dictionem*. They are: Accident, *Secundum Quid*, *Ignoratio Elenchi*, *Petio principii*, *Non Causa pro Causa*, Consequent, and Many Questions.*

Following in the main the terminology of Aristotle, later writers have introduced other classifications. Whately, for instance, divides fallacies into *logical* and *material*, and subdivides logical fallacies into *purely logical* and *semilogical*. Joseph and Latta and Macbeath adopt Aristotle's scheme. Creighton and Smart divide fallacies first into *errors of interpretation* and *fallacies in reasoning*, again divide fallacies in reasoning into *formal* and *material*, and further divide material fallacies into *fallacies of equivocation* and *fallacies of presumption*. Errors on interpretation arise through wrongly understanding the premises that constitute the data of inference. Properly speaking, they are not logical fallacies. Yet we shall study them because, if the premises are not correctly interpreted, our conclusions would go wrong. Fallacies in reasoning may be either formal or material. Formal fallacies are violations of the rules governing the various forms of inference. Material fallacies do not refer to the form of reasoning but to the content. These may arise either out of the ambiguity and looseness of the language used (equivocation) or out of unwarranted assumptions made by the one who reasons (presumption).

iii. Errors of Interpretation

(1) *Illogical Education*: In Chapter X we studied the different forms of immediate inference or education. If the rules of obversion and conversion are not observed, the education will be illogical. In obversion the logical contradictory of the predicate should be substituted for the original predicate. If, instead, the contrary is substituted, the obverse will be fallacious. Thus, it will be wrong to infer from the proposition 'Honesty is always a good policy' that 'Dishonesty is always a bad policy.' We are not interested here in the truth

*Joseph thinks that the fallacy of Many Questions might perhaps be referred more naturally to the other group. See p. 534.

‘आपत्कल्पो ब्राह्मणस्याब्राह्मणाद्विद्योपयोगोऽनुगमनं शुश्रूषा
समाप्ते ब्राह्मणो गुरुः’

इति । विद्याग्रहणमन्येषां रत्नप्रभृतीनामुपलक्षणार्थम् । अत एव
देवलः—

रत्नान्यापः स्त्रियो विद्या धर्मश्शौचं सुभाषितम् ।
विविधानि च शिल्पानि समादेयानि सर्वतः ॥

मनुरपि—

श्रद्धानशुभां विद्यामादधीतावरादापि ।
अत्यापदि परं धर्मं स्त्रीरत्नं दुष्कुलादापि ॥
विषादप्यमृतं ग्राह्यं बालादापि सुभाषितम् ।
अमित्रादापि सदृत्तममेध्यादापि काञ्चनम् ॥

तथा विद्याफलमपि स एवाह—

तपो विद्याऽपि विप्रस्य निश्श्रेयसकरं परम् ।
तपसा कल्मषं हन्ति विद्ययाऽमृतमश्नुते ॥

देवलोऽपि—

विद्या वित्तं तपश्चेति त्रीणि तेजांसि देहिनः ।
इह चामुत्र च श्रेयस्तदेतैस्साध्यते त्रिभिः ॥
विद्यया निर्मलं ज्योतिर्वित्तत्यागात्सुखोदयम् ।
तपसा विमलां भूतिं प्राप्नुयान्मानवान्निभिः ॥

इति । विद्यादानस्य फलमाह यमः—

regards Greek words. But in English there are no words which change their meanings when differently accented. And so, modern writers apply the name 'fallacy of accent' to the misconception that arises when the accent or emphasis is placed on the wrong word in a sentence. Thus, in the sentence, 'And he spake to his sons, saying, Saddle me the ass; and they saddled *him*', if the emphasis is placed on the last word, the meaning would be quite different from what is intended by the sentence. The commandment, 'Thou shalt not bear false witness against thy neighbour,' may be made by a slight emphasis of the voice on the last word *neighbour* to imply that we are at liberty to bear false witness against other persons.* The fallacy of accent is committed also when an author is not correctly quoted by omitting relevant passages or by italicizing words which are not italicized in the original.

iv. Formal Fallacies

Formal fallacies result from violations of the syllogistic rules. We have considered these already in our treatment of the forms of syllogism. We shall, however, recapitulate them here and give fresh illustrations.

(1) *Quaternio Terminorum*. The first rule of the syllogism states that 'a syllogism must contain three, and only three, terms.' A violation of this rule results in the fallacy of four terms or *quaternio terminorum*:

Cultured men are reasonable

Logicians are hair-splitters

∴ Logicians are reasonable.

The absurdity of this argument is so patent that no one would be misled by it. In fact, it is no argument at all, not even a semblance of an argument in spite of the 'therefore.' In some arguments, however, there may be an apparent middle term, but used in two different senses. These two are cases of fallacious syllogisms each having four terms. The fallacy has a special name, *viz.*, the *ambiguous middle*: *e.g.*,

All *criminal actions* ought to be punished by law

Prosecutions for theft are *criminal actions*

∴ Prosecutions for theft ought to be punished by law.

The ambiguity may lie in the use of the other terms, *viz.*, the

* See Jevons: *Lessons in Logic*, p. 174.

major and the minor. Even, then, the reasoning will be guilty of four terms.*

(2) Undistributed Middle: The fallacy of undistributed middle arises if the third rule of the syllogism, *viz.*, that the middle term must be distributed in one, at least, of the premises, is not observed. The reason for the rule and an example of the fallacy we have already given.† Here is another example:—

All Punjabis are Indians
All Bengalis are Indians
∴ All Bengalis are Punjabis.

The argument is fallacious because the middle term 'Indians' is not distributed even once. The middle term should be such that it relates the minor and the major terms; and this it will not be able to do if it is undistributed in both the premises.

(3) Illicit Major: This fallacy is committed when the major term is distributed in the conclusion without being distributed in the major premise. The fourth rule of the syllogism is that no term must be distributed in the conclusion which is not distributed in the premise. If the major term is distributed in the conclusion and not in the major premise, it means that we are inferring the more from the less, which is illogical. *E.g.*—

All cats are *mammals*
No dogs are cats
∴ No dogs are *mammals*.

(4) Illicit Minor: This is also an offence against the fourth rule. It occurs in an argument in which the minor term is distributed in the conclusion without being distributed in the minor premise. *E.g.*—

All generous people are loved by the poor
All generous people are *polite*
∴ All *polite* people are loved by the poor.

(5) Negative Premises: The fifth rule of the syllogism states that from two negative premises there can be no conclusion. A violation of this rule involves the fallacy of two negative premises. *E.g.*—

* See p. 78.

† See p. 79.

Anger is not good

Calmness is not anger.

From these we cannot infer whether calmness is or is not good.

(6) *Particular Premises*: The seventh rule is that from two particular premises there can be no conclusion. An argument which seeks to draw a conclusion from two particular premises is fallacious. It will be found that such an argument goes against one or more of the first six rules of the syllogism. *E.g.*—

Some radio broadcasts are false propaganda

Some radio broadcasts are educative

∴ Some educative broadcasts are false propaganda.
Here the middle term is undistributed, and hence the conclusion does not follow from the premises.

(7) *Denying the Antecedent*: This is a fallacy in hypothetical reasoning. Since in a hypothetical proposition the antecedent is only one of the conditions, we cannot say that because it is absent the consequent also must be absent. Hence the rule: Affirm the antecedent. Instead, if the antecedent is denied, the argument will be fallacious. *E.g.*—

If the study of logic furnished the mind with a multitude of useful facts, like other sciences, it would deserve to be cultivated;

But it does not furnish the mind with a multitude of useful facts;

Therefore it does not deserve cultivation.*

Here the reasoning is incorrect because denying the antecedent cannot lead to a denial of the consequent. The acquiring of a multitude of useful facts is not the only reason for undertaking the study of a science. The development of a critical attitude of mind is equally a good reason for recommending the study of logic.

(8) *Affirming the Consequent*: This is the other fallacy in hypothetical reasoning which occurs when the minor premise affirms the consequent. *E.g.*—

If there is rain, he will not go out;

He has not gone out;

∴ There is rain.

Here again, the antecedent is only one of the reasons and not

* See S. H. Mellone: *Elements of Modern Logic*, p. 163.

the sole reason. Rain is a cause of the person not going out; it is not the only cause; for he may not go out on account of other reasons like illness or preoccupation at home.

The fallacies of denying the antecedent and affirming the consequent result, as we have shown above, when condition and consequent are regarded as convertible. Aristotle refers to these fallacies by the term 'fallacy of the consequent.'

(9) *Improper Disjunction*: A disjunctive argument is fallacious when the alternatives are not mutually exclusive and totally exhaustive. *E.g.*—

Either Adams or Leverrier discovered the planet Neptune.

Leverrier discovered the planet

∴ Adams did not.

Here the disjunction is not properly made. For, there is nothing against two persons either jointly or independently being responsible for a discovery.

v. Material Fallacies

An argument may be true to form and yet invalid. Then the fallacy will be found to rest in the *matter* or *content* of the argument. It is not possible to detail all the circumstances which may give rise to material fallacies. It is not within the province of logic either to describe the faults that are peculiar to the different sciences and branches of learning. But there are two basic principles of logical reasoning by violating which an argument may go wrong in respect of its content. The first principle is that the terms used in an argument should be unambiguous and well-defined; and the second is that what is to be proved or the *probandum* should be strictly derived from the premises without being presupposed or presumed. If the first principle be violated, we have the fallacies of *equivocation*. If the second be transgressed, we have the fallacies of *presumption*.

(A) Fallacies of Equivocation

(1) *Ambiguous and Shifting Terms*: Under this head come all fallacies which are due to the ambiguous use of terms. Aristotle gave the name 'aequivocatio' or 'equivocation' to these. Instances of ambiguous and shifting terms we have already had in the fallacies of ambiguous middle, major

and minor.* The meanings of words may be changed because of a failure in defining them. De Morgan gives the example of a French word which was wrongly understood by an English sailor. 'The French,' said the sailor, 'call a cabbage a *shoe*; the fools! why can't they call it a cabbage, when they must know it is one?' Puns and quibbles are good examples of the fallacy of ambiguous and shifting terms.

No cat has two tails

Every cat has one more tail than no cat

∴ Every cat has three tails

He who is most hungry eats most

He who eats least is most hungry

∴ He who eats least eats most.

Cold can be expelled by heat

Govinda's illness is a cold

∴ Govinda's illness can be expelled by heat

It is unnecessary to multiply examples. Words are very good servants if they do the bidding of reason. But easily they become the masters to the ruin of logical thinking. Bacon says, "Men believe that their reason rules over words: but it is also the case that words react, and in their turn use their influence on the intellect."[†] A good reasoner, therefore, ought to be vigilant and should not accept words without examining their *bona fides*.

(2) *Fallacy of Figure of Speech*: This has nothing to do with the figures of speech in literature. To think that the two are the same is itself an instance of the fallacy of figure of speech. The term is applied by Aristotle to the error which might arise from supposing that words which are similar in form or derived from the same root are similar in meaning. Thus, to say that *important* has a negative meaning, because *imperturbable* or *impenitent* has a negative meaning, is a fallacy of figure of speech. Mill's argument in support of his hedonistic thesis is a good example of this fallacy. He says, "The only proof capable of being given that an object is visible, is that people actually see it. The

* We have stated that an argument may be fallacious in several ways and that the classification of fallacies does not give exclusive kinds.

† *Novum Organum*, I, 59.

only proof that a sound is audible, is that people hear it. In like manner, I apprehend, the sole evidence it is possible to produce that anything is desirable, is that people do actually desire it." Here Mill's mistake consists in thinking that *desirable* has a meaning similar to that of *visible* or *audible* because it has the same suffix. But really there is a difference. 'Visible' means 'able to be seen,' and 'audible' 'able to be heard,' while 'desirable' does not mean 'what can be desired' but 'what ought to be desired.' The fallacy of figure of speech occurs quite often through the use of paronymous terms. As Dr. Davis observes, "These have by no means similar meanings, e.g.—'Artist, artisan, artful'; 'Pity and pitiful'; 'Presume and presumption'; 'Project and projector'; what is 'imaginary' is unreal, but an 'image' formed of wood or stone is real; to 'apprehend' is to lay hold on, or to come to a knowledge of, while 'apprehension' often signifies fear or dread.*

(3) *Fallacy of Composition*: According to Aristotle, an argument in which words which should be taken separately are taken together, is guilty of the fallacy of composition; e.g. 'Platinum and iron are rare and useful metals; therefore platinum is a rare and useful metal, and iron is a rare and useful metal.' Here the words 'rare' and 'useful' should be taken separately, the former with 'platinum' and the latter with 'iron.' But, instead, they are combined and tagged on to both 'platinum' and 'iron.'

Modern logicians think that the fallacy of composition is not merely verbal, but that it consists in passing from a statement about a class of things distributively to the same statement about the whole considered collectively. In arguments that are vitiated by this fallacy a term is first used in a divided or distributive sense (*in sensu diviso*) and further on in a composite or collective sense (*in sensu composito*). The word 'all' facilitates this fallacy, for it has both distributive and collective signification. *E.g.*—

All the angles of a triangle are less than two right angles

A, B and C are all the angles of this triangle

∴ A, B and C are less than two right angles

Here the word 'all' is used in the major premise in the

* *Theory of Thought*, p. 270.

distributive sense, and in the minor in the collective sense. Mill's argument in support of utilitarianism is an interesting example of the fallacy of composition. He says, "No reason can be given why the general happiness is desirable except that each person, as far as he believes it to be attainable, desires his own happiness. This, however, being a fact, we have not only all the proof which the case admits of but all which it is possible to require, that happiness is a good: that each person's happiness is a good to that person, and that general happiness is, therefore, a good to the aggregate of all persons."* The substance of Mill's argument is that because each desires the happiness of each, all desire the happiness of all. A desires A's happiness; B desires B's happiness; C desires C's happiness; therefore A desires the happiness of $A + B + C$: is so does B; and so does C. This is a glaring case of composition. We add a few more examples of this fallacy: 'Every member of a jury is liable to be mistaken; therefore we can place no confidence in trial by jury'; 'A regiment of a hundred men is composed of soldiers who are all six feet high: therefore the whole regiment is six hundred feet high'; the spendthrift says, 'I can afford this luxury or that or the other; therefore I shall be able to afford them all.'

(4) *Fallacy of Division*: This is the converse of the fallacy of composition. Aristotle applies the term to the error which results from taking separately words which ought to be taken together; e.g. 'Hindus and Muslims are men and brethren; therefore Hindus are men and Muslims are brethren.' Modern writers on logic say that the fallacy of division consists in passing from a statement about a class considered collectively to the same statement about each and every member of the class taken distributively. It would be a fallacy of division if an uncharitable man were to argue, 'I cannot afford to help A and B and C and all who come to beg; so I am not obliged to help any of them.' The following are some examples of the fallacy of division: 'All the plays of Kalidasa cannot be enacted in a day; the *Sakuntala* is a play of Kalidasa; therefore the *Sakuntala* cannot be enacted in a day'; 'Eight is an even number; five and three are eight; therefore five and three are even number'; 'Those

* Utilitarianism, p. 53.

who contributed to the Andrews Memorial Fund gave five lakhs of rupees; A contributed to the Fund: therefore A gave five lakhs of rupees.'

(5) *Fallacy of Accident*: The fallacy of accident, according to Aristotle, consists in arguing that what can be predicated of the accidental qualities or relations of a thing can be predicated of the thing itself and *vice versa*. *E.g.*—'This dog is yours; this dog is a father; therefore this dog is your father.' The relation to the dog is an accident of yours; being a father is an accident of the dog: but it does not follow that the dog's being a father is related to you. Hence the argument is fallacious.

Modern writers on logic do not follow the usage of Aristotle in this respect. They identify the fallacy of accident with the fallacy of *secundum quid* which has two forms called by these writers (a) the direct or simple fallacy of accident, and (b) the converse fallacy of accident.

The direct fallacy of Accident: This consists in arguing that what is true as a general rule is true also under special circumstances. The following is an example of this fallacy: 'What is bought in the market is eaten; raw meat is bought in the market; therefore raw meat is eaten.' The logicians expressed this fallacy in the formula, *a dicto simpliciter ad dictum secundum quid*, which means that the first statement is true simply but the second needs qualification. It is generally true that what is bought in the market is eaten; but meat is eaten not in the condition in which it is when it is bought (*i.e.*, raw) but after undergoing some modification (*i.e.*, after being cooked). Welton and Monahan quote the following story from *The Decameron* as an amusing example of the direct fallacy of *secundum quid*: "A servant who was roasting a stork for his master was prevailed upon by his sweetheart to cut off a leg for her to eat. When the bird came upon the table, the master desired to know what had become of the other leg. The man answered that storks never had more than one leg. The master, very angry, but determined to strike his servant dumb before he punished him, took him next day into the fields where they saw some storks, standing each on one leg, as storks do. The servant turned triumphantly to his master; on which the latter shouted, and

the birds put down their other legs and flew away. 'Ah, sir,' said the servant, 'you did not shout to the stork at dinner yesterday: if you had done so, he would have shown his other leg too.'"*

(b) *The converse fallacy of accident*: As its very name implies, this is the converse of the direct fallacy of accident. It consists in arguing that what is true under a special circumstance is true also normally or generally. The ancient logicians expressed this fallacy in the formula: *a dicto secundum quid ad dictum simpliciter*, which means that the first statement is a qualified one, whereas the second is a general statement. *E.g.*—'When a person is unwell, staying in bed is good for his health, therefore staying in bed is always good'; 'Spirituuous liquors are of value in certain cases of disease; therefore, they must be beneficial to a person who is well.'

(6) *Dilemmatic Fallacy*: As we have seen, dilemmatic arguments are often fallacious. The compound hypothetical major premise generally does not contain a real connection of antecedent and consequent and the disjunction in the minor premise is not exhaustive of all the possible alternatives. And so dilemmas are ordinarily defective. The ways of meeting them we have studied already.

(B) Fallacies of Presumption

(1) *Petitio Principii*: We met with this fallacy in our discussion of Mill's charge against the validity of the syllogism. *Petitio Principii* or begging the question consists in assuming the conclusion in the premises or in proving the conclusion by premises which can only be proved by the conclusion itself. *E.g.*—"Virtue is right; to give to beggars is a virtue; therefore to give to beggars is right." Here the conclusion is only a restatement of the minor premise; and the major premise is a tautology. To call charity a virtue and to call it right are the same. And so, to say that to give to beggars is a virtue is not to *prove* that it is right.

In some arguments the conclusion may not be a restatement of one of the premises, but it may be based on premises which themselves stand in need of the conclusion for their proof. This particular form of the *petitio* is called arguing

* See James Welton and A. J. Monahan: *Intermediate Logic*, pp. 61 and 62.

in a circle or *circulus in probando*. De Morgan gives the following as an example of reasoning in a circle: "Porcelain says to Crockery, 'I am better than you.' 'I don't see that,' says Crockery. 'Why,' says Porcelain, 'am I not made of better clay?' 'How so?' says Crockery. 'Why,' returns Porcelain, 'my clay makes Porcelain, and yours only makes Crockery.'" Joseph gives another example: "If I argued that early Teutonic societies were originally held together by kinship, because all societies were so held together originally, I might be accused of arguing in a circle; for the major premise, it might be said, is only arrived at by enumeration; early Teutonic societies have to be examined in order to show that it is true."*

Another sub-form of the *petitio* is called *hysteron proteron* where the question is begged in a single step of inference. *E.g.*—'Opium induces sleep because it has a soporific quality'; 'The volume of a body diminishes when it is cooled, because the molecules then become closer.'

(2) *Complex Question*: This which is also known as the fallacy of many questions is committed when several questions are so combined in one that a simple answer in the form of 'yes' or 'no' involves the assumption of something more than what the respondent really means. *E.g.*—'Have you stopped beating your mother—yes or no?' 'Have you given up telling lies? 'Do the people in your part of the country still carry arms?' Charles II asked the members of the Royal Society, 'Why does a live fish, when placed in a bowl full of water, not cause the water to overflow, whereas a dead fish does?' The members of that learned Society offered various ingenuous explanations of the difference which in reality does not exist. The 'leading questions' used by lawyers are generally of the nature of complex questions; they are intended for misleading the witnesses in cross-examination.

(3) *Ignoratio Elenchi*: This is a very common fallacy especially among disputants. In the heat of debate what is sought to be established may be forgotten and arguments may be advanced which have no relevance to the point at issue. Or a clever debator may take advantage of the weak-

* An Introduction to Logic, p. 552.

ness of his opponent and put up a show of conclusive argumentation while in fact he is only deluding his opponent. All these are cases of *ignoratio elenchi*. The phrase means "ignoring the conclusion to be established or refuted." An *elenchus* is a syllogism with a conclusion contradictory of the thesis to be refuted. In order to refute a thesis, one must establish the contradictory thereof. If this is ignored and something else be proved, there is the fallacy of *ignoratio elenchi*. The English name for this fallacy is 'irrelevant conclusion,' i.e. establishing a conclusion which is irrelevant. Thus, e.g., to argue that 'philosophy bakes no bread' would be irrelevant and beside the mark. The advocates of a liberal education do not claim that philosophy is of direct use in practical life. Like so many other branches of study, they urge, philosophy broadens the mind; and what is more, it blesses its votary with a synoptic view of life and existence.' The opponent of philosophy ought to disestablish this character of philosophic knowledge. Instead, if he says, 'Philosophy bakes no bread,' his assertion would be an *ignoratio elenchi*.

There are several ways in which an irrelevant conclusion may be established. Some of them have acquired special names. We shall notice a few of them.

(a) *Argumentum ad hominem*: This is arguing about the person instead of about the proposition which he puts forward. *Argumentum ad hominem* is very common in political controversy. Recriminations and charges of inconsistency are usual with the disputants. The type of fallacy which these involve is also called by the name *tu quoque* which means 'you're another.' Thus if we say, 'He has no right to ask us to offer violent resistance to the aggressor, because he was formerly an advocate of all-out non-violence,' we are guilty of *argumentum ad hominem*.

(b) *Argumentum ad populum*: This consists in appealing to the passions and prejudices of people. The proper mode of establishing a thesis is to show its reasonableness. This has to be done by convincing others, by making their reason give its consent. But very often in controversies the passions are roused, and support is given to a move or opposition is organised on the basis of irrational impulses. A special form

‘कार्तिकी फाल्गुन्याषाढी पौर्णमासी । तिस्रोऽष्टका-
त्रिरात्रम्’

इति । उक्तपौर्णमासीरारभ्य त्रिरात्रम् । तथैव तिस्रोऽष्टकास्सप्त-
म्यादयस्तास्वपि त्रिरात्रमनध्ययनमित्यर्थः । एवं महानवम्या-
दिष्वपि द्रष्टव्यम् । तथा च पुराणम्—

महानवम्यां द्वादश्यां भरण्यां च महातिथौ ।
तथाऽक्षयतृतीयायां शिष्यान्नाध्यापययेद्बुधः ॥
माघमासस्य सप्तम्यां रथाख्यायां च वर्जयेत् ।
अध्यापनमथाभ्यक्तस्नानकाले च वर्जयेत् ॥
नीयमानं शवं दृष्ट्वा महीस्थं वा द्विजोत्तमः ।
अकालगार्जितादौ च परस्वा¹ शौचकादिषु ॥
अनध्यायं बुधः कुर्यादुपरागादिकेऽपि च² ।

महानवम्याश्वयुजशुक्लनवमी । महाद्वादशी कार्तिकशुक्लद्वादशी ।
महाभरणी प्रौष्ठपद्यनन्तरा³ । अक्षयतृतीया वैशाखशुक्लतृतीया ।
रथाख्या माघशुक्लसप्तमी । नारदीयेऽपि—

अयने विषुवे चैव शयने बोधने हरेः ।
अनध्यायस्तु कर्तव्यो मन्वादिषु युगादिषु ॥

पुराणान्तरेऽपि—

युगादिषु च सर्वेषु तथा मन्वन्तरादिषु ।
अनध्यायं प्रकुर्वीत या च सोपपदा तिथिः ॥

¹ सरस्वा—क.

² परागदिने तथा—ख.

³ पितृपक्षान्तवर्तिनी—ग.

that which ought to have been proved is, that there are more or stronger objections against the receiving than the non-receiving of it." This fallacy is most commonly committed by anti-innovators who are interested in raising objections against an innovation and do not weigh the *pros* and *cons* of the question.

(f) *Argumentum ad baculum*: This is an appeal to the 'big stick.' It is made by a person who imposes his views on others through threats. "To knock a man down when he differs from you in opinion may prove your strength, but hardly your logic."*

(4) *Non-Sequitur*: This is otherwise known as the fallacy of false cause, *non causa pro causa*. It is committed whenever the conclusion does not follow from the premises. E.g.—'It is ridiculous to suppose that the world can be flat; for a flat world would be infinite, and an infinite world could not be circumnavigated, as this has been.' Here what is sought to be disproved is that the world is flat; and a *reductio ad absurdum* proof is offered. 'If the world were flat, it would be infinite; if it were infinite, it could not be circumnavigated. But it is circumnavigated; and so it cannot be flat.' In this argument the supposition on which the conclusion is based is fallacious. It is supposed that if the world were flat, it could not be circumnavigated. This is wrong. It is true that if the world were infinite, it could not be circumnavigated. But if the world were flat and finite, circumnavigation is possible. And so, the argument contains the fallacy of *non sequitur*.

There is an inductive fallacy corresponding to *non sequitur*. It is called *post hoc ergo propter hoc*, and consists in mistaking sequence for consequence. This fallacy will be explained later in the chapter on fallacies of Inductive reasoning.

* Stock: Deductive Logic, p. 313.

PART TWO

INDUCTIVE LOGIC

CHAPTER XVIII

THE PROBLEM OF INDUCTION

i. Introduction

The systematic nature of reality is the basis of all inference. If the universe consisted of elements which had no interconnection whatever, knowledge and inference would be impossible. It is by virtue of the interconnectedness of things that we pass in knowledge from one element to another. Thus, it is the whole system of reality that is the ground of inference. In the language of Hegel, the great German philosopher, we repeat, the truth is the whole.* The whole which is the real is not a heap, but a system; and it consists of various sub-systems. Humanity, for instance, is a sub-system which is realised in all the individual men. And because mortality is an adjunct of human nature, we infer in respect of any individual man that he is mortal. This inference, then, is made possible because of the system, *viz.* humanity. When we reason—

Man is mortal

Socrates is a man

∴ Socrates is mortal,

we predicate mortality of Socrates on the ground that he is a member of the system called humanity and mortality is an integral content of human nature. All the inferences which we have studied in the first part of this book proceed on the basis of system. In some of them the system is explicitly stated; in some others it is not expressed because it is so obvious. In all of them the procedure, however, is the same. The principle of deductive inference is the application of a universal or system to any member thereof. But this presupposes a process by which the universal or system is established. In the present part of our book we shall study this process which is called induction.

ii. What is Induction?

Induction is the method by which universals are discovered. Deductive reasoning, we have seen, consists in

* See page 83.

applying a universal to an instance coming under it. Given a universal, we proceed to draw a conclusion regarding the nature of an individual that is an instance of the universal. The universal therefore, is the soul of deductive reasoning. That is why one of the rules of syllogism states that at least one premise must be universal. And in the first figure which is the perfect figure, according to Aristotle, the major premise must be universal. But how is this universal premise obtained? Deductive reasoning does not tell us how it is got. On the contrary, we have to assume the truth of the premises and then draw the conclusion that they warrant. The only aim in deductive logic is to see if the conclusion is *consistent* with the premises. Deduction cannot guarantee the *material truth* of the inference. If the premises are true, the conclusion will be true, provided it is drawn *from* them. But if the premises are false, the conclusion too will be false, even though it be consistent with them.

Let us illustrate this point. The argument—

All men are mortal

All dictators are men

∴ All dictators are mortal

is a syllogism in the mood *Barbara*, and is valid both formally and materially. It is formally valid because *Barbara* is a valid mood and is in accordance with the rules of syllogism. It is materially valid because the premises are true. That all men are mortal and that the dictators are men are propositions which cannot be contradicted. Suppose we change the major premise and have the argument thus:—

All men are immortal

All dictators are men

∴ All dictators are immortal.

This is also syllogism in *Barbara*, valid by all the canons of syllogistic reasoning. And yet the conclusion is wrong because the major premise is wrong. But how to know the truth or falsity of the premise? Deductive logic does not tell us how. Hence deduction has been called formal logic or the logic of consistency.

It may be said that the premise of a syllogism may be proved by another syllogism. In a chain of reasoning we have a connected series of arguments where each pro-syllo-

gism proves a premise of its epi-syllogism. But it will be easily seen that this is purely a formal process leading us nowhere; and it will leave us ultimately with universal propositions which it cannot prove. To arrive at universals we must employ some other process; and this is called induction. The problem of induction, then, is this: 'How are the premises of deductive reasoning established?' Since induction provides the material truth of inference, it has been called material logic or the logic of truth.

The problem of induction may be stated also in another way. The aim of science is to explain individual facts that are experienced. To explain a fact is to relate it to the law or laws by which it is governed or to the system of which it is a member. The falling of bodies towards the earth and the revolving of the planets around the sun are observed phenomena. They are explained in terms of the law of gravitation. Thus it is the laws or systems that make the individual facts intelligible. The purpose of science is to discover these laws. Just as a fact needs to be explained in the light of a law, a particular law may itself require to be explained. This is done by discovering the wider law of which it is an aspect. Thus the adventure of science progresses from facts to laws and from narrower systems to wider systems. How is this adventure conducted? What are the methods adopted for discovering universal laws? Science generalises from particular facts which are experienced. The process of generalisation is called induction. From this point of view, inductive logic may be described as the logic of the scientific method.

It is not the task of inductive logic to give us an account of the various laws that are discovered. There are the different sciences to deal with the respective laws. Biology, for instance, has to study the laws that govern the phenomena of life. Astronomy has to tell us about the system of heavenly bodies. But whatever be the science, there is a common method of generalisation. It is this method that is the subject of study in inductive logic.

Induction, then, is the process by which facts are *induced* to disclose the universal law that connects them while deduction is the process of reasoning which *deduces* from the system or universal law the character of a member of the system

or a fact coming under the law. Such is the distinction usually made between induction and deduction. But the truth is, the two are not independent processes of thought; they are but aspects of one and the same process of reasoning. This we shall explain a little later.

iii. Inductive Syllogism

In the Middle Ages logic became abstract and formal, and was mostly confined to the syllogism. Bacon (1561-1626) raised the banner of revolt against the logic of the Scholastics and taught what he called the new logic consisting of the inductive methods. But induction was not so new after all. Aristotle, the Father of Western logic, discusses the method of induction. He recognises this as a process which has to precede deduction. In deduction a principle or universal is applied to individual cases. The principle itself is not obtained through deduction. It is discovered by an examination of particular instances wherein it is realised.

Aristotle makes a distinction between the logical order and the order of experience. In the logical order, the general principle or universal is prior to the particular facts which are given in sense-experience. In the order of experience, the particular facts come first. The syllogism, which to Aristotle is the type of deductive inference, is expressive of the logical order, where the reasoning is from the principle to facts. But in experience we do not meet with principles; we came across only particular facts. It is from these that general principles are discovered. The method of discovery is induction.

The procedure of induction, then, is the reverse of the procedure of the ordinary syllogism. In the syllogism we go from the universal to the particular. In induction our thought progresses from the particular to the universal. According to Aristotle, induction begins with the *infima species*, and proves that what is true of each species is true also of the genus. We shall explain this by taking the example given by Aristotle himself.

Man, horse, mule, etc. are long-lived

Man, horse, mule, etc. are gall-less

∴ All gall-less animals are long-lived.

The characteristic of long life is found in each of the species of gall-less animals, like man, horse, mule, etc.;

अन्तर्गतशवे ग्रामे वृषलस्य च सन्निधौ ।

अनध्यायो रुद्यमाने समवाये जनस्य च ॥

वसिष्ठोऽपि—

‘सन्ध्यास्वन्तश्शवदिवाकीर्त्तिं’

इति । दिवाकीर्त्त्यश्चण्डालः । अन्तश्चण्डाले ग्रामे नाधीयीतेत्यर्थः ।

एतच्चतुष्पथादिष्वपि द्रष्टव्यम् । तथा च प्रचेताः—

‘चतु^१ष्पथमहापथरथ्यासूद्यानेषु न देवसमीपे’

अधीयीतेति शेषः । गौतमोऽपि—

‘श्मशानग्रामान्तमहापथाशौचेषु’

इति । श्मशाने तु विशेषमहापस्तम्बः—

‘श्मशाने सर्वतश्शम्याप्रासात् ग्रामेणाधचवसिते

क्षेत्रेण वा नानधचायः । श्मशानवच्छूद्रपातितौ ।

समानागार इत्येके । शूद्रायां तु प्रेक्षणप्रतिप्रेक्षणयो-

रेवानधचायः’

इति । तथाऽश्वारूढोऽपि नाधीयीत । तथा च मनुः—

नाधीयीताश्वमारूढो नवृक्षं च न हस्तिनम् ।

न नावं न खरं नोष्ट्रं नेरिणस्थो न यानगः ॥

शयानः प्रौढपादश्च कृत्वा चैवावसक्थिकाम् ।

नाधीयीतामिषं जग्ध्वा सूतकान्नाद्यमेव च ॥

them, we have 'perfect induction.' We count, for instance, the number of days in each month of the year, and make the general statement that all months of the year contain less than thirty-two days each. We examine the fruits in a basket and say that all the fruits are mangoes. These are cases of perfect induction, according to the Scholastics. Jevons, among the modern writers, accepts this view. "An induction . . . is called perfect," he says, "when all of the possible cases or instances to which the conclusions can refer have been examined and enumerated in the premises." 'Perfection' here means completeness of counting. There is absolute certainty about the conclusion. In the words of Jevons, "Perfect Induction . . . gives a necessary or certain conclusion."

The view of Jevons and of the Scholastics has been criticised by most of the writers on scientific induction. The business of induction, they say, is not to count instances. Counting, however complete, cannot give an explanation of facts. By enumerating the days of each month we may know *that* all months of the year contain less than thirty-two days each, but not *how* or *why*. The conclusion gives nothing more than the premises; it is not more general than they are. In spite of the 'all,' it is not universal. 'All months of the year contain less than thirty-two days each' is only a summary statement of the twelve individual propositions relating to the twelve months: 'January contains less than thirty-two days; February contains less than thirty-two days, etc.' In this sort of summary induction, there is "no real inference, no march of information, no addition to our knowledge." It is "a mere shorthand registration of facts known." And so, the conclusion which is reached through the so-called perfect induction is a general proposition only in appearance. The method of complete enumeration is not induction; nor has it anything like perfection. Moreover, this method can be applied only in cases where there is a limited totality. It is impossible, for instance, to count all men and make a generalisation about them. Especially in the sciences where a law has to cover all possible cases, past, present, and future, there can be no exhaustive counting. Thus, the use of the method of 'perfect induction' is restricted to a very narrow sphere; and even here it does not explain but merely records and summarises.

(2) *Imperfect Induction*: The Scholastics called the method of incomplete counting by the name 'imperfect induction.' It is also known as induction by simple enumeration. Where it is not possible to count all the instances, what we do is this: we count some of them and make a general statement covering not only the observed cases but also the unobserved ones. We count several crows, notice their black colour and frame the general proposition, 'All crows are black.' We observe quinine being administered to certain patients suffering from malaria and then generalise 'Quinine cures malaria.'

Here, it will be noticed, some of the defects of the 'perfect induction' are not present. The method of simple enumeration is not restricted to the spheres of limited totalities. The conclusion is truly general in that it is meant to cover all the possible instances. From an observation of a certain characteristic in some instances we infer that that characteristic will be found in the unobserved instances also. This is called the inductive leap or hazard. We make a leap from the known to the unknown cases. Hence there is a march of information, new knowledge.

Yet simple enumeration is not the method of induction that is employed by science. Counting can give a *that* and not a *how*. It cannot explain facts. By enumerating several crows we can only know *that* crows are black and not *how* or *why* they are black. Enumeration can reveal only a conjunction of qualities and not their connection. And so, there is no necessity about the generalisations made through simple counting; and consequently the generalisations are not truly universal. There is always the possibility of a contradictory instance. A white crow is not inconceivable. The conclusion obtained through simple enumeration is only probable and never certain. We can only say 'All crows are probably black' and not 'All crows must necessarily be black.' As Bacon remarks, "Induction which proceeds by merely citing instances is a childish affair (*respeurilis*), and being without any principle of inference, it may be overthrown by a contradictory instance."

We are not, however, saying that enumeration has no value in induction. Usually, it starts the inductive procedure, and makes us aware of conjunctions of attributes on the basis of which we construct explanatory theories. The place and

function of enumeration we shall show in a later chapter. What we would urge here is that simple enumeration is not the method of scientific induction, though sometimes it is an aid to it.

v. Parity of Reasoning

Under the heading 'Inductions improperly so-called', Mill includes, besides perfect induction, two other processes called 'parity of reasoning' and 'colligation of facts.' Induction by parity of reasoning is the process of establishing the truth of a general proposition on the ground that the same reasoning which proves a particular case will also prove every other similar case covered by the general proposition. The angles of a particular triangle, say, ABC, are known to be equal to two right angles. On the basis of this knowledge we say that all triangles have their angles as equal to two right angles. The principle of inference here is that the method of proof in the case of one triangle is the same as that in any other. This is called induction by parity of reasoning.

Mill characterises this method as 'an induction improperly so-called' because the essential of induction, *viz.*, observation of facts, is not present here. Geometrical reasoning does not depend on observation or sense-experience. ABC is not a particular triangle. It is a symbol of triangle *as such*. In fact, a real triangle can never be drawn, for a line should have length alone and no breadth, but the lines that we draw have breadth. The diagrams that are used in Geometry serve the purpose of illustration in order that the reasoning may be easily understood. As Carveth Read observes, "Diagrams are not used as facts of observation, but merely to fix our attention in following the general argument." Hence parity of reasoning is no induction. On the contrary, it is purely a deductive process. The characteristic of possessing angles which together are equal to two right angles is derived from the nature of the triangle as such which is a three-sided rectilinear figure.

vi. Colligation of Facts

Colligation of facts is making a mental union of facts observed by means of a general conception. Kepler observed the planet Mars in different positions in its orbit and brought those positions together under the general conception of an ellipse. Mill defines colligation as "that mental opera-

tion which enables us to bring a number of actually observed phenomena under a description; or which enables us to sum up a number of details in a single proposition."

According to Mill, colligation of facts is not induction proper. He thinks that it gives us merely a summary of the various observations, like the so-called perfect induction. Colligation is descriptive of facts; it does not explain them. It is true that this mental process involves great strain, as in the case of Kepler's discovery of the orbit of Mars. Still it is not a complete inductive method. It does not give us any means for testing the general conceptions which it yields.

vii. Scientific Induction

We have considered above certain methods of generalisation which are called, but are not, inductions. What, then, is the proper method of induction? The definition of induction we have already given. Induction is a process of reasoning whereby generalisations are made on the ground of the observed particulars. True generalisations or universals, we have said, cannot be reached through mere counting of instances. The scientific method insists on the analysis of the instances observed. It makes a generalisation not from the *number* of instances but from a knowledge of their *nature*. If a number of instances are examined sometimes, it is only to facilitate the process of analysis and to help in excluding irrelevant circumstances and defining what is relevant. Thus induction, as it is employed in science, is the method of analysing the nature of facts for the purpose of understanding their governing principle or law.

Four stages may be distinguished in the complete inductive method: (1) Observation and analysis of facts; (2) Formulation of a hypothesis; (3) Deduction of consequences from the hypothesis framed and verification; and (4) Proof.

(1) Reflection starts when facts are observed which require to be explained. Thus induction begins with the observation of facts. There is no use in merely counting these facts. They must be analysed and their nature determined. Observation, therefore, is not a passive process. As Mill says, "The observer is not he who merely sees the thing which is

before his eyes, but who sees what parts that thing is composed of".

(2) The next stage in induction is the formulation of a hypothesis. A hypothesis is a tentative suggestion or guess that is put forward for explaining the facts observed. It is the result of contemplating the facts. The investigator does not approach facts with an empty mind. His own past experience and the knowledge he had already gathered help him in framing a hypothesis which might possibly explain the facts.

(3) The third stage is mainly a deductive process. It consists of two steps. First, assuming the hypothesis to be true the investigator *deduces* in his own mind what consequences should have followed. The reasoning is of the form: if the hypothesis were true such and such facts should have occurred. Then, the inquirer proceeds to observe again the actual facts. If these facts correspond to the mentally deduced consequences, then the hypothesis is verified. If not that hypothesis is to be either altered or given up in favour of a more satisfactory one in the light of fresh observation, and the other stages have to be repeated.

(4) The verification of a hypothesis is not the same as its proof. By verification we only show that the hypothesis explains the facts, but not that it is the only explanation. To prove that the hypothesis is the *sole* explanation, it is not enough that we show that the deduced consequences and the observed facts tally; we should do much more, and conclusively demonstrate that no other hypothesis can explain the facts. This is extremely difficult and is seldom accomplished in science.

Sometimes two or more rival hypotheses may be suggested at the same time. When there are such alternative conceptions, equally plausible, the investigator searches for a crucial instance or devices a crucial experiment which will disqualify the hypotheses which are not true and point the way to the right explanation. A crucial instance or experiment is a pointer in the right direction even as the cross placed at the meeting of roads helps the traveller to choose his path.

It has been said by some writers that the process of induction might be represented in the form of a disjunctive argument:

P is either H_1 , H_2 , or H_3

It is not H_1 or H_2

\therefore It is H_3

Here P is the phenomenon to be explained, and H_1 , H_2 , and H_3 are the three possible hypotheses. In the minor premise H_1 and H_2 are rejected because they do not square with the facts. The conclusion, therefore, establishes that H_3 is the explanation of P. It is true that the process of reasoning in the inductive method may be put into the form of a disjunctive argument. But the inductive method is not so formal as the argument would make it out to be. We do not start with all the alternative hypotheses disjunctively set forth. Usually we begin with one; and if it proves to be unsatisfactory we frame another; and this second hypothesis might be either the old one in a modified form or a new one in the formulation of which the old hypothesis has had much to do. Thus in induction we do not rely on empty forms. Yet it must be admitted that the inductive method involves the disjunctive principle, *viz.*, the basis of a system.

Now we have shown that the method of induction is the complete process of explaining facts in terms of the law that governs them. It is not peculiar to science alone. Even in ordinary thought-life we make use of it. The only difference is a difference in degree as between the exactness and necessity achieved by the ordinary man in his thought-processes and those achieved by the scientist in his investigations.

viii. Deduction and Induction

Now that the nature of the inductive method has been explained, it will be easy to understand the relation between deduction and induction. In section *ii* of this chapter we said that induction is the process by which facts are *induced* to disclose the universal law that connects them, while deduction is the logical process of *deducing* from the system or universal law the character of a member of the system or an instance of the universal; and we also warned that the distinction between the two should not be mistaken for a difference. But very often a contrast is drawn between deduction and induction; and they are treated as though they were entirely different and opposite processes. Bacon describes induction as an ascending process and deduction as a descending pro-

cess. That is, in induction we mount up from the particulars to the universal whereas in deduction we get down from the universal to the particulars. Jevons regards induction as the inverse operation of deduction. Fowler thinks that induction proceeds from effects to causes, and deduction from causes to effects. These ways of expressing the relation between deduction and induction are not satisfactory.

The term 'induction' may be understood in two senses. It may mean the process of observing facts and making a generalisation about them. Or, it may signify the entire scientific method from the observation of facts to the verification (and proof, where possible) of the suggested theory. In the former sense, a contrast may be drawn between deduction which starts with a universal and applies it to a particular case and induction which starts with the particulars and proceeds to frame a universal. But the task of science does not end with the suggestion of a universal, or what we have called, the formulation of a hypothesis. The scientific method will be incomplete if the hypothesis is not tested and verified; and this process of verification, as we saw in the last section, is deductive in character. And so, induction in the second of the senses given above, which is the proper sense, includes deduction.

The nature of inference consists in seeing things as members of a system. This is found both in deduction and induction. In deductive reasoning we infer something about a particular case when we know the system to which it belongs. In induction we arrive at a universal or system through an analysis of the instances which are expressions of the system. Without deduction, the conclusion of an inductive reasoning will not be universal and necessary; without induction, deductive reasoning will not have content wherewith to proceed. Deduction without induction is empty; induction without deduction is blind. The only distinction between the two is as regards the starting point and mode of procedure. Deduction starts with a universal and proceeds to see if it is realised in a particular case. Induction starts with the particular cases and constructs the system of which they are members. The result, however, is the same, *viz.*, an insight into the interconnectedness of things.

ध्यानविशेषैराग्रेयैरहरहराग्रिमिन्धेतामन्त्राय गच्छेद्वा-
हृत्य निवेदयेत् '

इति । एवं कुर्वतः फलमाह मनुः—

आ समाप्तेशरीरस्य यस्तु शुश्रूषते गुरुम् ।

स गच्छत्यञ्जसा विप्रो ब्रह्मणस्सद्यः शाश्वतम् ॥

याज्ञवल्क्योऽपि—

अनेन विधिना देहं साधयन्विजितेन्द्रियः ।

ब्रह्मलोकमवाप्नोति न चेहाजायते पुनः ॥

यमोऽपि—

आ निपाताच्छरीरस्य ये चरन्त्यूर्ध्वरेतसः ।

ते यान्ति ब्रह्मणस्स्थानं जायन्ते न पुनर्भुवि ॥

यत्तु हारीतेनोक्तम्—

मृत्खोः परस्ता¹ दमृता भवन्ति

ये ब्राह्मणा ब्रह्मचर्यं चरन्ति ।

इति, तद्ब्रह्मविनैष्ठिकविषयं,

‘सर्वे ते पुण्यलोका भवन्ति ब्रह्मसंस्थोऽमृतत्वमेति’

इति श्रुतेः । सर्व एते चत्वार आश्रमिणः कर्म कुर्वाणाः पुण्यलो-
का भवन्ति । यः पुनरेषां मध्ये ब्रह्मसंस्थो ब्रह्मनिष्ठस्सोऽमृतत्व-
मपुनरावृत्तिलक्षणं फलमेति । एतच्च नैष्ठिकत्वं कुब्जादीनां
नित्यमित्यहं विष्णुः—

and understanding would be terms without meaning. To understand a fact is to apprehend it in its place in a system. It is the system or law that renders a thing intelligible. If there were no system, if the universe were not a system of interconnected things, then we should despair of understanding anything. Therefore, induction postulates that the world is an organic unity, that it is a cosmos and not a chaos, a universe and not a multiverse. The things that constitute the world are not an aggregate of unrelated particulars. "All speech vanishes altogether," says Plato, "if each thing be severed from everything else." Each thing is so organically connected with the rest of the universe that if we could know its nature in full we would know the whole universe. Tennyson expresses this truth in the lines:

Flower in the crannied wall,
I pluck you out of the crannies,
I hold you here, root and all, in my hand,
Little flower—but if I could understand,
What you are, root and all, and all in all,
I should know what God and man is.

Even a tiny flower will reveal to us the nature of the entire universe, if we could but understand it in full. This is because the universe is a system, identical in and through the changes that take place within it according to laws determined by its own nature.

This principle or postulate of the systematic nature of the universe has been expressed in several ways, of which two are important. They are: (1) the law of Universal Causation, and (2) the principle of the Uniformity of Nature.

ii. The Law of Universal Causation

One way of expressing that the universe is systematic is to say that everything which has a beginning must have a cause. Anything that is produced must have a parent. We may not know the cause of a particular event. But that does not mean that that event has no cause. Out of nothing, nothing comes *Ex nihilo nihil fit*. As Bain observes, "No change arises out of ~~of~~ vacuum or stillness; there must be a prior event, change or movement, as the *sine qua non* of any new event." There can be no uncaused event. Whatever be the event—the flight of an aeroplane, the booming of guns, the broadcast

over the air, pestilence or panic among a people—it must have a set of conditions without which it will not occur, and given which it needs must occur. This is known as the law of Universal Causation.

iii. The Uniformity of Nature

Another way of giving expression to the systematic character of the universe is to say that nature is uniform that what is once true is always true. This principle is also known as the reign of law. The law of Universal Causation says that every event must have a cause. The principle of uniformity adds, the same event must have the same cause. What happens at one time is a guide to what happens at other times. This principle means that nature is consistent and not chaotic, intelligible and not meaningless. The phenomena that constitute nature are not an unco-ordinated lot occurring or existing independently of one another. As Bradley observes, "Objects have ragged edges which imply other existences from which they have been torn and without which they do not exist."* Everything is related to a system as an element therein or to a law as an instance thereof.

The principle of the Uniformity of Nature is wider than the law of Universal Causation. All things in the world are not related as causes and effects. There are other relations like those of co-existence, whole and part, system and member, and so on. All these also come under the principle of Uniformity. A chemical whole and its constituents, an organism and its members, the foundation of a house and the superstructure, and the gravitational system are instances where the wider relation obtains. The causal relation involves an element of succession. Though there is no time-interval between a cause and its effect, we think of the latter as succeeding the former. This feature of succession is absent from many systems which consequently cannot be regarded as causal. But these also, as stated above, are covered by the principle of Uniformity which postulates that the universe is self-consistent and orderly.

There are some logicians who think that the law of Causation does not imply the principle of Uniformity, that is to say that every event has a cause is not the same as saying that the

* **Appearance and Reality**, p. 176.

same cause produces always the same effect. This view is wrong. Though the principle of Uniformity of Nature is wider in its scope than the law of Universal Causation, it is implied in the latter. A cause which is not uniform in its productive capacity is no cause. If like conditions do not produce like consequences, there is no meaning in saying that one thing is the cause of another thing. If to-day fire were to burn and to-morrow cool, then we cannot say, of which fire is the cause. The very terms 'causation' and 'production' would lose their significance; and all changes in the universe would have to be considered fortuitous. As Joseph writes, "Flint and steel may produce seed instead of a spark, and oil raise the waves or quench a conflagration." But in Nature "we do not expect to gather grapes of thorns, or figs of thistles; . . . If the same seed produced now one plant, and now another, there is no sense in saying that one thing was the cause of another."* And so, causation, without uniformity is void of meaning. To postulate causation is to postulate uniformity of connection. To say that every event has a cause is to say that every event has the same cause. Otherwise there would be a violation of the law of Identity. "To say that the same thing acting on the same thing under the same conditions may yet produce a different effect, is to say that a thing need not be what it is."† And to say so would be a blatant contradiction. Hence it is implied in the conception of cause that the cause is uniform in its activity.

iv. Can the Postulate be proved?

It has been held by empiricists like Mill that the postulate of induction is itself the result of an induction *from* experience. We observe A and B connected together several times and conclude that their connection is universal. We also experience other such uniformities also and then draw the conclusion that Nature is uniform. Thus the postulate of induction, *viz.*, that like conditions have like consequents, is an induction from experience; it is a generalisation reached through simple enumeration. "From instances which we have observed, we feel warranted in concluding that what we have found true in these instances holds in all similar ones, past,

* **An Introduction to Logic**, p. 374.

† Joseph: **An Introduction to Logic**, p. 378.

present and future.”* When such evidence of uniformities accumulates, we frame the general principle of the Uniformity of Nature. This is the view of Mill and other empiricists.

The empiricist view, however, will not stand examination.† In the first place, it involves a *petitio principii*. Because A has constantly come along with B, C with D, E with F, and so on, the empiricist argues, for everything there is a similar condition from which it invariably follows. But this argument begs the question. A might have been found constantly with B, C with D, and E with F. But unless we *assume* that the unseen will resemble the seen, we have no right to argue that M will be found with N, O with P, and so on. Thus in order to prove the principle of Uniformity the empiricist has to assume the principle. He assumes the principle at every stage. He finds A and B together and concludes that even in the future they will be together. That they will be together in the future is not certainly a knowledge derived from his limited experience, for the future is not within his present experience. That knowledge is based on the faith that the future will resemble the present, that is to say, it is grounded on the principle of Uniformity. Having observed particular, limited uniformities, the empiricist uses them as a guide to general uniformity. Here again he begs the question. That all other facts will be related in the same way as those which we have observed is a faith, an assumption; it is not proved by the limited uniformities of our experience.

The argument of the empiricist is defective also in another way. It says that we in our limited experience meet with uniformities and infer therefrom the uniformity of Nature in general. But is it true that we are presented with uniformities alone? Do we not experience disharmonies also? As Laird observes, “There can be no doubt whatsoever that, while much in our experience suggests regularity of sequence, much suggests irregularity also. Unsupported apples fall regularly to the ground; but some sparks fly upwards. The seasons come and go in a stately, inevitable succession, but the wind

* Mill, *Logic*, Bk. II, ch. iii, sec. 3.

† For a detailed criticism of the empiricist view, see Brand Blandshard, *The Nature of Thought*, Vol. II, ch. xxviii.

(even now) appears to blow where it lists. The *prima facie* evidence is therefore conflicting."* If we look for orderliness behind the apparently chaotic facts, it is because of a faith in the systematic character of Nature. And so, this faith itself cannot be the result of experience.

The principle of Uniformity is a postulate and cannot be proved because, like the laws of thought, it is the basis of all proof. It cannot be derived from anything because there is nothing more certain in science and thought than the principle of Uniformity. The only proof of the postulate, if proof it may be called, is negative. That is, if one refuses to accept the principle, he loses all claims to think. If the principle of Uniformity be not assumed, then all attempts at intelligibility would be futile, and thought would commit suicide. If experience is to become intelligible, *i.e.*, if there is to be the possibility of knowledge, one must accept the principle of Uniformity as a fundamental postulate of thought. It is not the task of induction to prove uniformity. It cannot do it. Assuming general uniformity in nature, induction proceeds to discover particular uniformities. Hence the principle of Uniformity has been called *the ultimate major premise of all induction*.

v. The Postulate and the Laws of Thought

In Chapter VIII we gave an account of the laws of thought. We discussed the implications of the laws of Identity, Non-contradiction, Excluded Middle, and Sufficient Reason, and said that all of them are but different ways of representing the demand of intelligence for understanding reality. The postulate of induction also expresses the same principle. Only, while the laws of thought are more formal, the postulate of induction has a more direct bearing on the content of the universe.

* Knowledge, Belief, and Opinion, p. 434.

CHAPTER XX

CAUSE

i. Introduction

In the last chapter we saw that induction assumes the course of nature to be uniform; it postulates that every event has a cause, that like conditions produce like consequences. Making this assumption, the sciences proceed to discover the connections between phenomena, the causes of effects and the effects of causes. Before we go on to consider the methods adopted by the sciences for discovering causal connections, we shall have to understand the implications of the causal relation. This we shall do in the present chapter.

ii. Primitive Notion of Cause

The idea of causality, it is thought, arose in man from his consciousness of his own voluntary action. He wills to lift his arm, for instance, and his arm rises. The lifting of the arm, then, is caused by his will. The pre-historic man understood causality only in this way. Will, according to him, was the only cause of phenomena. Motion and change without life were inconceivable to him. Natural phenomena were regarded as the effects of the volition of beings analogous to man. Indra wields the thunderbolt; Varuna sends down showers.* This is known as the anthropomorphic view which consists in representing physical forces, etc., in human form. Sometimes the primitive man invested the natural phenomena themselves with life. This is called the animistic view. It lingers still in our language as *e.g.*, when we say 'the sun rises,' 'the wind blows,' etc. In these stages, then, volition was considered the sole efficient cause of all phenomena. Natural events were sought to be explained in terms of supernatural agencies.

The dawn of scientific thought saw the gradual discrediting of the earlier mythical view of causality. The mechanical conception of cause superseded the mythical notion. Natural effects, it was declared, should be explained by natural causes. There is no miracle in nature by which its laws are set aside. Nothing that happens is without law. Thus arose the scien-

* There are deeper philosophical meanings for these with which we are not concerned here.

tific view with its insistence on a thoroughly physical or material conception of causality.

iii. Aristotle's View

As the earliest rational exposition in the west of the implications of the notion of cause we may examine Aristotle's view. Aristotle classified cause into four kinds: material, formal, efficient and final. The material cause is the matter or substance of which a thing is made. The formal cause is the form, pattern or shape into which a thing is made. The efficient cause is the agency which produces the thing, the labour, skill or energy expended, in the production. The final cause is the purpose for which the thing is made. Thus, in respect of an article of clothing, the material cause is cotton thread, silk or wool which constitutes the stuff of the cloth, the formal cause is the fashion or pattern the weaver has in his mind, the efficient cause consists of the agencies or instruments involved in the production of the cloth, and the final cause is the end or purpose for which the cloth is woven, say, to earn a livelihood or to clothe someone. Of these four kinds of causes, the material and the formal causes, are called intrinsic, since they enter into the constitution of the effect, and the other two, *viz.*, the efficient and the final causes, are called extrinsic, since they remain outside the effect and yet influence it.

iv. Definition of Cause

The definition of cause which has found favour for the purposes of science is that given by Mill. Cause, according to Mill, is the unconditional and invariable antecedent of an event. 'Cause' and 'effect' are relative terms. An event may be the cause of one phenomenon and the effect of another. A cause and its effect are related as antecedent and consequent. Antecedence implies priority in time, a relation of *before* and *after*. But this is not the meaning when we call the cause antecedent. The cause does not first take place, then stop and give rise to the effect. There is no time interval between cause and effect. Very often the two are simultaneous or contemporaneous, as fire and warmth. What we really mean by causation is that 'the beginning of a phenomenon is what implies a cause,' and that 'causation is the law of the succession of phenomena.'

Any and every antecedent is not the cause of an event.

So many incidents might precede the occurrence of a phenomenon without being connected with it as cause. The return to England of the Prime Minister from a visit to the United States of America, the tuning-in of the radio-set to a foreign station, the meeting of prominent leaders at Bombay are events antecedent to the writing of these lines: but they are not causally connected with the act of writing. "The sun's light may be an antecedent to the burning of a house, but not the cause, because the house would burn equally well in the night."* It is not enough, then, that the cause is antecedent to the effect: it must be the invariable antecedent. To assume that every antecedent of an event is its cause is to commit the fallacy of arguing *post hoc ergo propter hoc* (after this, therefore because of this). Sequence is no proof of consequence or causal sequence. Simply because A occurs *after* B we cannot say that A is *because* of B. That the causal connection is invariable follows from the principle of uniformity, *viz.*, the same cause must have the same effect, and the same effect must have the same cause.

Even if two events invariably occur one after the other we cannot say that they are causally connected. From the beginning of the world, night and day have succeeded each other invariably. But we do not say that each is the cause of the other. "It is necessary to our using the word cause," as Mill says, "that we should believe not only that the antecedent always *has* been followed by the consequent, but that as long as the present constitution of things endures, it always *will* be so. And this would not be true of day and night. We do not believe that night will be followed by day under all imaginable circumstances, but only that it will be so *provided* the sun rises above the horizon. If the sun ceased to rise, which, for aught we know, may be perfectly compatible with the general laws of matter, night would be, or might be, eternal. On the other hand, if the sun is above the horizon, his light not extinct, and no opaque body between us and him, we believe firmly that unless a change takes place in the properties of matter, this combination of antecedents will be followed by the consequent day; that if the combination of antecedents could be indefinitely prolonged, it would be always day;

* Jevons.

and that if the same combination had always existed, it would always have been day, quite independently of night as a previous condition. Therefore is it that we do not call night the cause, nor even a condition, of day. The existence of the sun (or some such luminous body), and there being no opaque medium in a straight line between that body and the part of the earth where we are situated, are the sole conditions; and the union of these, without the addition of any superfluous circumstance constitutes the cause. This is what writers mean when they say that the notion of cause involves the idea of necessity.”*

Invariability of sequence, then, is not what determines causal relation. The effect follows *from* the cause and does not merely follow it; it is *propter hoc* and not merely *post hoc*. The sequence, therefore, besides being invariable, must be unconditional or necessary. Regular sequence is not enough to establish a causal relation. There must be some intrinsic connection between cause and effect. Otherwise, there is no meaning in causality. We say, *because* *b* has followed *a* in the past, it will continue to do so.† But what is the ground of this assertion? It cannot be mere conjunction. Merely because *a* and *b* have been unvaried, we cannot say that they are invariable. There must be an intrinsic connection between the two over and above the *de facto* conjunction. Only then can they be regarded as causally related. That is, cause and effect must be unconditionally or necessarily related. By ‘unconditional’ is meant ‘not dependent on any other condition.’ Whether any other circumstances are present or not, in the presence of the cause, the effect must take place, and in the absence of the cause, the effect must not take place. This is the meaning of ‘unconditionality’ or ‘necessity.’ We may, then define the cause of a phenomenon, in the words of Mill, as ‘the antecedent, or the concurrence of antecedents, on which it is invariably and *unconditionally* consequent.’‡

Seldom is there a case in which a single antecedent brings about a phenomenon. That is why we have, in the definition given above, the phrase ‘concurrence of antecedents.’ For the

* Mill: *Logic*, Bk. III, ch. V, sec. 6.

† Blandshard: *Nature of Thought*, Vol. II, p. 507.

‡ *Logic*, Bk. III, ch. V, sec. 6.

production of an effect, usually many circumstances are necessary; and strictly speaking all these circumstances must be regarded as the cause. "Thus the cause of the loud explosion in a gun is not simply the pulling of the trigger, which is only the last apparent cause or *occasion* of the explosion: the qualities of the powder, the proper form of the barrel, the proper arrangement of the percussion cap and powder, the existence of the surrounding atmosphere, are among the circumstances necessary to the loud report of a gun: any of them being absent it would not have occurred." Commonly, the last of the circumstances is called the cause, and the others are termed conditions. Sometimes, all the conditions are not stated because some of them are evident and no one would pass them by. When it is stated, for instance, that the cause of a man's death was that his foot slipped in climbing a ladder, it goes without saying that the man's weight was also a condition of the effect. What one chooses to call the cause depends also on his interest and the purpose of his inquiry. But from the logical or philosophical point of view all the conditions must be included in the cause. Some of the conditions may be positive and the others negative. The positive conditions are those which bring about the effect by their presence. The negative conditions are those which should be absent if the effect is to be produced. The cause, then, should be defined as "the sum total of the conditions, positive and negative taken together; the whole of the contingencies of every description which being realised, the consequent invariably follows."*

Besides being the unconditional and invariable antecedent, the cause must also be equal to the effect in regard to the matter contained and energy embodied. When oxygen combines with hydrogen to constitute water, or with mercury to form oxide of mercury, the weight of the compound is exactly equal to the weight of the elements combined in it. The quantity of a piece of cloth is equivalent to the quantity of cotton expended in its production. This is what is called the conservation of matter. As a corollary to it there is the law of the conservation of energy. The total amount of energy in the universe is a constant quantity. Causation does not mean adding to the quantum of energy. It implies only trans-

* Mill: **Logic**, Bk. III, ch. V, sec. 3.

ference of energy from cause to effect. Hence the quantity of energy in the effect is exactly equal to the amount of energy in the cause. As between cause and effect there is quantitative equality.

v. The Doctrine of Plurality of Causes

It is believed by the popular mind that an identical effect may have many causes. By this it is not meant that several conditions combined and produce an effect. The cause of a phenomenon, as we have seen, is an assemblage of conditions. The doctrine of plurality of causes is quite different. According to this doctrine, the same effect may at different times or in different cases be produced by different total causes. The effect A may sometimes arise from B and sometimes from C. For the generation of heat, for instance, there are many causes like the sun, friction, percussion, electricity, chemical action, etc. Similarly, death may be produced by many causes like drowning, disease, old age, gun-shot, etc. Given the cause, we may say the effect has occurred. But given the effect, we cannot say what has caused it.

The doctrine of plurality of causes is not consistent with the scientific view of causal connection. In the view of science, causal connection is a reversible or reciprocal relation. The same cause has the same effect: and the same effect has the same cause. This would not be the case, if the popular view were correct. The popular view, however, is based on a superficial knowledge of the causal relation. It is true that death may be the result of drowning, gun-shot, etc. But this 'death' is a general and not a particularized effect. The nature of death varies according to the kind of cause that brings it about. "There are many cases of death only because there are many kinds of death."* The truth of this statement is borne out by the fact that it is possible by *post-mortem* examination to determine in each case of death the specific cause thereof. In the case of the generation of heat also, it can be shown that there are not really a plurality of causes, the ultimate source being one and the same. As Carveth Read says, "If we knew the facts minutely enough, it will be found that there will be only one cause (sum of conditions) for each

* Mellone: *Elements of Modern Logic*, p. 202.

‘ अभिसन्धिमात्रपुत्रिकेत्येकेषाम् ’

इति । सा च कथं पुत्रिका भवतीत्यपेक्षिते मनुराह—

अपुत्रोऽनेन विधिना सुतां कुर्वीत पुत्रिकाम् ।

यदपत्यं भवेदस्यां तन्मम स्यात्स्वधाकरम् ॥

इति । वसिष्ठोऽपि—

अभ्रातृकां प्रदास्यामि तुभ्यं कन्यामलङ्कृताम् ।

अस्यां यो जायते पुत्रस्स मे पुत्रो भवेदिति ॥

इयमेव पुत्र इति वा पुत्रिकाकरणं,

‘ शासद्वहिर्दुहितुर्नप्त्यं गात् ’

इति श्रुतेः । एतच्च निरुक्ते व्याख्यातं—

‘ प्रशास्ति वोढा सन्तानकर्षणे दुहितुः पुत्रभावम् ’

इति । असमानार्षगोत्रजां, समानमार्षं प्रवरो यस्य स समानार्षः । तस्माज्जातां समानार्षजातां नोद्वहेदित्यर्थः । अत एव गौतमः—

‘ असमानप्रवरैर्विवाहः ’

इति । समानता च नामतो वेदितव्या । यदाह बोधायनः—

एक एव ऋषिर्यावत्प्रवरेष्वनुवर्तते ।

तावत्समानगोत्रत्वमन्यत्र भृग्वङ्गिरोगणात् ॥

इति । समानगोत्रत्वं समानप्रवर इत्यर्थः । कथं तर्हि भृग्वङ्गिरोग-
णेष्वित्यपेक्षिते सङ्ग्रहकार आह—

have combined, it is called homogeneous. If it is of a different nature, it is called heterogeneous. The inter-mixture of effects then, may be either homogeneous or heterogeneous. The former obtains in mechanical causation; the latter in chemical and organic changes. If two locomotive engines pull a carriage, the speed capacity of the train would be the sum of the speed of the two engines. Here the joint effect is of the same kind as the separate effects of the two causes. In the case of chemical combinations, the inter-mixture of effects is heterogeneous. When two substances combine and produce a third substance, the properties of the third are different from those of either of the substances separately, or both of them taken together. "Not a trace of the properties of hydrogen or of oxygen is observable in those of their compound, water."* Similarly, in the case of combinations of elements which go to make organised bodies, the inter-mixture of effects is heterogeneous.

CHAPTER XXI

OBSERVATION AND EXPERIMENT

i. Introduction

We have now defined the problem of induction and the ground on which it is based. Induction, we said, is the method of establishing universal laws from particular facts or instances. But if universal laws did not govern the particular facts, they could not be discovered. Hence the need for assuming the orderly nature of the universe. That all phenomena are governed by laws, that all products are related to their causes necessarily and universally, is the fundamental postulate of induction. In the last chapter we investigated into the nature of causal connection; and we are now ready to consider the details of the inductive process by which causal connections between phenomena are established. The first stage in induction, we had occasion to point out earlier, is the observation and analysis of facts. We shall study in the present chapter the nature of scientific observation and of experiment which is only controlled observation.

* Mill: *Logic*, bk. III, ch. vi, sec. 1.

ii. Observation

If the postulate of uniformity provides us with the formal ground of induction, observation supplies us the data for inductive generalisation. Without observation scientific investigation cannot begin; and without observation it cannot be sustained. Observation "constitutes the beginning, it supports the middle and it determines the end of every inductive enquiry. Without it a problem cannot arise, a hypothesis cannot be formed, an inference cannot be confirmed."* Very often it is the observation of an unexplained phenomenon that starts us on an inductive inquiry. It is further observation that gives us all the relevant material wherewith we build a theory. It is observation again that is needed for the final check of the theory.

Observation is not mere sense-activity. It does not mean passive reception of facts. A good deal of interpretation and inference is involved in observation. However far down we may go in perceptual experience, we cannot find a simple sensation uninfluenced by thought. The perception of blue, for instance, involves implicit comparison and identification which are processes of thinking. Observation, therefore, is not the bare reception of impressions through the senses. It is not a vacant gazing at facts. It means rather keeping an object before the mind (from *ob*, before, and *servare* to keep). "It is the intelligence of man, that is the true observer; and the senses are but instruments of the observant mind."†

That observation is not a passive experience will be evident also from the fact that we do not observe all that is presented to our sense-organs. Having eyes, we may not see, having ears, we may not hear. When one is engaged deeply with a particular piece of work, one does not observe so many facts that occur but are not connected with the work. The example of Sakuntala's failure to notice the presence of Durvasa is in point. Sakuntala was so much lost in the contemplation of her absent lord that she did not observe Durvasa who had come to the forest abode on a visit, and as a consequence she had to receive the curse of the irate rishi. Much of what is presented passes unnoticed. All observation is

* Crumbley

† G. H. Joyce: **Principles of Logic**, p. 311.

selective. We often see what we expect to see. "An officer who superintended the exhuming of a coffin rendered necessary through a suspicion of crime, declared that he already experienced the odour of decomposition, though it was afterwards discovered that the coffin was empty."* "The only things," says William James, "which we commonly see are those which we preperceive."

What we choose to observe depends on the purpose we have in mind. It is theory that guides the selection in observation. We do not approach facts with an empty mind. The militarist sees the details of strategy, the relative strength of armies, the modes of weapons used, etc., in a war. The pacifist sees the waste and inhumanities, the quantity of blood spilt and the number of lives lost in the war. Without knowledge and insight, observation would be a dull affair. "The true 'seer,' indeed, is the rarest of all discoverers," writes Mackenzie, "but the true seer is one who brings to his observation more than he finds in it. The drudgery of the patient interrogator of nature is made divine only when it is inspired by ideas which are not objects of observation."† A good observer, then, must be a good theoriser.

But are we not arguing in a circle? We have to observe facts in order to discover a theory. Now we seem to say that we cannot observe without a theory, "We observe as a means to a theory, but unless the theory is there already we do not know what to observe."‡ The solution to this paradox is not difficult to find. The knowledge or theory at the commencement of observation is general. We search for relevant facts in the light of this knowledge. The facts observed again suggest a theory that is more specific. This theory in turn serves to narrow our attention further, and leads to a still more specific theory. Thus the adventure of knowledge consists in progressive realisation of intelligibility. What we call facts are themselves the results of a great deal of theorising. At no stage in observation can we dispense with theory.

There is, however, one danger against which an observer should guard himself. That is, he must not allow his theory

* Sully: *Illusions*, p. 88.

† *Introduction to Social Philosophy*, p. 13.

‡ Blandshard: *Nature of Thought*, Vol. II, p. 86.

to tyrannise over the facts. Prejudice and bias cling so close and fast that it is hard to get rid of them. Each person has his own pet notions which he hugs with all his heart. "It is not easy", says Jevons, "to find persons who can, with perfect fairness, register facts both for and against their own particular views." A good observer must have in him something of the spirit of renunciation. He must be prepared to renounce his theory if he finds facts contradictory thereto. And he must also observe all *relevant* facts whether they be favourable or unfavourable to his theory. Opinions and beliefs must be secondary to the scientist. He should love truth more than anything else.

iii. Errors of Observation

Observation becomes erroneous if due to some bias or prejudice relevant facts are overlooked or if wrong interpretations are put upon the observed facts. Thus "a fallacy of mis-observation may be either negative or positive; either Non-observation or Mal-observation. It is non-observation, when all the error consists in overlooking, or neglecting facts or particulars which ought to have been observed. It is mal-observation, when something is not simply unseen, but seen wrong; when the fact or phenomenon, instead of being recognised for what it is in reality, is mistaken for something else."*

(1) *Non-observation*: This is a negative fallacy consisting in the omission to observe relevant facts. On account of bias or pre-conceived notions, one may fail to observe the right things and as a result arrive at a wrong conclusion. The belief in the prophetic nature of the dreams of the early morning, and the superstition about the unlucky nature of number thirteen are cases of generalisations based on non-observation. The story is told of a priest who in Pagan times took an unbeliever into a temple and showed him a picture of all the persons who had been saved from shipwreck after paying their vows. The sceptic pertinently asked the priest: 'Ay, but where are they painted that were drowned after their vows?' Doubtless there were cases of sailors finding a watery grave even after professing their faith in the gods. But it was not to the purpose of the priest to cite them or commemorate them in the picture. As Bacon says, "Men mark when they hit,

* Mill: *Logic*, Bk. v, ch. iv, sec. 1.

and never mark when they miss." Brand Blandshard gives a few more examples of non-observation "There are those who believe that socialists generally are tub-thumping psychopaths; others who believe that Englishmen generally omit their h's and lack humour; others who think that all Jews are of dark hair and complexion; others who are convinced that honesty is the best policy, in the sense that it is always the most likely way to advance one's material interests; others who believe that the good are invariably happy and the wicked unhappy, that haste always makes waste, and that every thing worth doing is worth doing well. Those who accept such generalisations need the warning of another and similar saying: all that glitters is not gold. There is no logical path from 'some' to 'all'. A little observation would overthrow any of these sayings, and if, in some minds, they have become established, it is pretty certainly not because the negative instances have offered no corrective, but because the corrective has been ignored or forgotten."*

(2) *Mal-observation.* This is an error of commission, which consists in observing the wrong things or in interpreting the observed facts in the wrong way. Mistaking the rope for snake, the shell for silver, and the post for man are cases of mal-observation. The error in each of these instances lies in failure to interpret correctly what is observed. Here again it is prejudice or wrong theory that is responsible for the fallacy. The generality of mankind either fails to observe the right things or observes in the wrong way. The master-mind, on the contrary, chooses the right things and arrives at the right conclusions. As William James says, "The genius is simply he to whom, when he opens his eyes upon the world, the 'right' characters are the prominent ones. The fool is he who, with the same purposes as the genius, infallibly gets his attention tangled amid the accidents."†

iv. Experiment

A distinction is sometimes made between observation and experiment. Observation it is said, is *passive* experience, while experiment is *active* experience. This mode of describing the distinction is rather misleading. Observation, as we

* *The Nature of Thought*, Vol. II, p. 91.

† *Psychology*, II, p. 336.

तृतीयां मातृतः कन्यां तृतीयां पितृतस्तथा ।

शुल्केन चोद्ग्रहिष्यन्ति विप्राः पापविमोहिताः ॥

मातृतो मातृपक्षे तृतीयां मातुलसुतां, पितृपक्षे तृतीयां पैतृष्वसे-
यीमित्यर्थः । शुल्कं मूल्यम् । अनेन कूटस्थ^१मारभ्य गणयेदि-
त्युक्तं भवति । तथा च पुराणे ययातिवाक्यम्—

यो मे त्वं हृदयाज्जातो वयस्त्वं न प्रयच्छसि^२ ।

पापान्मातुलसम्बन्धात्प्रजा वै ते भविष्यति ॥

सुमन्तुरपि—

‘पितृपक्षस्सर्वा मातरः; तद्भ्रातरो मातुलाः;

तद्गृहितरश्च भगिन्यः; तदपत्यानि भागिनेयानि;

अन्यथा सङ्करकारीणि स्युः’

इति । उच्यते; सत्यमेतानि मातुलदुहितृविवाहनिषेधपराणि ।
तथाऽप्यासुरादिविवाहेन पित्रादिभिरनिवृत्तसपिण्डभावायाः पु-
त्रस्य मातुलसुता परिणेतुर्मातुश्च सपिण्डेति सा तावदविवाह्या ।
तथैवंविधायाः पितृष्वसुर्दुहिता परिणेतुः पितुश्च सपिण्डा
सगोत्रा चेति साऽपि न परिणेया । या तु ब्राह्मादिविवाहोदा-
तस्यास्स्वपित्रादिभिस्सापिण्डचानिवृत्तेस्तत्पुत्रस्य मातुलसुताप-
रिणयनं न^३ वार्यते । एवं पैतृष्वसेय्यामपि द्रष्टव्यम् । तदि-
दमुक्तं मनुना—

^१ मूलस्थ—ख; मूलस्थकूटस्थ ग. ^२ प्रयच्छति—ख. ग. ^३ केन—ग.

Geisher, and may thus so vary the points of observation as to render our procedure experimental. We are wholly unable either to produce or prevent earth-currents of electricity, but when we construct long lines of telegraph, we gather such strong currents during periods of disturbance as to render them capable of easy observation." In these natural experiments, the scientist has a greater facility for studying phenomena than when he is thrown upon simple observation. Still he has no complete command over facts—the command which he has by right in experiment. In natural experiment as in ordinary observation he has to wait on nature.

vi. **Relative Merits of Observation and Experiment**

It is claimed, not without some justification, that material progress has been made possible because of the experimental sciences. Physics and Chemistry which are the experimental sciences *par excellence* share between them the honour of having contributed most to this progress. Psychology, which was till the other day a mental science making use of the introspective method of looking within, is now fast becoming an experimental science. It is no doubt true that where it is possible and legitimate, experiment has a great value. The chief advantages of experiment are these: (1) If one experiment proves unsuccessful, we can repeat the experiment as often as we like till the correct results are got. This is not possible where we have to depend on simple observation. If we want to observe a comet, for instance, we cannot produce one. We must wait till a comet occurs; and it may occur only once or twice in a life time. There is no such abject dependence on nature in regard to data which are experimentable. A scientist who desires to understand the properties of a chemical compound, for example, can produce the compound as often as he likes. (2) Secondly, it is a particular advantage of experiment that it enables us to isolate the conditions which we want to observe from all other circumstances and thus contribute to the acquisition of definite and precise knowledge. By means of experiment, *e.g.*, we can ascertain that it is the presence of oxygen in the atmospheric air that makes the burning of a substance like the candle possible. By an experimental analysis of air we can separate its constituent gases, fill different jars with the different gases, and by

inserting a burning candle into each of the jars we can know that it is the oxygen-content of air that makes the burning possible. (3) Thirdly, in experiment we can make variations in the circumstances of the phenomenon under investigation with a view to determine the causal connection. Such variations help in the process of eliminating the unessential circumstances and determining the necessary conditions. Suppose we desire to know the condition which makes the audition of sound possible. We may increase or decrease the amount of air in a bell-jar, and by striking the bell each time show that for the audition of sound the presence of a medium like the air is necessary. (4) Lastly, in experiment we can afford to be cool and calm, unruffled and at perfect ease. As Carveth Read says, 'Experiment enables us to observe coolly and circumspectly and to be precise as to what happens, the time of its occurrence, the order of successive events, their duration, intensity and extent.'

Experiment is not without limitations, and observation is not devoid merit. (1) First, it must be noted that there are several kinds of phenomena which do not admit of experimental study. One cannot produce an eclipse or an earthquake. One must wait for their occurrence and then observe. Thus observation is wider in its range than experiment. (2) Secondly, in experiment we can reason only from cause to effect and not *vice versa*. Given a cause, we can, where possible, experiment and determine its effect. In simple observation there is not this limitation. We can reason from effect to cause as well as from cause to effect. (3) Finally, observation has to precede experiment. It is only after simple observation has analysed the given whole of complex matter and selected the relevant parts or aspects of it for consideration that scientific experiment can begin. As we said earlier, experiment is not different in kind from observation. Observation has to prepare the way for experiment. Experiment itself is only observation under control.

CHAPTER XXII

ENUMERATION—STATISTICS—PROBABILITY

i. Enumeration

In Chapter XVIII we said that induction which proceeds by mere counting is a childish affair and added that to say so was not to deny all value to counting. Number and measurement have come to stay in the physical sciences. Without these our everyday life would be miserable and our civilisation impossible. "We have only to picture a race which cannot count or measure trying to run the Bank of England, or control the milk market, or even understand the sporting columns of the daily press, to realise how deeply rooted numbers are in the complex activities of the modern world."* Precision and quantitative measurement are the guiding principles of scientific knowledge. It was the discovery of accurately measuring instruments that set the pace for modern science. The movement for quantitative calculation began in the physical sciences. But it has now entered even the province of psychology, the study of mental life. Thus the man of affairs and the man of science depend alike for a good part of their knowledge on number and measurement. "The desire for precision, in fact, leads investigators of all kinds, from the atomic physicist to the business man, to express the facts about that part of the universe which interests him in a quantitative way. Numerical data have come into being not only in the laboratory and the study, but in the counting house, the sales department, the Board Room and the legislative assembly. It is difficult to see how our society could be organised without them."†

Organisation of facts is essential for ordered existence and systematic knowledge. Counting, when it is not blind, effects such organisation. The welter of facts assumes some order when we begin to count. When we group things of the same kind together and note their number, we employ also

* G. Udny Yule and M. G. Kendall: *Introduction to the Theory of Statistics*, p. 1.

† *Ibid*, p. 1. Lord Kelvin says that one's knowledge of science begins when he can measure what he is speaking about and express it in number.

इति मातृपक्षे पञ्चमादूर्ध्वमुद्राहविधानात्कथं मातुलदुहितृपरिणय-
नम्? उच्यते—‘मातुरसपिण्डामुद्रहेत्’ इत्यनेन मातृबन्धुष्वप्य
विशेषेण विवाहप्राप्तौ तन्मा प्रसाङ्गीदिति मातृतो मातृबन्धुभ्यः
पञ्चमादूर्ध्वमुद्रहेदित्यविरोधः । एवं पितृबन्धुष्वपि द्रष्टव्यम् ।
अत एव गौतमः—

‘ऊर्ध्वं सप्तमात्पितृबन्धुभ्यो बीजिनश्च मातृबन्धु
भ्यः पञ्चमात्’

इति । बीजिनो यो नियोगादुत्पादयति तस्मादप्यूर्ध्वं सप्तमा-
दित्यर्थः । बान्धवा अपि स्मृत्यन्तरे दर्शिताः—

पितुः पितृष्वसुः पुत्राः पितृमातृष्वसुस्सुताः ।

पितुर्मातुलपुत्राश्च विज्ञेयाः पितृबान्धवाः ॥

मातुः पितृष्वसुः पुत्रा मातुर्मातृष्वसुताः ।

मातुर्मातुलपुत्राश्च विज्ञेया मातृबान्धवाः ॥

यत्पुनर्वसिष्ठेनोक्तम्—

पञ्चमीं मातृबन्धुभ्यस्सप्तमीं पितृबन्धुतः ।

इति तद्गौतमीयैकवाक्यत्वाय पञ्चमीं सप्तमीं चातीत्योद्बहेदित्ये
व-परम्, न पुनः पञ्चमीसप्तम्योरेवोद्बहावधिपरम् । यदपि
नारदेनोक्तम्—

पञ्चमात्सप्तमादवर्गिवन्धुभ्यः पितृमातृतः ।

अविवाह्या सगोत्रा च समानप्रवरा तथा ॥

इति, यदपि विष्णुना—

phenomena from the results obtained by the analysis of an enumeration or collection of estimates. Secrist holds statistics to be a method involving the collecting, sorting, classifying, tabulating, summarising, and comparing enumerated facts for the purpose of describing or explaining phenomena with which the enumerations deal. From these definitions it will be clear that the name 'Statistics' is applied both to the method and the results obtained by the method. As a method or methods, statistics mean counting and classifying phenomena according to certain definite principles with a view to describe or explain them. It is by statistics that we know, for instance, that one man in every five is an Indian, or that 'the ordinary peasant in our country with a wife and three children has to live along with his family on Rs. 27 a month—just under a rupee a day.* Especially in trying to solve problems in sociology, education, economics, psychology, biology and astronomy, we have the utmost need of statistical methods. Of late these methods have begun to play an important role even in physical theory.

Statistical methods are applied in cases where the subject of investigation is complex and where the governing law is not apparent. Statistics are required mainly for resolving the complexity of phenomena. If the things that are to be observed are too simple or too few, they will not need statistical treatment. It is only where facts are unwieldy and bewildering at the first sight that there is any necessity to classify them and arrange them. Classification and arrangement imply certain standards or units. If the complex material is not resolvable into simpler classes or in terms of certain units of measure; then, statistical treatment would be impossible. So the first characteristic of the class of facts to which statistics are applied is that they be complex and at the same time capable of measurement in terms of units or standards. The second condition for the application of statistics is that the law governing the facts is of such a nature that it is not directly known. The numeration of particular objects and events becomes interesting only when we are not able to reduce them to rules and laws. If, for instance, we had the knowledge of the law governing the number of deaths in a

* Minoo Masani: *Our India*, p. 28.

community, we would not take the trouble of registering the individual cases of death for the purpose of determining the death-rate. "There was some interest in counting how many eclipses of the moon and sun took place year by year, so long as they occurred unexpectedly and inexplicably; since the rule has been found according to which they occur, and can be calculated for centuries past and to come, that interest has vanished. But we still count how many thunderstorms and hail-storms occur at a given place, or within a given district, how many persons die, and how many bushels of fruit a given area produces, because we are not in a position to calculate these events from their conditions."*

Dr. Boyce Gibson gives an example to illustrate the value of statistics. He compares the fauna and flora of the mainland with those of (1) islands which have long been separated from the mainland by great distances of sea, and (2) islands which have only recently been separated from the mainland. As a result of statistics it is found that in class (1) islands there are many species of animals and plants which are not found in the mainland, whereas in class (2) islands there is not much variation. The statistics thus bear out Darwin when he says: "Wherever there is evidence of land-areas having been for a long time separated from other land-areas, there we meet with a more or less extraordinary profusion of unique or peculiar species, often running up into unique genera; . . . there is everywhere a constant correlation between the *degree* of the peculiarity on the part of the fauna and flora and the *time* during which they have been isolated."

Statistics are useful in several ways: (1) In the first place, the facts under investigation are presented in a neat and tidy manner, with their complexities removed and tangles broken. The census reports, *e.g.*, give us details regarding a particular group or population which otherwise it will be impossible to gather. (2) Secondly, statistics furnish a basis for predicting the probable course of events in the future. Most of our programmes and plans are based on the statistics of previous events. The Government of a country is able to present a budget every year on the strength of the knowledge

* Sigwart: *Logic*, Vol. II, p. 483.

it has of the revenue and expenditure of the preceding years, making provision, of course, for contingencies. (3) Thirdly, statistics often enable us to detect relations between phenomena, which would not have been observed otherwise. They reveal quantitative correspondences between two sets of facts, and thereby suggest some law of connection between the two. For instance, statistical records will make us realise the connection that there is between famine and epidemics on the one hand and the population strength on the other. "In 1876, when famine, with its companion, cholera, was already beginning to be felt, the births registered in Madras numbered 632,113, and the deaths 618,381. In 1877 the year of famine, the births fell to 477,447, while the deaths rose to 1,556,312. In 1878 the results of the famine showed themselves by a still further reduction of the births to 348,346, and by the still higher number of 810,921 deaths. In 1879 the births recovered to 476,307, still below the average, and the deaths diminished to 548,158. These figures are only approximate, but they serve to show how long the results of famine are to be traced in the vital statistics of a people."* India experienced in 1876 the most widespread and prolonged famine. The statistics of births and deaths in Madras taken during those years show how greatly famine affects the vitality of a people.

There is another use of statistics which we can refer to only in brief. Statistics are employed for determining the average of a large number of instances of a particular kind, e.g., the average rainfall of an area, the average height of a group of men, etc.

There are several sorts of average. (a) The commonest of these is called the *mean* or *arithmetic average*. In popular usage the mean is called "the average." It is obtained by dividing the total of the given numbers by the number of the individuals of which or of whom account is taken. Let us work out the arithmetic average of the marks secured by a

* W. W. Hunter: **The Indian Empire.**

dozen students in a class:—

A	90%	G	45%
B	84%	H	45%
C	80%	I	45%
D	72%	J	42%
E	60%	K	32%
F	59%	L	30%

The mean is to be found by adding together the marks of the several students and dividing the total by their number, *viz.*, twelve. The total is 684; therefore, the arithmetic average or mean is $684 \div 12 = 57\%$. The average, it should be noted, affords us an index as to the nature of the class as a whole; it does not tell us anything about any individual of the class. In this case, *e.g.*, 57% is the average; but it is the mark obtained by none of the students.

In some cases the arithmetic average might give quite a misleading description of the whole. Suppose a class consists of students a few of whom are very brilliant and the rest very backward, then the arithmetic average of the marks obtained by them might not properly represent the character of the class as a whole. In order to correct or supplement the conception as to the nature of the group given by the arithmetic average, certain other conceptions are used, *e.g.*, *median* and *mode*.

(b) The *median* is the condition of the individual at the middle, when the group is arranged either in the progressive or in the regressive order. It is the middle quantity in a scale having as many quantities above as there are quantities below it. If the group or series consists of an even number of quantities, then the median is got by striking the arithmetic average of the middle two terms. Thus the median mark in the case of the dozen students is the arithmetic average of the two middle quantities, 59 and 45, *i.e.*, 52%.

(c) The *mode* is the condition which is repeated most often in the group examined. It is the most likely case. In the list of marks given above the percentage which repeats itself most often is 45 which occurs three times. So 45% is the mode.

(d) In some cases the various items may not be all on a par; one item may be more important than another. For

instance, if the average price of different kinds of articles is to be known, there is no use of treating these as having equal value, and dividing the total price by the number of the kinds of articles. So what the economist does is to give "weights" to the different articles in proportion to their importance and to divide the total price by the sum of the weights. This is called the *weighted average*.

(c) The *geometric average* is that quantity which may be substituted for each one of the several quantities of a multiplication and gives, when multiplied, the same product as the quantities for which it is substituted. Thus 8 is the geometric average of 4 and 16, since $8 \times 8 = 64$ and $4 \times 16 = 64$. The rule for finding the geometric average is to multiply the terms together and take the root of the product corresponding to the number of terms. The geometric average of two numbers is the square root of their product; of three numbers the cube root of their product; and so on.

Statistics are thus important for analysing facts and for interpreting them. They give us various sorts of average which help us in understanding the nature of groups or classes of phenomena. Though the statistical average do not give us fixed laws of rigid constancies, they serve as approximations to laws. For instance, we learn through statistics that "there is an annual tax (of human beings) which we pay with frightful regularity to the prisons, convict settlements, and scaffolds."* And as Welton says, "It is well known that the number of persons who commit certain crimes, who are born, or who die, in the course of a year bears a remarkably uniform proportion to the total number of the inhabitants of any given country; there is, as we say, a pretty constant *average* preserved in many of the phenomena of social life." We do not know what conditions are responsible for the uniformities; statistics cannot reveal the nature of those conditions. Nor can statistics tell us anything definite about the individual cases. Though we may be fairly accurate in our predictions about the proportion of births to deaths in a particular locality on the basis of past averages, it will not be possible for us to say who will be born or who will die. As regards a great

* Quetelet: *Système Sociale*; quoted by P. Coffey: *Science of Logic*, Vol. II, p. 289.

extent of natural and social phenomena our knowledge has necessarily to be probable, as there is an element of "uncertainty". And it is here that statistical methods help us in making our knowledge as dependable and accurate as possible.

iii. Probability and the Calculation of Chances

The region of certitude in knowledge is very limited. Regarding many things in life and in nature we are ruled by probability rather than certainty. If we insist on demonstration at every step and in every case, then we should despair of living and knowing. As Locke says: "Her that in the ordinary affairs of life would admit of nothing but direct plain demonstration would be sure of nothing in this world but of perishing quickly."* Though there is in man the thirst for certainty, he has often to be content with a high degree of probability which amounts to practical certitude. "The farmer sows his fields, the manufacturer erects his machinery, the merchant opens his business, the soldier goes to battle, young people marry, and statesmen legislate. Under the influence of what determining motives? What has the future to offer them? Hope of success; *probabilities*."† Hence it has been aptly said that probability is the guide of life. The moment we reflect on the enormous range of phenomena of which we have neither complete knowledge nor total ignorance, the importance of probability will be evident. The region of probability lies between the two extremes of intellectual life—perfect certitude and absolute ignorance.

Many things happen around us, and we do not know how or why. We cannot say when or where or in what circumstances they will recur, because we have no scientific knowledge of them. In such cases we have simply to go by the probabilities or chances of their occurrence. By 'probability' or 'chance', it should be noted, we do not mean absence of causality. When we say that an event is probable or that it is a 'chance' occurrence, we do not intend to urge that it is uncaused. Our faith in the uniformity of nature will not allow us to think that anything is uncaused. Nothing is causal in the sense of being causeless.‡ "There is no doubt in lightning

* Essay Concerning Human Understanding, IV, xi, 10.

† P. Coffey: *Science of Logic*, Vol. II, p. 264.

‡ *Ibid.*, p. 269.

as to the point it shall strike; in the greatest storm there is nothing capricious; not a grain of sand lies upon the beach but infinite knowledge would account for its lying there; and the course of every falling leaf is guided by the same principle of mechanics as rule the motions of the heavenly bodies.”* ‘Chance,’ then cannot stand for chaos in nature. When we regard anything as a chance occurrence, we only mean that we are ignorant of its conditions, and not that it has no cause. ‘Chance is not the denial of causality; it is a name for our ignorance.

In the absence of certain knowledge about a phenomenon, we calculate, where possible, its probability. The estimation of probability is made by assessing the chances for and against the occurrence of the phenomenon. If a coin is tossed, the probability of its turning head is $\frac{1}{2}$, because the chances are equally balanced as between its head and tail.

The calculus of chances is not applicable to all data of our experience. There are certain conditions which alone make the application of the calculus possible. In the first place, we should have a knowledge of the definite number of alternative ways in one or other of which the phenomenon must happen. In the case of tossing the coin, for example, we know that there are only two ways of its falling. The coin must turn up either head or tail. The second condition for applying the calculus is that, while we know the definite number of alternative ways in which the phenomenon may happen, we should have no assurance as to in which of these ways it will happen. If it is possible to know that at the next throw the coin will lie with its head up, there is no need to calculate the probabilities; for the head turning up will then be a certainty. Thirdly, the data whose probability is calculated must conform to the same set of conditions throughout. If the influences that bring them about change in the middle, then all our calculations would be upset. In throwing dice, for instance, we know that the die should fall on one of its six faces, and so we calculate that the chance of its falling on any particular face is $1/6$. But if the die is “loaded,” the normal conditions are not present and the chances become unequal, making our calculation of probability impossible.

* Jevons: *Principles of Science*, Vol. I, p. 225.

In mathematics rules are given for the calculation of chances. It is needless for us to go into the details; for in logic we are interested mainly in the theory of probability and the basis for its calculation. The rule for estimating the probability of a simple event will be evident from the examples we have already given. The probability is "expressed by a fraction whose numerator is the number of favourable alternatives and denominator the total number of alternatives."* In the case of tossing the coin, the probability of the coin turning up head is $\frac{1}{2}$, because the favourable alternative is one and the total number of alternatives are two. In a single throw of the dice the chance of a die turning up the six is $\frac{1}{6}$, because the favourable alternative is one out of six total possibilities. "As there are fifty-two cards in a pack, but each particular value or number is present in four different ways, the probability in favour of drawing some particular value will be $\frac{4}{52}$, and the probability against it is $\frac{48}{52}$, as there are four chances in favour of any particular value and forty-eight against."† If in an urn there are 11 black and 39 white balls, the chance of taking a black ball is $\frac{11}{50}$, while the probability of drawing a white ball is $\frac{39}{50}$.

The examples we have given above are of simple events. If the principle of calculating their probabilities has been understood, it will not be difficult to calculate the chances of compound events. Thus if we throw three dice, the probability that all of them will show sixes is

$$\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$$

The calculus of probability, it should be borne in mind, does not apply to every or even to any individual case. I may spin a coin, say, 40 times without its turning up head even once. All that the calculation of chances means is that if coins are tossed an infinite number of times, 'heads' will turn up just as often as 'tails.'

The results of the calculation of chances may appear to be vague; but they are not without their use. The life insurance companies, for example, are run on the principle 'what

* P. Coffey: *Science of Logic*, Vol. II, p. 276.

† *Ibid.*, p. 276.

has happened in the past in a large number of cases will happen in the future.' Their business is conducted on the assumption that there will be an approximately constant death-rate. By examining a large number of cases, and taking into account all relevant factors, the probable number of deaths every year in each age-group are calculated and the rates of premium are fixed. Like statistical analysis, the calculus of chances plays an important part in everyday life.

CHAPTER XXIII

ANALOGY

i. Introduction.

We saw in the last chapter how enumeration can help in the organisation of our experience by suggesting probable causal connections. This it is able to do because in counting we also classify and analyse. There is a method, however, which, while not depending on enumeration, suggests causal connections on the basis of resemblance. In the present chapter we shall explain this method which is called 'analogy' in inductive logic.

ii. Reasoning by Analogy

Originally the term 'analogy' was used in the sense of identity of relation. Four entities were said to be analogous when the relation of the first to the second was the same as the relation of the third to the fourth. Thus 2 and 4, 3 and 6 are analogous; and we can infer that because 4 is twice 2, 6 is twice 3. The reasoning may be stated in the formula $a : b :: c : d$. In this sense 'analogy' meant equality of quantitative relations or ratios.* Subsequently the term was applied to qualitative relationships also. One might argue, for instance, that because the relation of a colony to its mother country is the same as that of a child to its parent, the duties of the colony are the same as those of the child. But the difference between the two uses of the term 'analogy' will be evident. Analogy in mathematics is what is usually called proportion; and reasoning based thereon is necessary. Strictly speaking it is a case of deductive or demonstrative reasoning. When we go beyond the realm of abstract mathematical relations and compare sets of terms which are not purely

* Aristotle employed the term 'analogia' in this sense.

quantitative in their relationships, we find ourselves on uncertain grounds. It is only a figure of speech to say, *e.g.*, that a colony is a child of the mother country. It does not follow from this that the duties of the colony should be the same as those of the child. Alexis de Tocqueville said of colonies, that they were like fruit which drops off from the tree when it is ripe.* This analogy would suggest a conclusion which is quite the opposite of the former one. Such analogies are not only inconclusive but oftentimes misleading.

In inductive logic 'analogy' stands not for identity of relations, but for a method of reasoning whereby from partial resemblance of two things to each other we infer a more complete resemblance. Mill states the principle of analogical reasoning thus: 'Two things resemble each other in one or more respects; a certain proposition is true of the one, therefore, it is true of the other.'† *E.g.*, the present war which began in 1939 resembles the war of 1914-18 in that it is a world war; the last Great War was followed by a period of economic depression; therefore, we have reason to believe that humanity will pass through a long spell of depression after the present war terminates. This is reasoning by analogy. At this stage we are not aware, for certain, of the causal connection between a great war and economic depression. Had we this knowledge, we need not compare the present war with the war of 1914-18; we may make a universal and unconditional statement that any world war should be followed by economic depression. In the absence of such a knowledge, we guess a causal connection on the ground of similarity. The two wars resemble each other in several respects. A further point is observed in the case of the war 1914-18, *viz.*, that it led to economic depression. Therefore, we expect that even the present war will lead to such a disastrous consequence, the presumption being that probably war and depression are related by way of cause and effect. Thus analogy is a distinct form of inference. It "supposes that two things, from resembling in a number of points, may resemble in some other point, which other point is not known to be connected with the agreeing points by a law of causation or of co-existence."

* Joseph: *An Introduction to Logic*, p. 494.

† Mill: *Logic*, Bk. III, ch. xx, sec. 2.

The reasoning by analogy may be expressed symbolically in the following form:

S_1 resembles S_2 in certain respects M .

S_1 exhibits the character P .

Therefore, S_2 will exhibit the character P also.

S_1 and S_2 are the two things resembling each other in being M . P is found in S_1 . Hence we argue that it may be found in S_2 also. That is, partial identity of content is made a ground for inferring further identity of content.

A classic example of analogical reasoning is the argument from the habitability of the earth to that of Mars. The two planets resemble each other in several respects. They revolve round the sun, borrow all their light from the sun, turn round on their axes, possess similar land and sea areas, etc. The earth is known to be characterised by another feature, *viz.*, habitability. Therefore, we infer that Mars too may be inhabited.

It might be noted from what we have said about analogy so far that the conclusions we arrive at through this kind of reasoning are only probable and never certain. The results which analogical reasoning yields are not conclusive. They are more or less probable.

The degree of probability is to be measured, according to Mill, by "the extent of ascertained resemblance, compared first with the amount of ascertained difference and next with the extent of unexplored region of unascertained properties." Mill explains the implications of this statement as follows: "Where the resemblance is very great, the ascertained difference very small, and our knowledge of the subject-matter tolerably extensive, the argument from analogy may approach in strength very near to a valid induction. If, after much observation of B , we find that it agrees with A in nine out of ten of its known properties, we may conclude with a probability of nine to one, that it will possess any given derivative property of A ."*

Mill is wrong in this view of his. Mere number cannot be a reliable guide in any kind of inference. The logical value of analogical reasoning does not depend on the number of resemblances. Two boys, let us say, resemble in many

* Mill: *Logic*, Bk. III, ch. xx, sec. 3.

अत्युच्चमतिह्रस्वं च अतिवर्णं च वर्जयेत् ।
 हीनाङ्गमतिरिक्ताङ्गमामयाविकुलानि च ॥
 श्वित्रिकुष्ठिकुलादीनां कुर्याद्विपरिवर्जनम् ।
 सदा कामकुलं वर्ज्यं लोमशानां च यत्कुलम् ॥
 अपस्मारिकुलं यच्च यच्च पाण्डुकुलं भवेत् ।

इति । अनार्षेयमविज्ञातप्रवरम् । अत्युच्चमतिदीर्घा यस्मिन्कुले
 तदत्युच्चम् । अतिह्रस्वमतिकुब्जा यस्मिन् । अतिशयितो वर्णो
 यस्मिन् तदतिवर्णम् । एतच्च रोगिकुलवर्जनं तद्रोगसङ्क्रान्तिभ-
 यात् । अत एव याज्ञवल्क्यः—

स्फीतादपि न सञ्चारिरोगदोषसमन्वितात् ।

इति । स्फीतात्समृद्धादित्यर्थः । दोषाः पातित्यम् । मनुरपि—
 महान्त्यपि समृद्धानि गोऽजाविधनधान्यतः ।
 स्त्रीसम्बन्धे दशैतानि कुलानि परिवर्जयेत् ॥
 हीनक्रियं निष्पुरुषं निश्छन्दो रोमशार्शसम् ।
 क्षय्यामयाव्यपस्मारिश्वित्रिकुष्ठिकुलानि च ॥

इति । हीनक्रियं यागाद्यनुष्ठानरहितम् । निष्पुरुषं पुरुषशून्यम् ।
 निश्छन्दो वेदहीनम् । रोमशं बहुरोमशम् । अर्शसं व्याधिविशे-
 षोपेतम् । एवमितराण्यपि द्रष्टव्यानि । एतच्च हीनक्रियादिवर्जनं
 तथाविधापत्यपरिहारार्थम् । श्रूयते च तथाविधापत्योत्पत्तिः
 पुराणे—

was known to Aristotle. But he called it by a different name, *viz.*, argument from example. We are said to argue from example when we infer from one phenomenon to another similar to it "by bringing the one under the same universal to which the other is known to belong." The example which Aristotle gives is this: "The war between the Thebans and Phocians was a war between neighbours, and was an evil; therefore war between the Thebans and Athenians, being a war between neighbours, will also be an evil." The principle of such arguments is described by Aristotle as "proving the major term of the middle by a term resembling the minor." Here in the above example, 'evil' is the major term; and it is proven of the middle term 'war between neighbours' on the ground of the term 'war between Thebans and Phocians' which is similar to or resembles the minor term 'war between Thebans and Athenians.' It will be easily seen that the principle of the argument from example is the same as that of the reasoning by analogy. The two wars, the one between Thebans and Phocians and the other between Thebans and Athenians, resemble each other in being wars between neighbours. A further feature is observed about the war between Thebans and Phocians, *viz.*, that it was an evil. The same may be predicated of the other war. We add another example given by Aristotle Pisistratus of Athens asked for a bodyguard, and made himself tyrant. Dionysius of Syracuse asked the people for a bodyguard. Therefore, one might infer, Dionysius had the intention of becoming a tyrant.

iv. Analogy and Simple Enumeration

Analogy resembles simple enumeration in being inconclusive. The results obtained through either are only probable, and stand in need of proof. Both the analogy and simple enumeration "stick in the particular instances." In the latter we find a character repeated in a few instances of a kind, and jump to the belief that the character might be present in all the instances. In the former, we observe, some resemblance between two particulars and conclude that the resemblance might be deeper, that a fresh feature noticed in the one might be present in the other also.

Though analogy and simple enumeration agree in certain respects, the two are not on the same level. Analogy is more

scientific in character than the method of simple counting because it carries the process of analysis farther still. By means of enumeration we may guess that there is some connection between, say, x and y , and that is all. But in analogical reasoning we very often get to know the nature of the connection and the basis thereof. As we have already pointed out, in analogy, we do not count instances, as we do in enumerative induction, but we weigh properties. Unlike enumeration, analogical reasoning proceeds by analysis of content. Hence it marks a higher stage in the inductive process and is of greater value than simple counting.

v. Value of Analogy..

Analogy is of supreme value in scientific discovery. Many of the important laws in science were first suggested by analogy. Newton formulated the law of gravitation because of the similarity he perceived between the falling of an apple and the falling of the moon. Darwin observed the competitive spirit that animated the industrial world; and this suggested to him that in the animal world evolution takes place through natural selection. These are cases of happy guesses. But analogy is not always the source of sound hypothesis. We know of several analogies that were tried and discarded by scientists in the past because their suggestions turned out to be false. The false suggestions and wrong hypotheses, however, are not without value; for they afford clues for advance and open up avenues for further search.

Thus analogical reasoning has a primary place in the logic of discovery. 'Discovery,' says Jevons, 'is most frequently accomplished by following up hints received from analogy. The task of the scientist, it is true, does not stop with discovery.*' The hypothesis suggested must be proved. The results of analogical reasoning, as we have seen, are only probable. They stand in need of verification and proof. An inquirer who thinks that he has come to his journey's end with the suggestion of a hypothesis through analogical reasoning does not know his business. Analogy, as Mill remarks, is "a mere guide-post, pointing out the direction in which more vigorous investigations should be prosecuted." "It is in this last respect," adds Mill, "that considerations of analogy have

* Logic, Bk. III, ch. xx, sec. 3.

the highest scientific value. The cases in which analogical evidence affords in itself any very high degree of probability are, as we have observed, only those in which the resemblance is very close and extensive but there is no analogy, however faint, which may not be of the utmost value in suggesting experiments or observations that may lead to more positive conclusions."

CHAPTER XXIV

MILL'S METHODS

i. Introduction.

In the sciences certain methods are employed for discovering causal connections. The scientific investigator may not know the nature of these methods. Nevertheless he employs them in his laboratory. Even in ordinary thinking, which differs from scientific thinking only in degree, these methods are used. As the aim of inductive logic is to understand the ways of establishing universal laws that govern all phenomena, and as causal connections are the most widespread expressions of these universal laws it is a matter of utmost relevance that we should study the methods of causal investigation.

John Stuart Mill formulated these methods. Prior to him Lord Bacon and Sir John Herschel had attempted logical analyses of scientific procedure. But it was Mill that set forth clearly the principles involved in the methods, giving them each a canon, though his treatment is somewhat artificial and in certain ways defective. These defects we shall point out later on, after explaining first Mill's formulation of the methods.

Mill, in his *System of Logic*, speaks of four methods of experimental inquiry. Actually, however, he explains five methods. They are, the Method of Agreement, the Method of Difference, the Joint Method of Agreement and Difference, the Method of Concomitant Variations, and the Method of Residues. Mill calls them experimental methods. But even, according to him the first is purely an observational method; and the others too are applicable in investigations where experiments are impossible.

ii. Analysis and Elimination

Mill's Methods (we shall call them so hereafter) are methods of discovering causes of effects and effects of causes.

तत्राप्यात्मन एवोत्पत्तिः । अतो नायं हेतुः किं त्वयमर्थवादः
इति मन्तव्यम् । यत्पुनर्यमेनोक्तं—

ब्राह्मणो वृषलीं गत्वा त्र्यहं भवति सूतकी ।

अथास्यां गर्भमाधत्ते ब्राह्मण्यादेव हीयते ॥

इति ; यदपि मनुना—

शूद्रां शयनमारोप्य ब्राह्मणो यात्यधोगतिम् ।

जनयित्वा सुतं तस्यां ब्राह्मण्यादेव हीयते ॥

इति, तेनापि व्युत्क्रमेणोद्गाहकर्म गमनं तस्यामेव सुतोत्पत्तिर्निषि-
ध्यते ; न पुनस्तत्र सुतोत्पत्तिमात्रम् । तथात्वे तु,

‘ चतुस्त्रिद्वैचकभागिनो वर्णतो ब्राह्मणात्मजाः ’

इति विभागवचनं न स्यात् । अत एव मनुरपि केवलशूद्रापत्य-
तया दोषमाह—

शिल्पेन व्यवहारेण शूद्रापत्यैश्च केवलैः ।

गोभिरश्वैश्च यानैश्च कृष्या राजोपसेवया ॥

अयाज्ययाजनैश्चैव नास्तिक्येन च कर्मणाम् ।

कुलान्याशु विनश्यन्ति¹ यानि हीनानि मन्त्रतः ॥

इति । गोभिरश्वौर्वेक्रीयमाणैरित्यर्थः । तथा मतान्तराण्यपि स
एवाह—

शूद्रावेदी पतत्यत्रेरुतथ्यतनयस्य च ।

शौनकस्य सुतोत्पत्त्या तदपत्यतया भृगोः ॥

¹ कुलान्यकुलतां यान्ति — क & ग.

do not satisfy the conditions of being the cause, and thereby exhibit the true cause. We have already studied the features that characterise causal relations. The cause of a phenomenon is its invariable and unconditional antecedent. In its presence, the phenomenon should occur and in its absence it should not. If any circumstance is not so related to the phenomenon whose cause we seek, then we should eliminate it as not being its cause. The grounds on which we may eliminate are the following:*

- (1) That is not the cause of a phenomenon in the absence of which the phenomenon occurs.
- (2) That is not the cause of a phenomenon in the presence of which the phenomenon fails to occur.
- (3) That is not the cause of a phenomenon which varies when it is constant, or is constant when it varies, or varies in no proportionate manner with it.
- (4) That is not the cause of a phenomenon which is known to be the cause of another phenomenon.

These principles or rules of elimination are but negative ways of stating the conditions of causal relation. A cause and its effect must be co-present and co-absent; and any change that occurs in the one must be reflected in the other. Since from the scientific standpoint there can be only one cause for one effect, the cause of one phenomenon cannot be the cause of a different phenomenon. From these conditions we can easily see that nothing can be the cause of a phenomenon, say P, which is absent when P is present, or is present when P is absent, or between whose changes and those of P there is no correlation, or which is the cause of another phenomenon, say Q.

The five methods of Mill involve the application of the principles of elimination. The Method of Agreement employs the first of the four principles. The Method of Difference proceeds on the basis of the second principle. The Joint Method of Agreement and Difference is a combination of the first two Methods. The Method of Concomitant Variations makes use of the third principle of elimination. And the Method of Residues applies the last of the four rules stated above.

* Joseph: *An Introduction to Logic*, pp. 403 and 404.

THE METHOD OF AGREEMENT

iii. Nature of the Method

The Method of Agreement, as its name suggests, consists in comparing several positive instances of the phenomenon under investigation and discovering its cause or effect, as the case may be, by seeing in what circumstance they agree. We know that in the presence of the cause the effect must be present, and *vice versa*. Let us suppose that we are in search of the cause of a phenomenon. Then, whenever the phenomenon occurs, its cause must have occurred. There will, of course, be other circumstances in the 'midst of which the cause is cast. But if we examine a sufficient number of varied instances in which the phenomenon occurs it might be possible to see that, while one circumstance is constantly in attendance, the others are sometimes present and at other times not. On the principle 'that is not the cause of a phenomenon in the absence of which the phenomenon occurs', we may eliminate the inconstant circumstances and conclude that the circumstance or factor which is uniformly present in all the instances is probably the cause. The reasoning is the same *mutatis mutandis* in the case of discovering the effect of a phenomenon.

iv. Canon and Procedure

Mill states the canon of the Method of Agreement thus: '*If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree, is the cause (or effect) of the given phenomenon.*'* In this method, what we do is this. We observe two or more instances in which the phenomenon under investigation occurs. These instances should not all be alike; for if they were exactly the same no circumstance could be eliminated and consequently the cause would not be discovered. The positive instances should be of such a nature that they help us in eliminating all the circumstances except one, which one alone is present throughout. The procedure is according to the principle, as Mill explains, 'whatever circumstance can be excluded, without prejudice to the phenomenon, or can be absent notwithstanding its presence, is not connected with it in the way of causation.' And 'the causal circumstances

* Mill: *Logic*, Book III, ch. viii, sec. 1.

being thus eliminated, if only one remains, that one is the cause which we are in search of.' It may not be possible always that we find only one common circumstance. In that case, adds Mill, the agreeing circumstances 'either are, or contain among them, the cause.'

Let us suppose that the phenomenon under investigation is P. In order to discover its cause by the Method of Agreement we should examine a few instances in which P occurs and analyse the circumstances in each case. We may symbolically represent the results of analysis as follows:—

P ₁	<i>a, b, c, d</i>
P ₂	<i>o, c, p, q</i>
P ₃	<i>m, n, c, r</i>
P ₄	<i>x, c, y, z</i>

In the four positive instances of P analysed above, we find that only one circumstance is constant, and that is *c*, while the others are inconstant. So we conclude that *c* is probably the cause of P.

v. Examples

The method appears simple when the instances are represented by means of symbolic letters. But actual examples it is difficult to find because rarely is the Method applied exclusively by itself. This is a remark which holds good in regard to all the 'four' methods of Mill; and we shall expatiate on this in a later section of this chapter.

As an illustration of the Method of Agreement, we shall take the following case, though it is rather strained. Four lodgers in a hotel die. They are all different in nationality, profession, status, culture, etc. But a few hours before their death, they had their food, and an analysis of the food reveals poison. From this we may draw the conclusion that the four unfortunate persons died of poison in the food, and not of any other circumstance, since all other circumstances vary in the case of the four individuals.

In some text-books of logic the following example is given to illustrate the Method of Agreement. David Brewster took impressions of a piece of mother-of-pearl in bees-wax and resin, balsam, gum-arabic, lead, etc., and found the iridescent colours reproduced in each case. The different materials agreed with the mother-of-pearl after the impressions had been

made on them only in one respect. *vis.*, in bearing the same form. Hence Brewster concluded that the colours were caused by the form, and not by any other circumstance. The illustration appears simple enough. But really, as Latta and Macbeath point out, it may serve equally well as an example of the application of the method which we are going to discuss next, *vis.*, the Method of Difference. For, we not only *compare* the different materials after they have received the impression of the mother-of-pearl but also *contrast* the states of each of the substances before and after it has been 'impressed.' This only shows that it is extremely difficult to find an illustration of Method of Agreement by itself.*

vi. Limitations of the Method

The Method of Agreement is purely an observational method; and since it takes into consideration only positive instances, the results obtained through it can be only probable and never certain. We can say that *c* may be, and not must be the cause of *P*. That is, the Method of Agreement gives us conclusions which are problematic and not apodeictic. It informs us of invariability in the sequence of two events, and not of their necessary connection.

As we have shown above, it is very difficult to find cases where the Method of Agreement is exclusively applied. Even where we have a single common circumstance, it may be that that circumstance is not the cause. In such a case, the method would mislead us and show what has nothing to do with the phenomenon as the sole invariable antecedent. Suppose each of a few persons is given a glass of lemonade mixed with a different poison, and all of them die. it would certainly be wrong to conclude that they died of lemonade, the common factor.

Since the Method of Agreement functions through a vague and superficial analysis of its material, its usefulness is very limited. Especially where we have an apparent plurality of causes, the method can yield us no result. If heat be generated through combustion in one case, through friction in another, and through electricity in a third, how can the Method of Agreement go about its task here and discover the sole invariable antecedent? And even where cases of heat generated

* Latta and Macbeath: *Elements of Logic*, p. 335.

through one kind of agency alone are examined, we can only say that that agency is a cause of heat, and not that it is the sole cause.

The difficulties we have pointed out go to show that the Method of Agreement cannot take us very far in the investigation into causal connections. As one writer puts it, it is a mode of reconnoitering, suggesting well-grounded starting-points for experimental inquiry, but making no pretence at causal explanation. Like the methods of simple enumeration and analogy, the Method of Agreement will be found useful in the suggestion of a probable cause. As Mill himself recognises, this method cannot by itself prove causation; the chief use is to suggest hypotheses as to the cause.

THE METHOD OF DIFFERENCE

vii. Nature of the Method

The Method of Agreement takes into account only positive instances. In the presence of the phenomenon the supposed cause is present. But it should also be known whether the suggested cause is absent in the absence of the phenomenon. In a thousand instances *c* and *P* may be co-present. But if *c* is present even once in the absence of *P*, it cannot be its cause. So great is the importance of the negative instance. In a way the Method of Agreement too makes use of negative instances. The circumstances other than *c*, viz., *a*, *b*, *d*, etc., are eliminated because in their absence *P* is present. But the Method of Agreement does not take into evidence a negative instance from which *P* is absent, for it proceeds on the principle "Nothing is the cause of a phenomenon in the absence of which the phenomenon occurs." In order that the results obtained by this method might be rendered more probable, the scientific investigator should employ the complementary aspect of the same principle which we set forth in a previous section as the second rule of elimination, viz., "that is not the cause of a phenomenon in the presence of which the phenomenon fails to occur." This is what is done in the Method of Difference.

viii. Canon and Procedure

Mill states the canon of the Method of Difference thus: "*If an instance in which the phenomenon under investigation occurs and an instance in which it does not occur have every circumstance in common save one, that one occurring only in*

the former; the circumstance in which alone the two instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon."*

Two instances are examined, one positive and the other negative. These two should agree in all respects save one. That is, the circumstances that go with these instances should be the same except one. That one circumstance should be present in the positive instance and absent from the negative instance. Then, we may conclude that the circumstance is causally connected with the phenomenon.

It is not easy to observe two instances which have every circumstance in common save one, unless we make them so. Hence the Method of Difference requires that we have complete control over the circumstances. This is possible only in experiments. So the Method of Difference is essentially the method of experiment. If we start with a positive instance, we can create a negative instance by removing a circumstance. Or if we have a negative instance to start with, we can make a positive instance by introducing a circumstance. And if we notice that the phenomenon disappears when a circumstance is removed or appears when the circumstance is introduced, we may say that that circumstance is causally connected with the phenomenon. Mellone explains the method thus: "When the addition of an agent is followed by the appearance, or its subtraction by the disappearance, of a certain event other circumstances remaining the same, that agent is causally connected with the event."

Let us suppose that P is found in the context of Q and R, and is preceded by certain circumstances, *c*, *d* and *e*. Now the cause of P should be sought for among the preceding circumstances. The problem is: which of the three, *c*, *d* and *e* is the cause? That which is the cause among them must be such that if it is removed P disappears. Let us remove *c* and see what happens. If on the removal of *c*, P goes out, then, we may conclude that the two are causally connected.

Antecedents

c, d, e
d, e

Consequents

P, Q, R
Q, R

∴ c is the cause of P.

* Mill: *Logic*, Bk. III, ch. viii, sec. 2.

ix. Examples

Any experiment will serve to illustrate the Method of Difference:

(1) Galileo's experiment to show that air has weight is a simple illustration of the Method of Difference. We first weigh a vessel emptied of air, and then filling it with air weigh it again. It will be noticed that the vessel when filled with air weighs more. This difference must be due to the greater quantity of air in the vessel. Hence air has weight.

(2) What is known as the coin and feather experiment is another example of the Method of Difference. If a coin and a feather are allowed to fall from the same height, the coin reaches the ground much quicker than the feather. This is due to the fact that the resistance offered by the air is greater in the case of the feather than in that of the coin. If no such resistance were present, both would reach the ground at the same time. This can be shown by means of an experiment. A coin and a feather are dropped in the receiver of an air-pump. The feather flutters to the bottom of the receiver after the coin. Then the receiver is emptied of air, and the coin and the feather are again dropped. This time the two reach the bottom together. Therefore we have to conclude that the presence of air is the cause of the difference in the rates of falling.

The experiment may be analysed thus:—

Instance one: A coin and a feather are dropped at the same time in a jar *filled with air*; result: they reach the bottom of the jar at *different times*.

Instance two: A coin and a feather are dropped at the same time in a jar *emptied of air*; result: they reach the bottom of the jar at *the same time*.

x. Limitations of the Method

The Limitations of the Method of Difference are those which are but natural to experiment. The method can be pressed into service only in circumstances which are under our complete control. The production of two instances agreeing in all respects save one implies perfect command over the circumstances. Hence in the wider reaches of life where the observer has to be at the mercy of phenomena, the Method of

Difference cannot be employed. Under good experimental conditions alone it will be found useful.

The method presupposes extensive knowledge on the part of the experimenter. He should see that no disturbing or counteracting influences are present during the experiment. In introducing or removing an element he should make sure that nothing else is brought in or taken away. He should also take cognisance of the time factor which may cause changes without his knowledge. In a word, the experimenter should be circumspect and should see to it that nothing of material value passes by without his notice.

Even where all precautions have been taken, the Method of Difference cannot give the whole and the sole cause of a phenomenon. For example, the method may show that a lighted match is the cause of explosion. But the lighted match is not the whole cause; for gunpowder is required for the explosion. Nor is the lighted match the only or sole cause for producing explosion, since the same phenomenon may be effected by other agencies such as an electric spark. Thus, like the Method of Agreement, the Method of Difference too is not able to get over the difficulty presented by a plurality of causes. We may know by the method that an element or circumstance is *a* cause of the given phenomenon but not that it is the *only* cause.

Another limitation from which the Method of Difference suffers is this. In experiment we can reason only from cause to effect and not *vice versa*. Hence, the method will be useful in determining the effects of causes. But where a cause is to be discovered, one must adopt the indirect method of taking a supposed cause and seeing whether it produces the given effect.

THE JOINT METHOD OF AGREEMENT AND DIFFERENCE

xi. Nature of the Method

If the Method of Agreement is defective because it considers only positive instances, the Method of Difference also has its limitations in so far as it takes into consideration only one positive instance and one negative instance and lays down as a condition that the instances be the same in all circumstances save one. It is well-nigh impossible, as we have stated, to find two instances which differ only in a single circum-

stance. To procure such instances we should either introduce or subtract a circumstance and note the result. This is feasible only in experiments. When we have to deal with phenomena which do not admit of experimentation—and they are far too many—the Method of Difference will be of no help whatsoever. And since in this method only two instances, one positive and the other negative, are contrasted, the result obtained may not be so reliable as it might be if many in each kind were compared. This is what is accomplished in the Joint Method of Agreement and Difference.

The Joint Method effects a combination of the Methods of Agreement and Difference employing the principles of both, *viz.*, “that is not the cause of a phenomenon in the absence of which the phenomenon occurs,” and “that is not the cause of a phenomenon in the presence of which the phenomenon fails to occur.” A number of instances wherein the phenomenon occurs are first compared together. Those circumstances which are present in some of the instances and absent from others are eliminated. And the circumstance which is present in all of them is, it is suggested, the probable cause. Thus far we have the Method of Agreement. Next, a number of instances wherein the phenomenon does not occur are compared. If it is noticed that while the other circumstances are sometimes present and sometimes absent, the circumstance which was uniformly present in the first set of instances is absent uniformly from these negative instances, we have corroborative evidence to show that our original result is probably valid. Here we have the principle of the Method of Difference, since we observe the difference between the two sets of instances, *viz.*, the uniform presence of a circumstance in the positive set and its uniform absence from the negative set. The Joint Method has also been called ‘the Double Method of Agreement’, because it bases its conclusion on a double agreement, agreement in presence and agreement in absence. That is, the positive instances agree in having a common circumstance, and the negative instances agree in not having it.

xiii. Canon and Procedure

Mill formulates the canon of the Joint Method thus:
“If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more

*instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which alone the two sets of instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon.**

From what we have stated already about the Joint Method, the meaning of the canon must be clear. Let us suppose the phenomenon under investigation is P. We examine, say, four instances of P and analyse the circumstances as follows:

Positive instances:

1	<i>a, b, c, d</i>
2	<i>e, c, f, g</i>
3	<i>o, c, r, q</i>
4	<i>x, y, c, d</i>

By comparing the four instances we find that they agree only in one circumstance, *viz.*, *c*, and so conclude that *c* is probably the cause of P.

Then we proceed to observe the negative instances, *i.e.*, instances of the absence of P, and analyse the circumstances in each case.

Negative instances:

1	<i>d, e, f, g</i>
2	<i>g, o, r, q</i>
3	<i>x, y, d, a</i>
4	<i>l, m, n, o</i>

These instances, we notice, exclude the circumstance *c* which was uniformly present in the first set of instances. *C* is present where P is present and is absent where P is absent. Therefore, we infer, *c* is probably the cause of P. The negative instances go to strengthen the result obtained previously by comparing the positive instances, and so the probability of *c* being causally connected with P is increased.

In the actual application of the Joint Method, it is important to remember that both the positive and the negative instances should belong to the same field of inquiry. If the subject of inquiry lie in the department of Chemistry, for instance, the negative as well as the positive instances must be sought in the same department. There is no use of selecting

* Mill: **Logic**, Bk. III, ch. viii, sec. 4.

any 'instances, in which the phenomenon does not occur.' "If we are investigating the cause of rust, the jumping of a horse over a hedge, the growth of flowers, a man falling off his bicycle, are 'instances in which the phenomenon does not occur' but they are not very helpful to us."* It is only where the positive and negative instances are from a definite and identical field that we can hope to make our inquiry fruitful. The positive and negative instances, in a word, must be *in pari materia*.

xiii. Examples

As an example of the Joint Method Mill cites Wells's investigations into the formation of dew. Wells exposed a number of polished surfaces of different substances, and compared those on which there was an abundant deposit of dew and those on which there was little or no deposit. "It appears that the instances in which much dew is deposited, which are very various, agree in this, and, *so far as we are able to observe, in this only*, that they either radiate heat rapidly or conduct it slowly; qualities between which there is no other circumstance of agreement than that by virtue of either, the body tends to lose heat from the surface more rapidly than it can be restored from within. The instances, on the contrary, in which no dew, or but a small quantity of it, is formed, and which are also extremely various, *agree (as far as we can observe) in nothing except* in not having the same property. We seem therefore to have detected the characteristic difference between the substances on which the dew is produced and those on which it is not produced."

Another illustration of the Joint Method we may find in the reasoning that a man suffering from insomnia would adopt for detecting the cause of his disease. The patient would compare carefully the circumstances preceding his sleepless nights with those preceding nights of satisfactory rest. The sleepless nights are the positive instances. Let us suppose that the patient examines the circumstances preceding four such nights thus:

First night: prolonged study, heavy meal, mental quiet, a cup of strong coffee.

Second night: prolonged study, light meal, mental unrest, a cup of strong coffee.

* Latta and Macbeath: *Elements of Logic*, p. 336.

इति । एवं च वाग्दानप्रभृत्या सप्तमपदादोषदर्शने मरणादौ वा कन्यामन्यस्मै दद्यात्, नोर्ध्वमित्युक्तं भवति । अत एव कात्यायनः—

अनेकेभ्यो हि दत्तायामनूढायां तु यत्र वै ।

वरागमश्च सर्वेषां लभेतादिवरस्तु ताम् ॥

अथागच्छेयुरूढायां न तां पूर्ववरो हरेत् ।

पूर्ववरस्वदत्तं शुल्कमेव हरेन्न कन्यामिसर्थः । मनुरपि—

न दत्वा कस्य चित्कन्यां पुनर्दद्यात् विचक्षणः ।

दत्वा पुनः प्रयच्छेद्यः प्राप्नोति पुरुषानृतम् ॥

तदपि तेनैवोक्तं—

पञ्च पञ्चनृते हन्ति दश हन्ति गवानृते ।

शतमश्वानृते हन्ति सहस्रं पुरुषानृते ॥

पञ्च बान्धवानिसर्थः, तेषामेव प्रकृतत्वात् । वसिष्ठस्तु पाणिग्रहणादुपर्यप्यन्यस्मै दानमाह—

पाणिग्रहे कृते कन्या केवलं मन्त्रसंस्कृता ।

सा चेदक्षतयोनिस्स्यात्पुनस्संस्कारमर्हति ॥

नारदोऽपि—

उद्वाहिताऽपि या कन्या न चेत्सम्प्राप्तमैथुना ।

पुनस्संस्कारमर्हति यथा कन्या तथैव सा ॥

एतदपि दोषदर्शने वेदितव्यम् । अत एव यमः—

present in the positive instances and uniformly absent from the negative instances. This is not easy to obtain. Secondly, the incidence of a plurality of causes is not made impossible even in this method. In the case of sleeplessness, for instance, the method suggests that drinking strong coffee is the cause. But sleeplessness might be produced by other causes as well. *e.g.*, mental unrest or insufficient meal; and determining the cause of sleeplessness we should take into account conditions like temperament, constitution, general habits of life, etc. The Joint Method, like the others we have studied, can suggest only the probable cause. The degree of probability depends on the nature of the analysis of circumstances. If the analysis is superficial, the result will be unsatisfactory. In order to render the analysis adequate we should make use of experiments where possible; and where experiments are not possible, methods like statistics should be employed as instruments of analysis.

THE METHOD OF CONCOMITANT VARIATIONS

xv. Nature of the Method

The methods we have studied so far can be helpful in suggesting vaguely the causal connection between two phenomena. But the precise quantitative relation between the two phenomena cannot be determined through those methods. The aim of science is not to stop with the suggestion of a causal connection but to pursue further and understand the quantitative correspondence between cause and effect. In fact, the scientist often makes such correspondences between phenomena, when he observes them, the grounds for concluding that the phenomena so related are causally connected. If two things are related as cause and effect, variations in one of them must bring about variations in the other. Any circumstance which is unaffected by the variations of a phenomenon cannot be connected with it by way of causation. 'That is not the cause of a phenomenon which varies when it is constant, or is constant when it varies, or varies in no proportionate manner with it.' This principle of elimination is applied in the Method of Concomitant Variations.

Especially in cases where it is not possible to exclude a circumstance and thus obtain a negative instance, the Method of Concomitant Variations may be applied with profit. The

influence of gravitation on a body, for instance, cannot be eliminated. But it can be varied by moving the body from place to place. As Mill remarks, we cannot take away the earth from the pendulum, nor the pendulum from the earth, to ascertain whether it would continue to vibrate if the action which the earth exerts upon it were withdrawn. But by varying the circumstances we can show the influence of the earth on the oscillations of the pendulum.

Thus the Method of Concomitant Variations can be employed in cases where the conditions of the Method of Difference are not realisable. The Method of Difference requires the exclusion or introduction of a circumstance. Where this is not possible, the circumstance in question may be varied, and if the variation is followed by a corresponding variation in the phenomenon, then we may say that the circumstance and the phenomenon are causally connected.

The Method of Concomitant Variations has a further use. It may be employed as an auxiliary of the Method of Difference, "to give additional precision to a solution which that has found. When by the Method of Difference it has first been ascertained that a certain object produces a certain effect, the Method of Concomitant Variations may be usefully called in, to determine according to what law the quantity or the different relations of the effect follow those of the cause."*

xvi. Canon and Procedure

Mill states the canon of the method thus: *'Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation.'*

If a variation in *C* is followed by a corresponding variation in *P*, then we may say that the two are causally connected. The concomitant variations of two phenomena cannot by themselves show that one of the phenomena is the cause and the other the effect; for the varying phenomena might be the co-effects of a common cause. Hence the clause in the canon, 'or is connected with it through some fact of causation.'

* Mill: *Logic*, Bk. III, ch. vii, sec. 6.

The method may be symbolically represented as follows:

C_1	P_1
C_2	P_2
C_3	P_3
C_4	P_4

In this series we notice P increasing proportionately with the increase of C ; i.e., the variations are in the same direction. But this need not be always the case. The changes may also be in the inverse ratio. The essential condition of the method is that there shall be a determinate or definite quantitative relation between the variations of the events.

xvii. Examples

That friction is a cause of heat was proved by Mr. Joude through the Method of Concomitant Variations. By rubbing one substance against another heat is seen to be produced; and it is observed that the greater the quantity of force applied the greater is the amount of heat generated. The heat is greater or less in proportion as the force is greater or less. The concomitant variations in the quantities of friction and heat thus show that they are causally related.

As an example of the method Mill instances the case of determining the expansion or contraction of a body to be an effect of heat is to enlarge the dimensions of bodies; or what a body. but "we can modify it in quantity, we can increase or diminish it; and doing so, we find by the various methods of experimentation or observation already treated of, that such increase or diminution of heat is followed by expansion or contraction of the body. In this manner we arrive at the conclusion, otherwise unattainable by us, that one of the effects of heat is to enlarge the dimensions of bodies; or what is the same thing in other words, to widen the distances between their particles."

We said above that the Method of Concomitant Variations can be employed to reinforce a result obtained through the Method of Difference. It can be shown by means of an experiment that air is necessary for the audition of sound. When a bell is struck, it vibrates and sets up waves in the surrounding air. These waves strike the ear-drum which is set in motion. The mechanical motion of the ear-drum is

then transformed into electric currents of varying intensity in the nerves which lead to the brain. Finally these currents result in our hearing the sound.* This in outline constitutes the process of hearing. Now, if a bell-jar is taken, emptied of air, and struck, no sound will be heard. From this it is easy to conclude, on the principle underlying the Method of Difference, that the air is necessary for the transmission of sound. The same conclusion can be demonstrated by varying the density of the air in the bell-jar, instead of emptying the jar of the air. It will be found then that as the density of the air increases, the intensity of the sound increases, thereby showing that there is a causal relation between the two.

The Method of Concomitant Variations can be employed even in cases where experiment is not possible. Mill gives the following example: "Let us now suppose the question to be what influence the moon exerts on the surface of the earth. We cannot try an experiment in the absence of the moon, so as to observe what terrestrial phenomena her annihilation would put an end to; but when we find that all the variations in the *position* of the moon are followed by corresponding variations in the time and place of high water, the place being always either the part of the earth which is nearest to, or that which is most remote from, the moon, we have ample evidence that the moon is, wholly or partially, the cause which determines the tides."

xviii. Limitations of the Method

In many cases of causal investigation it will be found that the Method of Concomitant Variations is useful only as an auxiliary to one or the other of the methods studied already. When the present method is employed in experiments, we are really combining it with the Method of Difference. When it is applied to facts which are not experimentable, it is found combined either with the Method of Agreement or with the Joint Method.

In making use of the Method of Concomitant Variations it is essential that attention be paid to the nature of the variations. If the concomitant changes be superficial, the result will be misleading. To avoid error, as many varied instances of variation as possible must be observed and the

* See Sir James Jeans: *Science and Music*, p. 10.

nature of the correspondence in the variations must be precisely known.

THE METHOD OF RESIDUES

xix. Nature of the Method

The Method of Residues can be employed only at a very late stage in a causal investigation. If several parts of a complex phenomenon have been explained in terms of their antecedent circumstances, then the remaining part or parts of the phenomenon can be determined to be the effect or effects of the antecedent factors which are yet left over. Mill explains the principle of the method thus: 'Subducting from any given phenomenon all the portions which, by virtue of preceding inductions, can be assigned to known causes, the remainder will be the effect of the antecedents which had been overlooked, or of which the effect was as yet an unknown quantity.* If the portions of a complex phenomenon whose causes we already know are subducted from the whole phenomenon, we get the residual part which is to be explained. Similarly, if from the complex of antecedents, those which have been determined through previous inductions to be the causes of parts of the phenomenon are deducted, we are left with a balance of antecedents. The presumption then is that these antecedents are causally connected with residual phenomenon. Here too we base our conclusion on elimination, the principle being 'that is not the cause of a phenomenon which is known to be the cause of another phenomenon.'

From the nature of the Method of Residues set forth above it will be evident that the method involves an element of deduction. Observation gives us certain antecedents and consequents. Then we calculate the effects of known causes and subtract them from the total effect. The residual consequent that is got thereby we connect with the remaining antecedent. The process of calculation by which the results in this method are obtained is essentially *deductive* in character.

xx. Canon and Procedure

Mill states the canon of the method thus: '*Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents.*'

* Mill: *Logic*, Bk. III, ch. viii, sec. 5.

If it is known that the antecedents of P, Q, R are c, d, e , and if it is further known through prior inductions that R is the effect of e and Q the effect of d , it may be determined by the application of the Method of Residues that P is the effect of c .

c, d, e	..	P, Q, R
e	..	R
d	..	Q
$\therefore c$..	P

The Method of Residues comes in here only at the last stages. By previous inductions, *i.e.*, by the application of any of the methods considered already we must have known previously that e is the cause of R and d of Q. Then on the principle of elimination which is the basis of the Method of Residues we determine that c is the cause of P, for it alone is not the cause of any other factor in the complex of effects, while e which is the cause of R and d which is the cause of Q cannot be the cause of P.

The similarity between the present method and the Method of Difference can easily be discerned. Both require a positive instance and a negative instance. But while the negative instance is supplied for the Method of Difference by experiment, the Method of Residues gets it by previous inductions and through a process of subduction. On account of the similarity between the two methods Mill regards the Method of Residues 'as a peculiar modification of the Method of Difference.'

So far our explanation of the Method of Residues has proceeded on the assumption that all the factors of the causal as well as the effect complex are given at the start. But in actual scientific investigations this is hardly the case. We begin explaining a complex phenomenon and find that the antecedent circumstances we have examined provide only partial explanation. Then it is our duty to push the investigation further and discover the cause of the residual factor. Dr. Mellone offers the following explanation of the principle of the method: 'When any part of a complex phenomenon is still unexplained by the causes which have been assigned, a further cause for this remainder must be sought. Thus the method becomes a finger-post to the unexplained. As Latta and Macbeath re-

mark, the Method of Residues sets a problem rather than solves it*. The symbolic representation of the method would then take the following form:—

d, e	P, Q, R
e	R
d	Q
$?$	P

xxi. Examples

As examples of the results obtained by the Method of Residues we may cite two instances of astronomical discoveries.†

(1) There is what is known as Bode's Law in astronomy which helps us to remember the approximate distances of the planets from the sun.

If we give 10 (inches, millimetres or whatever we like) as the scale for Earth's distance from the sun which is 93 million miles, the distances of the other planets can be written down as follows:—

	Bode's Law	Distance
Mercury $4 + 0 =$	4
Venus $4 + 3 =$	7
Earth $4 + 6 =$	10
Mars $4 + 12 =$	16
(Minor Planets) $4 + 24 =$	(28)
Jupiter $4 + 48 =$	52
Saturn $4 + 96 =$	100
Uranus $4 + 192 =$	196
Neptune $4 + 384 =$	388

A look at the above table will enable us to know how the calculations are made. We write down a set of 4's, and add to them other numbers which are doubled every time, beginning with 3 for Venus.

It will be noticed that after Mars we have put down Minor Planets within brackets. Till about the end of the eighteenth century these had not been discovered. The Law worked all right up to Mars. But between Mars and Jupiter

* The Elements of Logic, p. 340.

† Sir J. Herschel says: "Almost all the greatest discoveries in astronomy have resulted from the consideration of residual phenomena of a quantitative or numerical kind." The two examples are adapted from H. H. Turner's interesting book, *A Voyage in Space*, Lecture IV.

there was a big gap. To fill this gap Bode thought that there must be a planet. He set a band of astronomers who were jokingly called the 'astronomical police' to look out for the missing planet. But as luck would have it, it was given to an astronomer (Piazzi) outside the band to discover the planet.* And further observations revealed not one planet but a good number of them. The first to be discovered, Ceres, was seen on the very first day of the nineteenth century, January 1, 1801. Three others, Vesta, Pallas and Juno, were all found by March 1807. Forty years later one more was discovered. And now we know nearly 1,000 of these little planets. What is important for us to note here is the circumstances that led to their discovery. Apparently Bode's Law† broke down between Mars and Jupiter. It could be made whole only if there were a planet (or planets) in between. While the other parts in the series were explained by the law, there was one part, viz., that between Mars and Jupiter, which could not be explained. This was the residual factor and needed for its explanation another planet (or planets) interposed between Mars and Jupiter. The Method of Residues thus played in the present case the role of a signpost to a new discovery.

(2) The fascinating story of the discovery of Neptune is another illustration of the application of the Method of Residues as a pointer to the unexplained. Taking into account the attraction of the sun and that of the other planets, it is possible to calculate the orbit of any planet. This was done for Uranus. But actual observations revealed that the planet did not follow the course calculated for it. There were perturbations in its path which could not be explained in terms of the attraction of the sun and that of the other known planets. These irregularities were the residual factors which needed explanation. Adams, an Englishman, and Leverrier, a Frenchman, calculated that there must be a yet undiscovered planet pulling Uranus out of the course which it should other-

*Piazzi was not looking for the planet, but he happened to find it, which was later on identified as the missing planet.

† Strictly speaking, the law should be called after the name of Titius who was the first to state it. But it is generally known as Bode's Law because of the sensational use Bode made of the law.

wise follow. Independently they calculated the whereabouts of this unknown planet.

J. C. Adams took his calculations to the Astronomer Royal at Greenwich; but as the latter was at dinner at that time, he could not be disturbed. Disheartened, Adams left a note for the Astronomer Royal, which read as follows: "According to my calculations, the observed irregularities in the movements of the planet Uranus may be accounted for by supposing the existence of an exterior body, the orbit of which is as follows." The Astronomer Royal did not take the note seriously. All that he did was to write to Adams, asking him what he regarded as a test question. Adams was disappointed, thought that his calculations must have gone wrong, and did not take further steps to verify them.

Meanwhile, Leverrier, who had become a famous astronomer by that time, unlike Adams, published several papers in which he explained his calculations and ended up by pointing to the place near which the new planet must be. The place of his calculation was very nearly the same as that of Adams. Leverrier set Galle, a German astronomer, to work, and on the very first night Galle found the planet.*

xxii. Limitations of the Method

The limitations of the Method of Residues we have already pointed out during the course of our explanation of the method. In the first place, the method will not be useful at the initial stages of an inductive inquiry. Only after a major portion of a complex phenomenon has been explained, the Method of Residues can be used for discovering the cause of the residual factor of the phenomenon. Secondly, it has been shown that the Method of Residues is not an independent method. It is a variant of the Method of Difference. Thirdly, a good part of the calculations involved in the application of the Method is deductive in character. Lastly, in actual practice the cause of the residue of the phenomenon is not readily present before us to be connected with the residue; it has to be discovered; and what the Method of Residues does is to warn us that we should not stop in our investigation till every part of the phenomenon has been explained.

*For a fuller account see H. H. Turner, *A Voyage in Space*, pp. 158-164.

A CRITICAL ESTIMATE

xxiii. Mill's Claim

Mill's claim on behalf of his methods is rather high-pitched. He says, "The four methods which it has now been attempted to describe, are the only possible modes of experimental enquiry—of direct induction *a posteriori*, as distinguished from deduction: at least, I know not, nor am I able to imagine, any others."* Again, "the business of Inductive Logic is to provide rules and models (such as the syllogism and its rules are for ratiocination), to which if inductive arguments conform, those arguments are conclusive, and not otherwise. This is what the Four Methods profess to be, and what I believe they are universally considered to be by experimental philosophers, who had practised all of them long before any one sought to reduce the practice to theory."†

xxiv. An Estimate

We shall examine the implications of Mill's claim and arrive at what appears to us to be a just estimate of the methods.

(1) Mill compares the status of the four methods to that of the syllogism and its rules, and says that the methods occupy a place in induction similar to that of the syllogism in deduction. Just as the Aristotelian logician would characterise deductive arguments that do not conform to the syllogistic rules as invalid, Mill would regard those inductive arguments as inconclusive which are not in conformity with the canons of his methods. In the chapter on 'The Limits of Syllogistic Reasoning', we disposed of the claim that the syllogism is the pattern of all mediate inference and showed that there are mediate inferences which are not syllogistic and cannot be expressed syllogistically. A similar answer must be given to the claim Mill makes on behalf of his methods. The principle which is the ground of the methods, *viz.*, elimination, is no doubt employed in inductive reasoning. But if it be maintained, as Mill does, that all inductive arguments must be capable of being expressed in one or the other of the forms of the four methods, then we must take exception to it. As:

* *Logic*, Bk. III, ch. viii, sec. 7.

† *Logic*, Bk. III, ch. ix, sec. 6.

Joseph remarks, the 'methods, as formulated by Mill, 'threaten us with a repetition of the same sort of mischief as arose from supposing that every argument could be put into the form of a syllogism'.*

(2) Mill characterises the 'four methods' as 'the only possible modes of experimental enquiry.' Evidently, he is wrong, and himself provides a refutation of his claim by recognising a 'fifth' method, *viz.*, the joint Method of Agreement and Difference, and by saying that the Method of Concomitant Variations is often employed to give additional precision to a solution which the Method of Difference has found, and that the Method of Residues is only a peculiar modification of the Method of Difference. Mill explicitly writes, 'The simplest and most obvious modes of singling out from among the circumstances which precede or follow a phenomenon, those with which it is really connected by an invariable law, are two in number. One is, by comparing together different instances in which the phenomenon occurs. The other is, by comparing instances in which the phenomenon does occur, with instances in other respects similar in which it does not. These two methods may be respectively denominated, the Method of Agreement, and the Method of Difference.'† It would be truer to say, stretching Mill's own logic still further, that the principle underlying the methods is one and the same, *viz.*, elimination of those circumstances that do not satisfy the conditions of being the cause of a phenomenon. The number, then, of the methods which Mill gives is arbitrary. Just as Mill combines the methods of Agreement and Difference in the Joint Method, we may combine any two or more in an inductive investigation.

(3) It would appear from Mill's account that the methods are self-complete and self-sufficient for purposes of inductive generalisation, requiring neither a before nor an after. In the opinion of their author, they are the only 'means which mankind possesses for exploring the laws of nature by specific observation and experience.' But this is far from the truth. We have already seen that the analysis of the given has to precede the application of the methods, and

* Introduction to Logic, p. 396.

† Logic, Bk. III, ch. viii, sec. 1.

that the work of analysis is the most difficult task to accomplish. It is only after the circumstances have been analysed and hypotheses as to the probable cause have been suggested that the rules governing causal relations may be applied and those circumstances which do not satisfy the conditions may be eliminated.

(4) Even then the methods do not prove a causal connection. Taking some of the examples given by Mill, Bradley shows how inconclusive they are. As a specimen of a truth ascertained by the Method of Agreement, Mill gives the proposition "Dogs bark," the instances upon which it is based being 'this dog, and that dog, and the other dog.' And as a truth made known by the Method of Difference, he cites the proposition "Fire burns," the argument for the conclusion being, 'Before I touch the fire I am not burnt; I touch it, and am burnt.*' As regards these examples Bradley has the following comment to offer: 'By seeing three barking dogs, I perceive that they "*have only one circumstance in common.*"' By standing in front of a burning fire-place, and then touching the fire and being burnt, I am to know that the two facts "*have every circumstance in common but one.*" Is this not preposterous? Surely, it is clear in the first case that Mr. Mill's way of arguing might prove just as well that all dogs have the mange, and in the second that every fire-place blisters. And these conclusions hardly seem to be sound."†

(5) We do not deny to the methods the value that properly belongs to them. Even before Mill formulated them, they were being used by scientists in their investigations, and they continue to be so used. But what we would like to urge is that Mill's claims on their behalf are extravagant. The methods can neither discover causal connections nor prove them. As Latta and Macbeath remark, "they can only give further confirmation or added probability to the connections already suggested. They narrow down the field of possible causes. And in some cases their main value is to suggest causes which can be further tested by other methods."‡

* Mill: *Logic*, Bk. III, ch. ix, sec. 6.

† *Principles of Logic*, p. 336.

‡ *Elements of Logic*, p. 326.

CHAPTER XXV

HYPOTHESIS AND EXPLANATION

i. Introduction

The goal of scientific induction is the explanation of facts in the light of the universal laws that govern them. That the universe is a systematic and coherent whole of inter-related parts, we have seen, is the fundamental postulate of induction. If facts are bound together, then, any given phenomenon must be explicable in terms of the connections that bind it. The quest that science is engaged in is for the discovery of connections amongst phenomena. Our knowledge of things can never be satisfactory if the systems to which they belong remain undiscovered. A thing becomes intelligible only when it is shown to be a case under a law or an element in a system. Isolated phenomena, unrelated particulars, incoherent facts—these constitute an idiot's lore and not the furniture of a sane mind. Hence the seeker after knowledge always strives to discern laws of connection.

Just as facts need to be explained in terms of laws, laws which are restricted and narrow require for their explanation wider and more general laws. Kepler explained the movement of Mars by formulating the law that all planets move in elliptical orbits. This law itself was explained by Newton in terms of the principle of gravitation which gives the reason for the revolution of planets in elliptical orbits. Thus the process of explanation has to go on till the completest systematization is reached. This is only an ideal for science. But, nevertheless, the scientific inquirer is guided by its light, and his one aim is to achieve ever-increasing orderliness.

It is not only the scientist that tries to explain. Even the man in the street endeavours to relate the facts of his experience. He knows, for instance, that quinine cures certain cases of fever, that some flowers are poisonous, etc. But his knowledge is empirical and the law that he formulates is an empirical law. He does not know *how* quinine cures malarial fever or *how* certain flowers are poisonous. Since he bases his generalisations on surface observation they are not universal in character. For his purposes which are mostly practical, they are enough. But the seeker after knowledge cannot

rest content with empirical generalisations. He wants to see all things together, to unify and to relate all phenomena. The difference, however, between empirical and scientific⁶ knowledge is not one of kind but of degree. The method of explanation is essentially the same whether employed by the common man in solving his everyday problems or by the scientist in his laboratory investigations of a highly complicated nature.

In this chapter we shall recapitulate what we said earlier regarding the stages through which the method of explanation has to pass, and point out the importance of hypothesis and its verification.

ii. The Method of Explanation

In Chapter XVIII we stated that the complete inductive method consists of four distinguishable stages: (1) observation and analysis of facts; (2) formulation of a hypothesis; (3) deduction of consequents from the formulated hypothesis; and verification, and (4) proof of the hypothesis. These constitute what Mr. Jevons calls the inductive method *par excellence*.

(1) With the observation of facts begins the inductive inquiry. The observer employs methods like enumeration and statistics to arrange and analyse the facts. The process of bringing order into facts starts even at the very beginning of induction. The investigator performs experiments wherever possible; and experiments as we have seen, are but controlled observations. Observation is not passive perception; it is active and selective. The facts that are relevant should not escape notice. No rules, however, can be given as to what facts are relevant. The trained eye of the scientist picks and chooses the right things.

(2) The next step in induction is the formulation of a tentative explanation or hypothesis on the basis of observed facts. The observer approaches facts not with a vacant mind. His past experience and the fund of knowledge he has to his credit suggest to him the possible or probable explanation of the new facts. Except in the light of a hypothesis or view, the facts will be meaningless. As T. H. Huxley says "Those who refuse to go beyond fact rarely get as far as fact." To go beyond fact is to read the law that governs it, to explain it in

terms of a principle. An attempt at explanation takes the form of a hypothesis.

(3) Since a hypothesis is only a tentative suggestion, it needs verification. Unverified hypothesis cannot satisfy the demands of science. The explanation provisionally offered must be tested. The testing of a hypothesis consists of two steps. First, the hypothesis is assumed to be true and its consequences are deduced. Then, the investigator proceeds to see if the consequences agree with the actual facts. If they agree, the hypothesis is verified. If not, it must either be rejected or modified in the light of fresh evidence.

(4) Verification is not proof. A hypothesis is verified when it is shown to explain the facts. It is proved only when no other hypothesis is found to explain them. We have dealt with this point already in Chapter XVIII and shall return to it later in this chapter.

Of the four stages in the method of explanation, the first was considered in detail in the chapter on Observation and Experiment, and by implication in our discussions of observational methods. We shall here deal with hypothesis and its verification and proof and then illustrate the method of explanation by means of examples.

iii. The Nature of Hypothesis

We have seen above that both in common life and in science we frame hypotheses for the explanation of facts, that it is impossible for man not to make suppositions to explain phenomena, that he is a theorising being whose observations are not random perceptions but perceptions with some purpose or other and that theorising implies the framing of provisional explanations or hypotheses. Thus in the inductive method hypotheses occupy a central place.

A hypothesis is a conjecture we make in order to explain a given fact or group of facts. It is "an attempt at explanation: a provisional supposition made in order to explain scientifically some fact or phenomenon."* The sight of a swollen river, for instance, makes us suggest that it must have rained in some part of the country drained by the river. If an individual suffers from typhoid fever, we guess that he should have been drinking impure water. Thus we are constantly

* P. Coffey, p. 121.

making hypotheses. The formation of hypotheses, as Creighton and Smart write, is simply the mind's response to the demand for explanation.

iv. The Formation of Hypothesis ✓

No rules can be given by means of which hypotheses may be formed. As Lloyd Morgan says, "It is idle to expect through the application of rules of scientific procedure to attain scientific insight; for the man of science, so far as he is creative is an artist."* And in the words of De Morgan, "A hypothesis must have been started, not by rule, but by that sagacity of which no description can be given, precisely because the very owners of it do not act under laws perceptible to themselves. The inventor of hypothesis, if pressed to explain his method, must answer as did Zerab Colburn (a calculating prodigy of the early nineteenth century) when asked for his mode of instantaneous calculation. When that poor boy had been bothered for some time in this manner, he cried out in a huff, "God had put it into my head, and I can't put it into yours."† It is the fertile brain that invents hypotheses; and no rules can be given for making the unfertile brain fertile."‡

This much however, may be asserted regarding the formation of hypotheses, *viz.*, that hypotheses are not received from without through sense-perception. They are creations of the imagination. Great scientific inventions have been made in moments of inspiration. It is only a scientific genius lit up with the spark of imaginative insight that can discover sound hypotheses. But not even a scientific genius can afford to dwell in the cloud-land of fancy without keeping close to the *terra firma* of facts. Uninformed imagination is worse than useless. It was against wild imagination that Newton took objection when he said '*Hypothesis non fingo*,' 'I do not imagine hypothesis.' The two, *viz.*, capacity to theorise and keenness to observe, should go together. Imagination must have as its material observation and experiment; only then can it become the means for building up theories. As Creighton and Smart put it, "the process of explanation may be

* **Comparative Psychology**, p. 307.

† **A Budget of Paradoxes**, Vol. 1, p. 86.

‡ **Pilsbury: Fundamentals of Psychology**, p. 407.

described as a fitting together of the facts given by observation, with the explanatory theories that the mind originates."

How the mind originates theories one cannot say. The creative mind of the scientist is like that of the artist. To the man of science one can only say as to the artists, in the words of Lloyd Morgan, "saturate yourself through and through with your subject and with all that bears, or may bear upon it, and *wait*.. If the flash of insight comes, treasure it, and then patiently work it out in all its bearings remembering that no art product is made convincing without labour. *Then* you may apply your rules of scientific method with profit and advantage. And if it does not come, still *wait*."*

v. The Function of Hypothesis

The hypothesis, we have seen, is a tentative theory invented for the explanation of observed facts. Its main function is to systematise the known facts. It also serves as a guide to further enquiry. A hypothesis may perform these tasks; but thereby it does not become true. If fresh facts are found to be incompatible with it, the hypothesis should either be rejected or modified. Such a hypothesis is called a *working hypothesis*.

It is accepted provisionally because it *works*, i.e., provides an explanation for the facts. Though likely to be rejected later, it has value because it forms a basis for further exploration. That electricity is a kind of fluid was a working hypothesis which has led to many an important result. The Ptolemaic theory, *viz.*, that the earth is the centre of the universe, explained all the astronomical facts known in those remote times, and on the basis of it even eclipses and other phenomena were correctly predicted. But as knowledge advanced, certain new facts were brought to light which could not be fitted into the Ptolemaic system. Hence it was rejected and the Copernican theory, *viz.*, that the sun is the centre of the universe and that all planets including the earth move round the sun, came to be adopted, as it introduced a new simplicity into the observed facts. From these considerations what we learn is this: a hypothesis, though it may be discarded later, has a double function to fulfil. It systematises

* *Comparative Psychology*, p. 307.

and brings into order known facts, and it shows which way the scientist has to follow in his search for truth.

vi. Requirements of a Good Hypothesis

Writers on logic prescribe certain conditions which a good hypothesis should satisfy. The canons usually laid down are: (1) that the hypothesis shall be conceivable and not absurd; (2) that it shall not violate any of the principles on which our knowledge as a whole rests; (3) that it shall be such that deductions can be made from it, and (4) that the relevant observed facts must be found to be in harmony with the consequences drawn from the hypothesis.*

The first condition, *viz.*, that the hypothesis shall be conceivable and not absurd, must be applied with caution. What seems conceivable to one may appear absurd to another. Thus for a long time it seemed incredible that the earth should be round and revolve on its own axis. Any new innovation or discovery puzzles the minds of the majority of people, and reads like a fairy tale. And so inconceivability as applied to a bad hypothesis ought not to mean difficulty in understanding it but self-discrepancy. That is to say, a good hypothesis is that which is self-consistent and not self-contradictory.

The second requirement, *viz.*, that the hypothesis should not contradict any of the known laws of nature, is not always a sound test of a good hypothesis. Many a discovery, when it was first made, seemed to contradict some law of nature known at that time. The Copernican theory, for instance, was in conflict with the Ptolemaic system. When there is conflict between a suggested hypothesis and a section of our present knowledge, it is the latter that has to give way. In such cases it is our past knowledge that must be readjusted in the light of the new hypothesis. There is, however, another way of interpreting the second requirement. It is this. There are certain fundamental assumptions for each science; and there are certain general principles of all knowledge. A hypothesis should not be incompatible with the assumptions of the science which makes it as also with the general principles of knowledge as such. As Joseph remarks, "It would be an

* Latta and Macbeath, p. 362 ff.

Creighton and Smart, p. 338 ff.

illegitimate hypothesis on the part of a bank clerk confronted with a small discrepancy in his books to suppose that on this occasion $2+2$ make 3."

The third rule, *viz.*, that the hypothesis shall be of such a nature that deductions can be made from it is the most important of all. A hypothesis to be of scientific value must be capable of proof or disproof. 'What is not proved,' as Bosanquet says, 'is not really discovered.*' Unless deductions can be made from the hypothesis, it is impossible either to prove or disprove it. Therefore a scientific hypothesis should furnish a basis for deduction. We must be able to say what should happen if the hypothesis were true. Any hypothesis that does not satisfy this condition is called a *barren hypothesis*. It would be a barren hypothesis, for instance, to suggest either that angels send us showers or that demons drop bombs. We do not know what these are and what they are capable of, and we can never conjecture as to what things they cause. A good hypothesis, then, should not be barren. It must admit of deductive development.

The fourth canon, *viz.*, that the consequences drawn from the hypothesis and the relevant observed facts should be in agreement, is also of importance. The main purpose for which a hypothesis is framed is to explain all the observed relevant facts. If even a single fact be contradictory to the hypothesis, then it is the hypothesis that must be given up. The soundness of a hypothesis depends on its being faithful to the relevant facts.

vii. The Verification of Hypothesis

It is not enough that a scientific investigator possesses a mind fertile in ideas. He must love truth more than his theories; and he should be ready to relinquish the latter when they contradict truth. Every hypothesis, therefore, should be subjected to vigorous tests. And the method of testing is called verification. There are two stages, as we have already said, in the process of verification: (1) Assuming the hypothesis to be true, we deduce the consequences that follow from it. (2) Then we observe facts with a view to see if they and the deduced consequences agree. If they do agree, the hypothesis is regarded as verified. If they do not, it is either discarded or so modified as to fit the facts.

* *Logic*, Vol II, p. 119.

The process by which it was demonstrated that the atmosphere has weight illustrates the method of verification eminently well. Galileo noticed that water raises in a pump to a height of about 33 feet. But he was not able to explain why it stopped at that point. Torricelli suggested that it was the weight of air on the surface of the water which pushed the water up the pump when there was no air in the pump pressing the water down. If this hypothesis were true, he argued, the weight of air must be able to lift mercury, which is 14 times heavier than water, to one-fourteenth of the height. He filled a tube about 34 inches long with mercury and turned it upside down into a basin containing mercury. The mercury in the tube began to settle and finally rested at a height of about 30 inches. This result was in agreement with the consequences of the hypothesis and so the hypothesis was verified to be true.

viii. The Proof of Hypothesis

The proof of a hypothesis is not complete with its verification. If the consequences of a hypothesis agree with facts, the hypothesis is verified, and not proved. We cannot argue that because the deduced consequences of the hypothesis agree with the relevant facts, the hypothesis is finally established. If we do argue that way, we shall be committing the fallacy of affirming the consequent thus:

If the hypothesis is true, such and such consequences should follow.

Such and such consequences are observed.

Therefore, the hypothesis is true.

In order to remove the defect which vitiates the above argument, it must be shown that the hypothesis in question is the only one that can explain the facts. This is, indeed, difficult, and is rarely accomplished in science. But the scientist should ever hold that complete explanation as his ideal. He must gather evidences from all related fields of inquiry. This is what is known as 'the consilience of results.' When several groups of facts gathered from different spheres of investigation point towards the hypothesis as the one conception capable of systematising them, the hypothesis can be regarded as established almost beyond doubt.

The verification and proof of a hypothesis is thus not an

easy task. The difficulty increases when we happen to meet with two or more rival theories which appear equally plausible. In such cases what we have to do is to search for a crucial instance (*instantia crucis*) or devise a crucial experiment (*experimentum crucis*) which will confirm one hypothesis and at the same time falsify the others. We may cite as an example the crucial experiment which disproved the caloric theory of heat and proved the theory that heat is a form of motion. According to the caloric theory, heat is an imponderable fluid held in the pores of substances. As a thing gets colder the fluid flows out of its pores, and as it gets warmer the fluid flows into them. If two bodies at different temperatures are kept in contact, the hot one gets colder and the cold one gets hotter. This is because the caloric passes from the hot body to the cold one till both of them attain the same temperature. The explanation that the caloric theory offered seemed plausible. But another theory was suggested, *viz.*, that heat is a form of motion. Sir Humphry Davy performed an experiment which provided a crucial test. Two pieces of ice were protected from all sources of heat and were kept rubbing together. Soon the ice melted and got transformed into water. Without heat water could not have been formed. As the cause of heat there was no hot body but only motion. This test served to justify the motion theory and discredit the caloric theory.

ix. Examples

We shall add here two examples to illustrate the complete method of explanation which is the inductive method.

(1) One morning I enter my study and find things thrown pell-mell, the inkpot overturned, sheets of paper lying on the floor, window panes broken, etc. These are the facts observed, calling for explanation. I am naturally annoyed and try to guess as to who or what should have been the cause of all this mischief. I think a burglar might have broken into my room and caused this disorder. This is the hypothesis I form at first. Then I argue within myself what the consequences should be if the burglar had entered the room. He must have tampered with the lock or made a way through the window in order to gain entry into the room. He must have opened the drawers or the safe and removed the

valuables kept therein. These are some of the consequences that may be deduced from the hypothesis. Then I go about the room and examine things closely to see if the consequences are to be found. But if I notice that the valuables in the room are safe and the lock has not been meddled with, I decide that I was wrong in my guess, and try to evolve a better explanation. I now think that a strong gust of wind might have been the cause of the disorder. If this hypothesis were true, I ask, what should be observed? Not only in my room, but even outside, there should be evidence of a recent cyclone. The trees around must wear a weather-beaten appearance, leaves should have fallen in profusion, etc. Then I go out and observe. If I find that the trees and other things stand witness to a storm overnight, I decide that my second guess is correct.

(2) Animals in the arctic regions are observed to be white in colour. This fact calls for an explanation. On a preliminary observation of some of these animals we might conjecture that the whiteness is caused by the action of the environment, which is white, on the animals. If this hypothesis be true, all the animals in those regions without an exception should be white. But we notice that some of the arctic animals are not white. Hence rejecting the suggested explanation we search for another. That the white colour of the animals checks radiation and helps to keep them warm is a plausible hypothesis. But even then, all the animals should be white, since all of them are equally exposed to the coldness of the arctic regions. Our observations, however, reveal some non-white animals. And so even the second hypothesis we discard. A third hypothesis is that the whiteness of the arctic animals is a protective colouring. The colouration is for easy concealment. If this explanation be sound, all those animals which need protection should be white. We observe again with patience and care and find that the facts agree with the consequence we have drawn from the hypothesis. Thus we are assured that there is truth in the theory that the colouration has a protective value.

x. Hypothesis, Theory, Law and Fact.

In the course of this chapter we have used the term 'theory' in some places for the sake of variation as an equi-

valent of 'hypothesis.' But strictly speaking, a distinction must be made between the two. The provisional suggestion or unverified guess that is made for explaining facts is a 'hypothesis.' When the hypothesis is verified and thus acquires a fair measure of probability, it is called a 'theory.' And when the workability of the theory has been sufficiently proved, and when the theory has been well established, it is called a 'law.' Thus the three terms, *viz.*, hypothesis, theory and law, indicate different stages in the process of explanation from the unverified tentative supposition to the well-established law of Nature.

The term 'fact' which we have used often is highly ambiguous. It is generally contrasted with theory. But in reality no such contrast exists. We cannot get at brute facts which do not involve a theory. We say the table in front of us is a fact. But do we see the other side of the table? When we interpret what is presented to us as a table, do we not make a series of suppositions? That there is no fundamental contrast between fact and theory will be evident even from our ordinary usage. We refer to indisputable laws as facts. We speak to-day of the fact of gravitation. But to the world of Newton it was only a theory.

CHAPTER XXVI

CLASSIFICATION, NOMENCLATURE AND TERMINOLOGY

i. Introduction

In the last chapter* we set forth the various stages involved in the inductive method and explained the nature and importance of hypothesis. It must be clear by now that the goal of induction is explanation. To explain a fact is to relate it to the system to which it belongs. We explain by generalising, by seeing phenomena in their proper perspective as belonging to systems. There are certain auxiliary processes without which generalisation is impossible. These we shall study in the present chapter.

ii. Classification

We are able to generalise and explain because each thing is not an isolated entity but is a member of a kind or class. The mind cannot rest content, as we have often said in the

course of this book, with the bare particular. It arranges things according to kinds, reduces them to unity and order. The process by which it accomplishes this is called *classification*. Classification is 'a mental grouping of facts or phenomena according to their resemblances and differences.'

In Chapter V we had occasion to discuss classification in its relation to division. While division starts with a genus and distinguishes the species within it, classification, we said, starts with the particulars and groups them into kinds. The end or aim of the two processes is the same, *viz.*, an intelligible arrangement of objects, discovering the system in them, or reducing them to order. The fact that the knowing mind employs both division and classification together is another evidence to show that deduction and induction are complementary processes of thought.

Generally a distinction is made between two kinds of classification: (1) artificial or diagnostic classification, and (2) natural or scientific classification. Artificial classification is based on some superficial quality in the things classified and is motivated by a practical purpose. Plants, for instance, may be classified into thorny and non-thorny, drugs into poisonous and non-poisonous, commodities into essential and non-essential, etc. In each of these cases of classification some special purpose is involved and the principle of classification is more or less arbitrary. The main object is to understand a thing in relation to a practical end. A catalogue of books arranged in the alphabetical order helps one to choose and identify the books one wants. *Paradise Lost* and *Paromithasara* have nothing in common except the fact that they begin with the letter P. But as they are arranged in a catalogue, it is easy to pick them out. All artificial or diagnostic classifications are of this sort. They are groupings of facts according to some point of similarity, chosen arbitrarily for a special purpose. As distinguished from these, natural or scientific classification is based on a consideration of essential attributes. It is 'a classification made for the purpose of expressing the actual order, unity or system of the things classified, their real fundamental relations to one another and the unity of which they are elements or differences.'*

*Latta and Macbeath: *Elements of Logic*, p. 154.

botanist, for instance, classifies plants according to their nature, out of a theoretic interest and not from any practical motive. Understanding and not utility is his aim. Scientific or natural classification exhibits real affinities among things. The distinction between the two kinds of classification, however, is not absolute. Even the artificial classification is based on a property that actually belongs to the things classified; only that property may not be an essential one. But often an artificial classification serves as a stepping stone to scientific or natural classification. Every natural classification, on the other hand, is to a certain extent artificial, for it is determined by the special purpose of the particular science which makes it. The same objects may be classified differently, *e.g.*, by physics and chemistry. It is with the progress of knowledge that the element of artificiality gets gradually eliminated.

iii. Nomenclature and Terminology

The act of naming is one of the first steps in knowledge. Civilised thought is impossible without the help of names. Great discoverers and pioneers in knowledge add fresh names to language. Giving names is both the cause and the consequent of an intelligible arrangement of things, qualities and relations. In the preceding section we saw how essential to thought is the process of classification. It is not possible to classify without the help of names; and when a new class of things is discovered it must be assigned a new name. Men of genius who see things clearly and in their context create names that endure. The languages of the West, *e.g.*, owe a great deal to Plato and Aristotle. "These two great minds," says Bosanquet, "mapped out the world of knowledge in its essential features much as we have it before us now, and gave to its main divisions the names which they still retain. Or, again, what a gigantic advance was made by the work of Linnaeus, though it now serves as the stock example of an 'artificial' classification! It was the indispensable starting-point for the more profound and rational researches of modern times, and thus if not one of the most arduous, at least one of the most valuable, of scientific achievement."*

* *Logic*, Vol. I, p. 6.

The progress of science, then, depends not a little on names. Names there must be for every important idea or meaning. And the meanings of names must be fixed and unambiguous. In the economy of thought names play a very high role. They subserve the same purpose in thought as; it has been well said, "the binding does to the books of a library; without such, the mind would resemble a library of books, all in separate leaves, confusedly mixed."

There should be a distinct name for each class or group of things that a science studies. The system of such names is called *nomenclature*. Every science has a nomenclature of its own. Psychology, *e.g.*, has its names for the laws of behaviour, the psychological types, instincts, emotions, etc. There are names for the various classes of animals in Zoology. Botany has its names for the different kinds of plants; and so on. The rule by which sound nomenclature proceeds is the same as the one which governs logical division, *viz.*, *per genus et differentiam*. The name of a class must indicate the essential nature of the objects that constitute the class. As we have observed above, the purpose of a good name is to economise thought and help us in understanding with ease the thing signified. If names are given to classes arbitrarily, it would be difficult to remember them. Hence good nomenclature consists in giving names that are significant to the fullest degree. Things are members of classes, and classes are related as higher and lower. The name of a lower group should be formed by adding to the name of the higher group the distinguishing feature or the differentia of the lower. There is no use in calling a new thing or class by the name of its discoverer. Such a procedure may have a historical value. Its significance for science is next to nothing. "What am I the better for hearing a rare moss called *Hedwigia Hornschuchiana*, beyond being led to infer that Germany has or had, two botanists, one called Hedwig and the other Hornschuch? On the other hand, when I am told that such a moss is called *Trichostomum lanuginosum*, I am, on supposition of previous knowledge of *Trichostomum*, presented with a definition, *lanuginosum* ('woolly') expressing the differentia of this species in the genus *Trichostomum*, even as *Trichosto-*

mum does that of the genus when viewed as species of the higher genus which contains it.”*

There should be names not only for classes of things but also for clearly describing the parts, qualities, and activities of things. A system of such names is called *terminology*. Each science must have a vocabulary of its own. In the science of Botany, for example, there is a system of terms with settled and significant meanings. “When a leaf is long and very narrow, it is said to be linear; when the length is three or more times as great as the breadth, and the broadest part is below the middle, while the summit is tapering, the leaf is described as ‘lanceolate’; when the broadest part is above the middle, and the blade tapers towards the base, the leaf is called ‘cuneate’; and when the blade is broadly cuneate with a rounded top we say that it is ‘flabelliform.’”†

CHAPTER XXVII

FALLACIES OF INDUCTIVE REASONING

i. Introduction

Our account of Induction will be incomplete if we do not discuss the nature and kinds of fallacies that one is likely to commit in inductive reasoning. What a fallacy in logic is we have explained already.‡ Thinking is a difficult task, more difficult than manual labour. The average man tries to avoid it, if he can. He accepts the current phrases and prevalent notions uncritically. He feels relieved when the task of thinking is done for him by some one else. Naturally, he is easily misled and falls into error. Mental indolence, thus, is the cause of most of man’s delusions. If these are to be removed, habits of thinking must be formed through cultivation of a vigilant and critical outlook. Incredulity within limits is a virtue. He who has not doubted can never know. Logic and philosophy would have ill-served their purpose if they do not rouse in our minds a questioning spirit. It is with a view to put us on our guard against the common tendency

* See Boyce Gibson: **The Problem of Logic**, p. 68.

† **The Problem of Logic**, p. 67.

‡ See Chapter XVII.

to be led into the worn-out ruts of uncritical thinking that an account of fallacies is given in treatises on logic.

ii. Bacon's Idols

Francis Bacon gave the name 'Idols' (*Idola*) to the false notions which are deep-rooted in the human understanding and beset men's minds. "The human understanding," he says, "resembles not a dry light, but admits a tincture of the will and passions, which generate their own system accordingly; for man always believes more readily that which he prefers. He, therefore, rejects difficulties for want of patience in investigation; sobriety, because it limits his hope; the depths of nature, from superstition, the light of experiment, from arrogance and pride, lest his mind should appear to be occupied with common and varying objects; paradoxes, from a fear of the opinion of the vulgar; in short, his feelings imbue and corrupt his understanding in innumerable and sometimes unperceptible ways."^{*}

The Idols are classified by Bacon into four kinds. He calls them (1) Idols of the tribe (*Idola tribus*). (2) Idols of the den (*Idola specus*), (3) Idols of the market (*Idola fori*), and (4) Idols of the theatre (*Idola theatri*). These Idols are prejudices that prevent man from understanding aright, cloud his vision and conceal the truth.

The idols of the tribe are inherent in human nature and the very tribe or race of man. They are illusions common to all men. There are certain prejudices shared by mankind as a whole. For instance, the human understanding, from its peculiar nature, usually supposes a greater degree of order and equality in things than it really finds. Another bias common to the entire human race is the tendency to ignore unfavourable instances and inconvenient facts. Like uneven mirrors, men's minds impart their own properties to the objects which they reflect and distort and disfigure them.

The Idols of the den are illusions peculiar to each individual. In addition to the errors common to the human race, each man has his own individual den or cavern from which he looks at the world. His disposition, education and intercourse with others, his reading and the authorities whom he reverences and admires—all these endow him with a parti-

^{*} *Novum Organum*, Book I, Aphorism, xlix.

cular point of view or angle of vision which distorts his knowledge. Bacon gives a few examples of the Idols of the den. "Some men become attached to particular sciences and contemplations, either from supposing themselves the authors and inventors of them, or from having bestowed the greatest pains upon such subjects, and thus become most habituated to them." If they apply themselves to philosophy and contemplations of a universal nature, they corrupt them by their preconceived fancies. Other instances of the kind of fallacy we are considering are these. Some men observe the differences of things, others their resemblances. "Some dispositions evince an unbounded admiration of antiquity, others eagerly embrace novelty." Thus each individual has his predisposition and prejudices that colour the results of his intellectual pursuits and beliefs.*

The Idols of the market are errors in conversation, prejudices embodied in words. We think that reason is the mistress of words, while, in fact, words every often tyrannise over reason. Words are generally formed in a popular sense. But when they are used in contexts which require precision and accuracy, they are misleading and serve as obstacles to thought. There are two kinds of Idols imposed upon the understanding by words. Names may be given to things which have no existence. Or there may be names which, though referring to actual objects, are 'confused, badly defined, and hastily and irregularly abstracted from things.'†

The Idols of the theatre are illusions 'which have crept into men's minds from the various dogmas of peculiar systems of philosophy, and also from perverted rules of demonstration.'‡ These Idols are not innate, nor do they enter the mind secretly. They are the results of theories and dogmas one consciously accepts; *e.g.*, those who favour the Dialectical Materialism of Karl Marx have their own dogmas, while those who accept a particular Church have quite a different set of beliefs.

The four kinds of Idols stand as hurdles in the way of a true understanding of Nature. They "must be abjured and

* *Novum Organum*, I, lii-lviii.

† *Ibid*, I, lix and lx.

‡ *Novum Organum*, I, xliv.

renounced," says Bacon, "with firm and solemn resolution; and the understanding must be completely freed and cleared of them, so that the access to the kingdom of man which is founded on the sciences, may resemble that to the kingdom of heaven, where no admission is conceded except to children."*

We have so far explained in this chapter Bacon's classification of the errors to which we are subject in inductive reasoning. But this classification is not a satisfactory one. It is difficult to say, *e.g.*, which illusion is an Idol of the tribe and which an Idol of the den. The first variety given by Bacon really includes all the rest. Therefore, taking our clue from the various processes involved in induction, we shall classify the fallacies into those of observation, analogy, and explanation.

iii. Errors of Observation

These we have already explained in the chapter on Observation and Experiment. Errors in observation are committed either due to insufficient observation or on account of confusing an observed fact with a preconceived theory. Over-looking facts which ought to be observed is non-observation. Wrongly interpreting the facts observed is mal-observation.

(a) *Non-observation*: Generally the error of passing over instances is due to bias. We have a tendency to consider only those facts which would support our theory and neglect those which would invalidate it. We direct our attention to positive instances and disregard negative instances. We mark when we hit and not when we miss. This is a characteristic of mankind as a whole, and is one of those idols which Bacon calls Idols of the tribe.

(b) *Mal-observation*: Wrong interpretation of sense-impressions is mal-observation. Here too there is the work of bias and pre-formed theories. As Welton says, "The rustic who takes a tombstone brightened by the rays of the moon for a ghost or who interprets a donkey's bray as the voice of a departed ancestor, falls into the fallacy we are now considering. 'All conjurer's tricks' appeal to the innate facility with which mankind observes badly."

iv. Fallacies of Analogy

(a) To the illusions arising through the wrong use of

*Ibid., I, lxviii.

words, Bacon gives the name 'Idols of the market.' A frequent misuse of words occurs in analogical reasoning. When metaphorical language is faultily employed, reasoning by analogy goes wrong. The metropolis, *e.g.*, is compared to the 'heart' or the 'head' of the body; and it is argued that just as, when the heart grows too big or the head dropsical, the other parts of the body decay, even so when the metropolis develops, the other parts of the country are impoverished. Here the error in the reasoning is due to the metaphorical use of the words 'head' and 'heart.' A little reflection will make us convinced of the difference that there is between the living body and a country.

(b) A second source of faulty analogy is failure to distinguish between essential and non-essential properties. Sound analogy must have as its basis resemblance between the two things compared in essential points—points essentially connected with the inferred feature. 'A child has come to know that, when the dog is pleased, he wags his tail. On this he argues that, when the cat wags its tail, it must be pleased.'* The child's argument here is a case of analogical reasoning. He observes a resemblance between the dog and the cat as regards wagging of the tail. He knows that the dog wags when he is pleased, and therefore concludes that the cat wags because it is pleased. But the resemblance is not an essential one such that it may serve as the ground of the inferred character in the cat. Hence the child's analogy is unsound.

v. Fallacies in Explanation

(a) Our generalisations are very often based on insufficient evidence. We observe a few instances, neglect the negative cases, and hastily generalise. The fallacy of non-observation which has already been explained is a species of illicit generalisation. All superstitions and sweeping statements are cases of hasty generalisation.

(b) When sequence is mistaken for consequence, succession for the causal connection, the fallacy known as *post hoc, ergo propter hoc* is committed. To argue that 'A is because of C since it is after C' is fallacious. 'It is the fallacy of false cause. The nature of this fallacy was discussed in Chapter XX. We add here an interesting example given by Minto.

* Bradley: *Principles of Logic*, p. 324.

"The believers in Kenelm Digby's 'Ointment of Honour' appealed to experience in support of its efficacy. The treatment was to apply the ointment, not to the wound, but to the sword that had inflicted it, to dress this carefully at regular intervals, and, meantime, having bound up the wound, to leave it alone for seven days. It was observed that many cures followed upon this treatment. But those who inferred that the cure was due to the bandaging of the sword, failed to observe that there was another circumstance that might have been instrumental, namely, the exclusion of the air and the leaving of the wound undisturbed while the natural healing process went on. And it was found upon further observation that binding up the wound alone answered the purpose equally well whether the sword was dressed or not."*

In Shakespeare's *Henry IV* we have an instance where the absurdity of the belief that what goes *before* must be the *cause* is pointed out by Hotspur in his retort to Glendower.

Glendower:

At my nativity
The front of heaven was full of fiery shapes,
Of burning cressets: and at my birth
The frame and huge foundation of the earth
Shaked like a coward.

Hotspur: Why so it would have done at the same season, if your mother's cat had but kittened, though yourself had never been born.

(c) Another fallacy which belongs to this group is using a generalisation which is restricted in scope forgetting the restriction. The law that water boils at 100 degrees Centigrade at sea-level is true. But one might fail to notice the important condition, *at sea-level*, and expect that water should boil at that degree even on a hill-top. The same fallacy results when there is an undue extension of respect for authority. A man may be an acknowledged authority in a particular branch of learning. But it is illegitimate to swear by his name in connection with some other subject.

(d) It is a mistake in inductive reasoning to give up a well-established law because we meet with an apparent excep-

* *Logic, Inductive and Deductive*, p. 296.

tion. While testing a law, there may be counteracting forces; and consequently the law may appear not to function in certain cases. On that account we should not think that the law is invalid. It is true that according to the law of gravitation bodies near the earth generally tend to fall towards the earth. But birds fly without falling, so also balloons and aeroplanes. If one believes that these are real exceptions not governed by the law of gravitation, one would be mistaken. These very instances, if analysed and studied carefully, would prove the principle and not violate it.

(e) The cause of a phenomenon is the sum total of conditions positive and negative which being realised the phenomenon follows and in the absence of which it does not. In ordinary usage we do not stick to this definition of cause, but regard that as the cause which is the most striking condition. For a communal riot, for instance, one may point out a particular incident as the cause. But strictly speaking, the incident was only the occasion, while in the cause must be included so many other conditions like the strained relations between the communities concerned, the unhealthy emotional climate, bad leadership, etc. Failure to take note of all the factors contributing to an occurrence will result in unscientific and erroneous causal explanation.

(f) Sometimes it may not be possible to say which is the cause and which the effect in a complex of phenomena. In such cases the investigator should not arbitrarily choose a factor and call it the cause. "That it is by no means easy to determine in all cases which are the determining and which the determined elements in a complex phenomenon is illustrated by the fact that meteorologists are not agreed whether the copious and sudden downfalls of rain which usually attend thunderstorms are the cause or the effect of the electric discharge. The common opinion is that they are the effect, but Sir John Herschel held that they were the cause."*

(g) In certain cases the determination of cause and effect may be made difficult because of the fact that the phenomena in question may be reciprocally related. Sir G. C. Lewis says, "It happens sometimes that, when a relation of

* Welton: *Intermediate Logic*, p. 464.

causation is established between two facts, it is hard to decide which, in the given case, is the cause and which the effect, because they act and react upon each other, each phenomenon being in turn cause and effect. Thus, habits of industry may produce wealth, whilst the acquisition of wealth may produce industry; again, habits of study may sharpen the understanding, and the increased acuteness of the understanding may afterwards increase the appetite for study."

(h) Individual prepossessions are responsible for a class of fallacies which Bacon calls 'the Idols of the den.' Each individual is shut up in his cave or den and views things from his own standpoint. The dreamy idealist and the hard-boiled realist come to different conclusions regarding the nature of reality because their approaches differ, their dispositions are at variance.

(i) Just as each individual has his prejudices and affections, each age and cultus have their pet notions and theories. Bacon calls them 'Idols of the theatre.' The Middle Ages in Europe had their set of dogmas. The modern era of science has a different set of dogmas. Theological conceptions hold sway and govern the actions of men at one time. At another, scientific notions grip men's minds and nothing is believed which is not measurable by the standards of science.

CONCLUSION

We have come to the close of our study of the fundamentals of logic. The nature of thought, the principles that ought to govern its processes, the mistakes in reasoning that we most commonly make when we stray away from the path of truth—these and other related topics have been discussed in this book. Thinking is what each one of us is intimately concerned with. Even without our knowing, we employ logical principles in our daily conversation and arguments. In one of Moliere's plays, a rich shopkeeper, who wishes to play the gentleman and to get himself educated, engages the services of a professor of languages. The professor teaches him that language is either prose or poetry, and that everything that is not poetry is necessarily prose. "And what am I speaking?" asks the astonished bourgeois. "You are speaking prose,"

replies the professor. "So I have been speaking prose all my life," says the shopkeeper, "without my knowing it," and in his delight he hastens to inform his wife and household of the new discovery.* Many students of logic may find themselves in the same state as the shopkeeper was in when he was told that he was speaking prose. The science of logic appears difficult and strange at first. But when once its principles are understood we realise that we have been using them, however imperfectly, in our commonest thoughts and expressions.

* Quoted by A. S. Rappoport in his **Primer of Philosophy**, p. 20.

QUESTIONS AND MODEL EXERCISES

In this section are given questions taken mostly from University question papers and arranged to suit the order of topics discussed in this book, as also directions for working exercises in Logic with a few model answers.

DEDUCTION

I

THE SCOPE OF LOGIC

QUESTIONS:

1. Frame a definition of Logic which you believe to be adequate, giving your reasons. (March '28)
2. Determine the scope of Logic in relation to those of Psychology and Ethics. (Sept. '34)
3. Logic is a "Normative Science." Explain this and illustrate the difference between Normative and Positive sciences. (Sept. '35)
4. Explain the relation of Logic to two other sciences allied to it in respect of (a) content and (b) method. (March '29)
5. Logic has sometimes been called the "Science of Sciences." In what sense are all the other sciences part of the subject-matter of Logic? (April '35)
6. Distinguish Art from Science, and discuss how far Logic can be regarded as an art. (March '37)
7. 'Logic deals with the form, and not with the matter of thought'. Discuss this statement. (Sept. '39)
8. How is Logic concerned with (a) thought, and (b) language? (March '40)
9. Explain and examine the statement: "Logic is the Grammar of Thought. Grammar is the Logic of Language." (Sept. '30)
10. Consider the claim of Logic to be a science. Where does Logic get its material for study? (March '41)
11. "Some who have studied Logic cannot reason well, therefore it is a useless study; some who have not studied Logic can reason well, therefore it is an unnecessary study." Discuss. (Sept. '38)
12. How does science differ from ordinary knowledge? (Sept. '40)
13. If some of your classmates, who have not learnt Logic, can argue as well as you can, what is the advantage of studying Logic? (Mysore '43)

II

THE DOCTRINE OF TERMS

MODEL EXERCISES:

Give the logical characteristics of the following terms in thick types.

- (a) **Mr. Asquith is the Prime Minister of England.**
- (b) **All British citizens are free.**
- (c) **The soul of a nation cannot be conquered.**
- (d) **This is the house that Jack built,**

- (e) Influenza is sometimes serious.
- (f) Piety is a virtue.
- (g) Milton was blind.

Answer

- (a) i. Mr. Asquith: Singular, Concrete, Positive, Absolute.
ii. The Prime Minister of England: Significant Singular, Concrete, Positive, Absolute.
- (b) All British citizens: General, Concrete, Positive, Absolute.
- (c) The soul of a nation: General, Abstract, Positive, Absolute.
- (d) The house that Jack built: Significant Singular, Concrete, Positive, Absolute.
- (e) Influenza: General, Concrete, Positive, Absolute.
- (f) i. Piety: Singular, Abstract, Positive, Absolute.
ii. A virtue: General, Abstract, Positive, Absolute.
- (g) Blind: General, Concrete, Privative, Absolute.

Note: It should be enough to identify a given term from the following standpoints:

- i. Singular, General or Collective.
- ii. Abstract or Concrete.
- iii. Positive, Negative or Privative.
- iv. Absolute or Relative.

Except where we have a possibility of more than one meaning, we need not identify terms as univocal (as different from equivocal terms). Similarly, except where we have a combination of words constituting a term, we need not have a recourse to the distinction, between simple and composite terms. The reason for this view is not far to seek. In Logic we are concerned only with words with a definite meaning. Wherever we come across a word or a combination of words, we must make sure that the meaning is unambiguous. Where it is not, Logic refuses to go further. A term must, in a given context, stand for something definite. Otherwise it is not a term.

As for the distinction between simple and composite, it is unnecessary to point out that a term is a simple one when it is made of only one word.

The distinction between categorematic and syncategorematic plainly refers to words—those which can stand by themselves and convey an intelligible meaning and those which cannot. Terms proper must always convey an intelligible meaning. If they do not, they are not terms.

One other distinction is also pointless. Every term has both connotation and denotation, though in a given context one or the other aspect may be more prominent. There is no point therefore in identifying a term as connotative or non-connotative.

QUESTIONS:

1. Distinguish between Name, Term and Concept.
2. (a) When is a term said to be (i) equivocal, (ii) general, (iii) negative, (iv) non-connotative.
(b) Discuss if connotation means all the known qualities of a thing. (March '34)
3. 'Extension and intension vary in inverse ratio'—Discuss. (March '32)
4. Distinguish a connotative from a non-connotative term, and discuss whether proper names are connotative or non-connotative. (Sept. '37)
5. Explain the difference between the distributive and the collective use of a name. State whether the subject of each of the following propositions is used distributively or collectively.
 - (a) The students surrounded the building.
 - (b) The students filled up class enrolment forms.
 - (c) All the fish weighed twenty pounds.
 - (d) All the angles of a triangle are less than two right angles. (Latta & Macbeath.)
6. State the logical characteristics of the terms in thick types:
 - (a) **Kautilya is the Indian Machiavelli.**
 - (b) **Germany's oldest lighthouse stands on Newark Islands.**
 - (c) **Great ships require deep waters.**
 - (d) **Forgiveness is greater than revenge.**
7. Give the logical characteristics of the terms in the following:

He is a Tilak.
The dumb are ignorant. (Annamalai, '38)
8. Arrange the following terms according to increasing denotation:—

marble statue, representation of the human figure, piece of art, marble statue of a king, human product.
9. Describe the logical characteristics of the terms in thick types in:—
 - (1) **The love of God is the crown of knowledge.**
 - (2) **The spread of violence is lamentable.**
 - (3) **A drop of water contains millions of microscopic germs.** (Annamalai, '40)
10. Give the logical characteristics of:—

window, the syndicate, nationality, the dumb millions. (Annamalai, '38)

III

THE PREDICABLES

MODEL EXERCISES:

1. Give propositions predicating a differentia, a property, a separable and an inseparable accidents of 'school-boys.'

Answer

All school-boys are those who go to school for receiving education (Differentia).

All school-boys are those who know their teachers (Proprium).

Some school-boys are mischievous (separable accidens).

All school-boys are liable to err (inseparable accidens).

2. Indicate the predicable used in the following propositions:

(a) Properly educated subjects are a great help to their rulers (Proprium).

(b) All Negroes have snub-noses (Inseparable accidens).

(c) Rivers are water-roads connecting various countries (Genus and Proprium).

(d) Man is a progressive animal (Proprium and Genus).

(e) Some men are incapable of generous feelings (separable accidens).

(f) A wilful man must have his way (Differentia).

Note: The answer is given within brackets at the end of each proposition.

3. Illustrate the five predicables with reference to the term 'College.'

Answer

All colleges are educational institutions (Genus).

Some colleges are Medical colleges (species of the genus 'college').

All colleges are educational institutions imparting higher education to students and training them for degree examinations (Differentia).

All colleges exercise a healthy influence on the minds of students (Proprium).

Some colleges have European Principals (Accidens).

QUESTIONS:

1. Give three examples of terms standing to one another in the following relations:—

(a) Species and genus, (b) species and accidens, (c) species and property, (d) species and differentia.

2. Which predicable does each of the following statements illustrate:—

(a) Isosceles triangles have two of their angles equal to each other.

(b) Preachers are teachers.

(c) Logic is a good mental discipline.

(d) Even men are animals.

(e) Two straight lines cannot enclose a space.

(f) An umbrella protects a man from the sun.

(g) All democracies are governments.

(h) Crows are black.

IV DEFINITION

MODEL EXERCISES:

Examine the following definitions:—

(i) **Logic is a mental science.**

This definition is too wide because 'mental science' is the genus of logic, and the differentia is not stated.

(ii) **A pump is a water-raising machine worked by a handle.**

This is too narrow, because there are pumps which are not worked by hand.

(iii) **Gold is a kind of metal.**

'Kind of' is a vague expression. So the definition is ambiguous.

(iv) **Thyroid is a glandular organ in the trachea.**

This definition is not readily intelligible, and hence is obscure.

(v) **Architecture is frozen music.**

Architecture is not music but is likened to music. So the definition is figurative.

(vi) **Man is a human being.**

'Man' and 'human' are much the same. Hence the definition is synonymous or the tautologous.

(vii) **Life is the opposite of death.**

What life is is not stated, but only what it is not. So it is a negative definition.

QUESTIONS:

1. Explain clearly what is meant by definition **per genus et differentiam**, and compare it with genetic definition. (1921)

2. State the main requirements of a good definition. (Sept. '25)

3. Test the following definitions:—

(a) Stupidity is the absence of intelligence.

(b) Fine is a pecuniary mulct.

(c) Iron is a cheap metal.

(d) Health is wealth.

(e) Music is expensive noise.

(f) Mind is a *tabula rasa*.

(g) A square is a four-sided figure.

(h) Man is a featherless biped.

4. Test the logical validity of the following definitions:—

(i) Duration is a temporal slab of nature.

(ii) Wisdom is the avoidance of folly.

(iii) Causality is the uniform antecedence in time.

(Annamalai, '39)

(iv) An idle person is one who does not like to work.

(v) Logic is a machine for combating fallacy.

(vi) A Legislator is a member of either the Legislative Council or Legislative Assembly. (Annamalai, '38)

- (vii) A wireless set is an electrical apparatus operated without wires.
- (viii) A nut is something with a shell and good to eat.
- (ix) A statesman is a politician who is not yet found out.
- (x) A gentleman is one who has no visible means of support.
- (xi) The University is the gateway to higher employment. (Mysore '43)

V

DIVISION

MODEL EXERCISES:

Examine the following divisions:—

(i) **Students into men, women, research students and under-graduates.**

The rule that there should be only one *fundamentum divisionis* in a single act of division is not observed here. There are two principles, viz., sex and nature of study. Hence this is a cross-division.

(ii) **Religions into Monotheism, Christianity and Presbyterianism.**

This violates the rule, *Divisio ne fiat per saltum*, i.e., division must not make a leap. A species and its sub-species are given here as if they were co-ordinate with the genus. The division is not also exhaustive.

(iii) **Tree into branches, stem and roots.**

This is a physical partition of 'tree' into its parts, and hence not a logical division.

(iv) **Terms into connotative and non-connotative.**

This is division by dichotomy (vide p. 35).

(v) **Human nature into body, mind and spirit.**

This is a conceptual analysis of human nature. Therefore it is a metaphysical division.

(vi) **Pound into a coin in British currency, a weight in measurement, and a place where stray cattle are kept.**

The various meanings of an ambiguous word are given here. Hence it is a verbal division.

QUESTIONS:

1. Distinguish between logical definition and division, and show how the two are related, illustrating your answer with a suitable example. (Sept. '27)

2. What is logical division? What are its laws? How is it different from verbal division and metaphysical analysis? (April '36)

3. Explain and illustrate the distinction between co-division and cross-division. (1920)

4. 'The method of dichotomy depends upon the law of Excluded Middle.' Explain. (1919)

5. Test the following divisions:—

- (a) Men into Hindus, Christians and Muhammadans.
- (b) Indians into North Indians, South Indians and Andhras.
- (c) Books into literary, philosophical, political, interesting and cloth-bound.
- (d) Terms into singular, general, concrete and negative.
- (e) Organisms into plants, animals and social organisms.
- (f) Substances into ponderable, imponderable and spiritual.
- (g) Mind into knowing, feeling and willing.
- (h) Vice into an immoral act and a mechanical instrument.
- (i) Games into those that are athletic and those that are intellectual.
- (j) Buildings into stone, mortar, tiles and timber.
- (k) Teachers into popular, impressive, poor, powerless and harmless persons.
- (l) States into monarchical, industrialist, progressive and imperialist polities.

VI PROPOSITIONS

MODEL EXERCISES:

Reduce the following to logical form:—

- (i) Every Spartan is not a hero.
L.F.—Some Spartans are not heroes. (O)
- (ii) Not every one is a good judge of his own interests.
L.F.—Some people are not good judges of their own interests. (O)
- (iii) All are not wise who read much.
L.F.—Some who read much are not wise. (O)
- (iv) A fool is not always wrong.
L.F.—Some fools are not wrong. (O)
- (v) The virtuous are seldom miserable.
L.F.—Some virtuous people are not miserable. (O)
- (vi) Few men really know their own mind.
L.F.—Some men are not those who really know their own mind. (O)
- (vii) A few students were present.
L.F.—Some students are those who were present. (I)
- (viii) Few men have not suffered disappointment.
L.F.—Some men are those who have suffered disappointment. (I)
- (ix) Many Americans are rich.
L.F.—Some Americans are rich. (I)
- (x) Only the wise are happy.
L.F.—Either: No non-wise persons are happy; (E)
Or: All happy persons are wise. (A)
- (xi) Nothing is beautiful except truth.
L.F.—Either: No non-truth is beautiful; (E)
Or: All beautiful things are truths. (A)

- (xii) All your anxiety will not save him
L.F.—None of your anxiety is one which will save him. (E)
- (xiii) Neither riches nor honour can banish anxiety.
L.F.—No riches are those which can banish anxiety. (E)
- and
- Honour is not that which can banish anxiety. (E)
- (xiv) Two lies do not make a truth.
L.F.—No lies are such that any two of them can make a truth. (E)
- (xv) Opinions differ.
L.F.—Some men are those who differ in their opinions. (I)
- (xvi) It never rains, but pours.
L.F.—All instances of rainfall are instances of heavy downpour. (A)
- (xvii) Many are called, but few are chosen.
L.F.—Some of those who are called are not those who are chosen. (O)
- (xviii) Firm at his post he stood.
L.F.—He is one who stood firm at his post. (A)
- (xix) The boy stood on the burning deck whence all but he had fled.
L.F.—(a) All persons other than the boy, are those who had fled. (A)
(b) No persons other than the boy are those who stood on the burning deck. (E)
- (xx) It snows.
L.F.—Snow is something which is falling now. (A)

QUESTIONS:

1. Examine the meanings of the following expressions and show their relation to one another—Judgment, Proposition, Concept, Term. (1922)
2. "Judgment is the unit of thought." Explain. (1919)
3. What is meant by the quantity and quality of propositions? (Sept. '25)
4. What are (1) Indesignate, and (2) Exclusive propositions? Show how each should be interpreted. (1919)
5. Reduce the following statements to logical form:—
 - (a) Any man but a saint would have lost his temper.
 - (b) Honesty is not always the best policy.
 - (c) None but the brave deserve the fair.
 - (d) It may rain this evening.
 - (e) Not all good writers are good speakers.
 - (f) Shall we submit to such a tyranny as this?
 - (g) They rode back, but not the six hundred.
 - (h) Beware of the dogs!

- (i) Money is the miser's end.
- (j) The more the merrier.
- (k) Except when they are naughty, children are invariably good.
- (l) They also serve who only stand and wait.
- (m) It is an ill-wind that blows nobody good.
- (n) All is well that ends well.
- (o) None but the educated are fit to vote.
- (p) Few important discoveries are due to accident.
- (q) He who digs a pit for others falls in himself.
- (r) Every mark of weakness is not a disgrace.
- (s) Not all are friends who profess to be so.
- (t) Only ignorant people believe in witchcraft.
- (u) Better late than never.
- (v) What has been may be again.
- (w) It needs two to make a quarrel.
- (x) Fine feathers do not make fine birds.
- (y) We smile not only when we are pleased.
- (z) Unasked advice is seldom acceptable.

VII

THE IMPORT OF PROPOSITIONS

QUESTIONS:

1. What various theories have been held regarding the import of propositions? Which do you favour and why?
(Sept. 1927)
2. What are the various ways in which a proposition can be understood? Discuss their relative merits.
(Annamalai '39)

VIII

THE LAWS OF THOUGHT

QUESTIONS:

1. Explain the nature and function of the laws of thought.
2. 'The Laws of Thought' express the one great principle that reality is systematic, coherent and intelligible. Discuss.

IX

THE OPPOSITION OF PROPOSITIONS

MODEL EXERCISES:

1. If the statement 'All men are liars' is false, what can you say about the truth or falsity of the following:
 - (i) No men are liars.
 - (ii) Some men are liars.
 - (iii) Some men are not liars.

Answer

- (i) is doubtful, since it is the contrary of the given statement.

(ii) is doubtful, since it is the subaltern of the given statement.

(iii) is true, since it is the contradictory of the given statement.

2. Elucidate the opposition between the first and the rest of the following propositions:—

(i) Only the faithful are happy.

(ii) Nobody can hold that those who are happy are faithful.

(iii) Indeed no people who are happy are at all faithful. (1913)

Answer

(i) L.F.—All happy persons are faithful.

(ii) L.F.—Some happy persons are not faithful.

(iii) L.F.—No happy persons are faithful.

(ii) is the contradictory of (i).

(iii) is the contrary of (i).

3. Find the relation of opposition between the first of the following and the rest:—

(i) I have met no case of Influenza which if taken in time was fatal.

(ii) I have met at least one case of Influenza which taken in time was not fatal.

(iii) I have met one case of Influenza which though taken in time was fatal.

(iv) I have not met a single case of Influenza which though taken in time was not fatal. (1919)

Answer

(i) L.F.—No cases of Influenza, taken in time, are fatal. (E)

(ii) L.F.—Some cases of Influenza, taken in time, are not fatal. (O)

(iii) L.F.—Some cases of Influenza, though taken in time, are fatal. (I)

(iv) L.F.—All cases of Influenza, though taken in time, are fatal. (A)

(ii) is the subaltern of (i).

(iii) is the contradictory of (i).

(iv) is the contrary of (i).

Note. For the purposes of this exercise the phrase 'I have met' is omitted.

4. Test the validity of the following argument:—

(a) Epimenides says all Cretans are liars.

(b) Epimenides is a Cretan, hence what he says is not true.

(c) Cretans, therefore, are not liars.

(d) But Epimenides is a Cretan, and what he says must, therefore, be true.

(e) Since Epimenides says all Cretans are liars, his statement must be true.

(From S. S. Suryanarayana Sastri's *Rudiments of Logic*, part I, p. 96).

Answer

The purpose of this argument is to establish the truth of the statement of Epimenides on the ground of its falsity. Evidently this is absurd. A careful student of Logic will find here an ingenious violation of the laws of opposition. The Second proposition, viz., (b) says that Epimenides' statement should be rejected on the ground of his own assertion. The rejection of the universal affirmative 'All Cretans are liars' implies the acceptance of its contradictory, and not that of its contrary. The proposition (c) therefore, really means 'Some Cretans are not liars.' From this it does not follow that Epimenides is not a liar. The propositions (d) and (e) would follow only if (c) were a universal negative. Thus the queer argument that the rejection of Epimenides' statement involves the acceptance thereof is not valid.

QUESTIONS:

1. Explain the phrase 'opposition of propositions' and compare contrariety with sub-contrariety. (Sept. '30)
2. State all the propositions in 'opposition' to the following, explaining the nature of the 'opposition' in each case:—Some of the ablest men I know are talkative. (April '36)
3. Given the proposition, 'some men are unjust' as true, state the propositions that can be inferred from it (a) as true, (b) as false and (c) as doubtful. (April '37)
4. (a) Explain the distinction between contrary and contradictory propositions.
(b) Give the contrary (or sub-contrary) and the contradictory of:
(i) No coward need apply.
(ii) Smoking affects the throat.
(iii) Some victories are worse than defeats. (Annamalai '38)

X

IMMEDIATE INFERENCE

MODEL EXERCISES:

1. State the obverse of each of the following:—
(a) If a body is heated, it rises in temperature.
Obverse. If a body is heated, it does not fail to rise in temperature.
(b) If a term is abstract, it is not singular.
Obverse. If a term is abstract, it is other than singular. (Sept. '37)
2. Convert the following propositions and state the peculiarities, if any, you notice in converting them:—
(a) Mr. Asquith is the Prime Minister of England. (S a P)
Converse: The Prime Minister of England is Mr. Asquith. (P a S)

This is the simple converse of the original proposition which is A, since the subject and predicate of the original A proposition are singular terms.

(b) All British citizens are free.

(S a P)

Converse: Some free people are British citizens. (P i S)

This is conversion by limitation of the original A proposition.
(1914)

3. Give, if possible, the contrary, the obverse and the contrapositive of:—

If it is right for a man to vote, it is right for a woman too.

Answer

Hypothetical propositions do not admit of distinctions of quantity. Moreover, they express a relation of dependence, and not that of concomitance. 'If S, then P,' means that P follows from S. We cannot be certain that where P exists, there S also must exist. Hence hypothetical propositions cannot be converted. It will be clear, then, that we cannot have contraposition for the given proposition.

The contrary is: If it is right for a man to vote, it is not right, for a woman to vote.

Obverse: If it is right for a man to vote, it does not follow that it is not right for a woman to vote.

4. Educe as many propositions as possible from 'the square of three is nine.'

Original: The square of three is nine. ((S a P)

Obverse: The square of three is not other than nine. (S e \bar{P})

Partial contrapositive: No number other than nine is the square of three. (\bar{P} e S)

Full contrapositive: All numbers other than nine are numbers other than the square of three. (\bar{P} a \bar{S})

Full inverse: Some numbers other than the square of three are numbers other than nine. (S i P)

Partial inverse: Some numbers other than the square of three are not nine. (S o P)

Converse: Nine is the square of three. (P a S)

Obverted converse: No nine is other than the square of three. (P e \bar{S})

Note. Since the terms 'the square of three' and 'nine' are singular terms, the conversion is simple.

5. Examine the logical relationship between the first and each of the rest of the following propositions:—

(i) None but graduates wore gowns.

(ii) Some graduates did not wear gowns.

(iii) Some who wore gowns were not graduates.

(iv) None who was not a graduate wore a gown.

Answer

The logical form of the first proposition is either 'All those who wore gowns are graduates', or 'No non-graduates are those who wore gowns.'

Let us have the E form and see how the other propositions are related to the first.

The eduction scheme for E is this:—

$$\begin{array}{ccc}
 & \text{S e P} & \\
 & \text{-----} & \\
 \text{S a } \bar{\text{P}} & & \text{P e S} \\
 \bar{\text{P}} \text{ i S} & & \text{P a } \bar{\text{S}} \\
 \bar{\text{P}} \text{ o } \bar{\text{S}} & & \bar{\text{S}} \text{ i P} \\
 & & \bar{\text{S}} \text{ o } \bar{\text{P}}
 \end{array}$$

Here: S=non-graduates

 S=graduates

 P=those who wore gowns

 P=those who did not wear gowns.

(ii) L.F.:—Some graduates are not those who wore gowns ($\bar{\text{S}} \text{ o P}$).

This is the sub-contrary of $\bar{\text{S}} \text{ i P}$ which is the partial inverse of S e P, the first proposition.

(iii) L.F.:—Some who wore gowns are not graduates (P o S).

This is the contradictory of P a S which is the obverted converse of S e P.

(iv) L.F.:—No non-graduates are those who wore gowns (S e P).

This is the same as the original.

6. Given the truth of the proposition 'All voters have a share in the management of the country,' what can you infer about (i) non-voters, (ii) those who have no share in the management of the country, and (iii) those who have a share in the management of the country.

Answer

The logical form of the given proposition is 'All voters are those who have a share in the management of the country.'

The meaning of the question is, 'What inferences can you draw from the truth of the given proposition with (i) S as the subject, (ii) $\bar{\text{P}}$ as the subject, and (iii) P as the subject?' The educts having S as the subject are full and partial inverses; those having P as the subject are full and partial contrapositives, and those having $\bar{\text{P}}$ as the subject are converse and obverted converse.

Eduction Scheme:

$$\begin{array}{ccc}
 & \text{S a P} & \\
 & \text{-----} & \\
 \text{S e } \bar{\text{P}} & & \text{P i S} \\
 \bar{\text{P}} \text{ e S} & & \text{P o } \bar{\text{S}} \\
 \bar{\text{P}} \text{ a } \bar{\text{S}} & & \\
 \bar{\text{S}} \text{ i } \bar{\text{P}} & & \\
 \bar{\text{S}} \text{ o P} & &
 \end{array}$$

(i) $\bar{S} \text{ i } \bar{P}$: Some non-voters are those who do not have a share in the management of the country.

$S \text{ o } P$: Some non-voters are not those who have a share in the management of the country.

(ii) $\bar{P} \text{ e } \bar{S}$: No persons who do not have a share in the management of the country are voters.

$\bar{P} \text{ a } \bar{S}$: All persons who do not have a share in the management of the country are non-voters.

(iii) $P \text{ i } S$: Some who have a share in the management of the country are voters.

(iv) $P \text{ o } S$: Some who have a share in the management of the country are not non-voters.

7. Consider the following:—

(a) (i) Warmth is agreeable: therefore cold is disagreeable.

This is material obversion or illogical obversion. It violates the rules of obversion that (a) the quality of the proposition must be changed and that (b) the contradictory of the original predicate must be given in the obverse. We notice here that the quality of the proposition is not changed and that the contrary (not the contradictory) of the predicate is given. Whereas in a valid obverse, the subject is not changed at all, here the subject is replaced by its contrary.

(ii) 'All brave men are generous. Therefore all generous men are brave'—(Creighton).

The correct logical converse will be 'some generous men are brave' as most A propositions must be converted only per accidens—not simply. Though we are not likely to commit such mistakes in working exercises, in the heat of debate we are likely to assume that what is stated universally about the subject is stated universally about the predicate also.

(iii) No dogs are bipeds. Therefore, all non-bipeds are dogs (i.e., $S \text{ e } P$, therefore $\bar{P} \text{ a } S$).

This is a fallacy of Contraposition. The correct contrapositive will be 'some non-bipeds are dogs' (i.e., $\bar{P} \text{ i } S$).

"A fallacy of contraposition is only possible when the given premise is an E or I proposition or hypothetical proposition of corresponding form"—(Welton and Monahan).

(iv) Thought is existence. Therefore what contains no element of thought is non-existence.

This is a fallacy of Inversion. In fallacies of Inversion we argue from All $S \text{ is } P$ that No $\bar{S} \text{ is } P$ or from No $S \text{ is } P$ that All $\bar{S} \text{ is } P$. It will be equally fallacious to argue thus: If $S \text{ is } M$, it is P . Therefore if $S \text{ is not } M$, it is not P .

'Inductively also we are likely to commit this error. Though a condition is known to produce a certain effect, its absence cannot be taken as ruling out the presence of that effect through some other agency. Because a certain flower is fertilised while it is visited by

insects, it does not follow that it could not be fertilized if no insects came near it.'—(Welton and Monahan)

(b) (i) A lawyer is a man. Therefore a good lawyer is a good man.

This is Immediate Inference by Added Determinants. The inference given above is invalid as the determinant 'good' qualifies Subject and Predicate, not alike but differently.

(ii) All jurors are tax-payers. Therefore, a majority of jurors is a majority of tax-payers.

This is Immediate Inference by complex conception. The inference here is invalid, as there is variation in significance. A majority of jurors cannot be a majority of tax-payers.

(iii) A is the grandfather of B. Therefore B is the grand-child of A.

This is valid Immediate Inference by Converse Relation.

In Immediate Inferences by Added Determinants and by complex conception caution must be exercised. There should be no variation in significance by adding the same qualification to S and P. Similarly in complex conception also, the new conception must not vary in significance. It is advisable to see in all such cases that the qualification is not accidental to the nature of S and P.

Immediate Inference by Converse Relation cannot be judged to be valid or invalid by a reference to logical rules only. They require knowledge of the particular spheres and relations (temporal, spatial, etc.) to which they refer.

QUESTIONS:

1. Distinguish between mediate and immediate inference and decide whether immediate inference is really inference. Would you prefer to designate the latter 'Interpretation of proposition'? If so, state reasons for so doing.

2. Explain the terms contrapositive and full inverse. Why is there no inverse for I and O. (Sept. '34)

3. Draw as many inferences as you can from:—

- All monochromatic light is coloured.
- No poets are guided by reason.
- Some have greatness thrust upon them.
- Some people are not considerate.

4. Assign the logical relation between the first of the following propositions and each of the rest:—

- Madrasis alone are progressive.
- Some non-Madrasis are not progressive.
- Some non-progressive people are Madrasis.
- No progressive people are non-Madrasis.
- All Madrasis are not progressive.
- Some Madrasis are not progressive.

(Sept. '34)

5. Given the truth of the proposition "All peace-lovers are blessed", what can you infer about (i) war-lords (to be treated here as the contradictory of peace-lovers), (ii) those who are blessed, and (iii) those who are not blessed?

6. Examine the following:—

(i) A false weight is an abomination to the Lord. Therefore a just weight is His delight. (March '34)

(ii) A tortoise is an animal. Therefore a swift tortoise is a swift animal.

(iii) Wealth is desirable. Therefore poverty is undesirable.

(iv) Protestants are Christians. Therefore a majority of Protestants are a majority of Christians.

(v) All equilateral triangles are equiangular. Therefore all equiangular triangles are equilateral.

(vi) A bad man must be miserable because happiness is the result of well-doing. (March '39)

(vii) All coals are atoms of carbon. Therefore some atoms of carbon are coals. (Creighton)

(viii) If we can be sure that the inconsistent is unreal, we must logically be just as sure that the real is consistent. (Bradley)

(ix) Existent circles are all imperfect. There must therefore be something other than existent circles which is perfect. (Latta and Macbeath)

(x) War is productive of evil. Therefore peace is beneficial. (March '40)

7. Give the contradictory, the contrapositive and the inverse of:

(1) He can never become a Hitler who has the slightest of the Gandhi in him.

(2) Only intelligent persons can rise to positions of eminence in the learned professions. (Annamalai '43)

8. State the logical relation between the first of the following propositions and each of the rest. If the first proposition is true, what can you infer about the truth or falsity of the rest?

(a) No sensible man ever questions a woman's word.

(b) Some who are not sensible question a woman's word.

(c) Some who do not question a woman's word are not sensible.

(d) All those who question a woman's word are persons who are sensible. (Annamalai '39)

9. Examine the following inferences:—

(a) "Some Irish women are pretty," inevitably implies "Some Irish women are not pretty."

(b) "Some smokers do not drink," therefore "Some drinkers do not smoke."

(c) "All rich men are generous," therefore "All poor men are niggardly."

(d) A poet is a man, therefore a good poet is a good man. (Annamalai '39)

10. Assign the logical relationship between the first and each of the following propositions:—

(1) None but the righteous command confidence.

(2) Some who command confidence are not righteous.

- (3) None who does not command confidence is righteous.
 (4) Some who are righteous do not command confidence. (Mysore '43)
11. Educe as many propositions as you can from the proposition "None but cowards claim to be virtuous." (Mysore '43)
12. (1) If it is true that 'there is no disgrace in losing when one has done one's best,' does it follow that 'those who win deserve no particular glory'?
 (2) Heat expands bodies, therefore cold contracts them. (Mysore '43)
13. Draw all the possible inferences from 'Only fools fail to learn from experience.' (Mysore '43 & '44)
14. Give the logical relation between the first proposition and each of the rest:—
 (1) All dealers are producers.
 (2) None who are producers are dealers.
 (3) Some who are not dealers are not producers.
 (4) Some producers are not other than dealers.
 (5) Some dealers are persons other than producers.
 (6) Some who are other than dealers are producers. (Mysore '43)
15. State the relation between the proposition 'No ambitious men are successful' and each of the following:—
 (1) Ambitious men are sometimes successful.
 (2) Only unsuccessful men are unambitious.
 (3) Not a few successful men are unambitious.
 (4) Some ambitious men are not unsuccessful.
16. Examine the following:—
 (1) Only fools say so; therefore every one who says so is a fool.
 (2) Boys love play; therefore, old men are averse to play. (Mysore '44)

XI

THE CATEGORICAL SYLLOGISM

MODEL EXERCISES:

State the following arguments in logical form, and give the symbolic equivalent in each case:—

1. Gold is not a compound substance; for it is a metal, and none of the metals are compounds. (Creighton and Smart, p. 145)

We must first find the conclusion and then arrange the premises thus:

Conclusion: Gold is not a compound substance.

So the syllogism will be:

No metals are compounds

Gold is a metal

∴ Gold is not a compound

M e P

S a M

∴ S e P

2. This man shares his money with the poor, but no thief ever does this, therefore this man is not a thief.

(Creighton and Smart, p. 145)

Conclusion: This man is not a thief.

The syllogism will be:

No thief is a person who shares his money with the poor. P e M

This man is a person who shares his money with the poor. S a M

∴ This man is not a thief ∴ S e P

3. All truly rich people are content with what they have. An envious man is not content with what he has; no envious man therefore is truly rich.

Conclusion: No envious man is truly rich.

The syllogism will be:

All truly rich people are those content with what they have. P a M

No envious man is content with what he has. S e M

∴ No envious man is truly rich. ∴ S e P

QUESTIONS:

1. (a) Why is the middle term so called?

(b) Why should the middle term be distributed at least once in the premises? (March '32)

2. Write short notes on: (i) Illicit Process, (ii) The *dictum omni et nullo*. (1917)

3. Arrange the following arguments in syllogistic form and test their validity, with reference to the general rules of the syllogism:—

(a) All suffragettes are women and all suffragettes are progressive; hence all women are progressive. (April '36)

(b) Birds have wings; bats are not birds; therefore bats have no wings. (March '24)

(c) 'Dear Alice,' said Peggy, 'I am going to tell you where father has gone. Father has gone to Heaven.' 'Must mother go there too?' asked Alice. 'Not yet, Alice dear. Mother isn't dead.' 'Then father is dead,' said Alice, with a rapidity of syllogism that took Peggy aback. (1911)

(d) Socrates must have been happy, for wise men alone are happy.

XII

THE FIGURES AND MOODS

Problems bearing on the syllogism cannot be worked out without a clear understanding of the General Rules of the syllogism. So, to work the problems correctly, the student must make sure that his knowledge of the General Rules is sound. Then he must proceed to apply these rules and derive the mood and figure of arguments. He must be in a position to arrive at the answer without reference to the mnemonic lines. He must not argue from the figure but must argue to it.

MODEL EXERCISES:

1. Given I as the major premise, what can you determine from general principles as to the figure and the mood?

(March 1929)

I is the major premise. I propositions distribute neither subject nor predicate. So whether we have it as M i P or as P i M, neither the middle term nor the major is distributed. In a valid syllogism, the middle term must be distributed at least once. So, it must be distributed here in the minor premise. If it is distributed as the predicate of negative proposition, it will give rise to the fallacy of illicit major thus: one of the premises being negative, the conclusion will be negative. So P the major term will be distributed in the conclusion. To avoid illicit major, it must be distributed in its own premise. But the major premise is an I proposition where no term is distributed. We must therefore have a universal affirmative proposition as the minor premise with the middle term as its subject. The following combinations are, then, possible.

M i P	P i M
M a S	M a S
∴ S i P	∴ S i P

With I as the major premise, we can construct an argument in **Disamis** in the third figure and **Dimaris** in the fourth figure.

2. Is it possible for both the major and the minor terms to be undistributed in the premises? (1920)

If both the minor and major terms are undistributed in their premises, they must be undistributed in the conclusion also to avoid the Illicit process of the minor and major terms. The conclusion, then, must be S i P. The major premise may be either M a P or M i P or P i M. If it is M a P, the minor may be M a S or M i S, or S i M as in none of the combinations S is distributed. Combining these, we get

M a P	
S i M	
∴ S i P	which is Darii in the 1st figure;
M a P	
M a S	
∴ S i P	which is Darapti in the 3rd figure, and
M a P	
M i S	

∴ S i P which is **Datisi** in the same figure. If we have M i P as the major, we can have only M a S as the minor.

This will give us

$$\begin{array}{c} M i P \\ M a S \\ \hline \end{array}$$

$\therefore S i P$ which is **Disamis** in the third figure. With $P i M$ as the major and $M a S$ as the minor we have

$$\begin{array}{c} P i M \\ M a S \\ \hline \end{array}$$

$\therefore S i P$ which is **Dimaris** in the fourth figure.

Thus we have **Darii** in the first figure, **Darapti**, **Datisi** and **Disamis** in the third figure and **Dimaris** in the fourth figure. In all these syllogisms, the Minor and Major terms are undistributed in the premises.

3. If in a valid syllogism we substitute for both the premises their contradictories, can we prove the contradictory of the original conclusion? (Sept. '22)

We cannot have **AA** as the two premises since their contradictories are **OO** and from two negatives, no conclusion can be drawn. Nor can **EE** be the premises for no syllogism can be constructed with two negatives. We cannot try **IE** as they will give rise to the fallacy of **Illicit Major**. We are then left with these possibilities.

(1) $\begin{array}{c} A \\ O \\ \hline O \end{array}$	(2) $\begin{array}{c} O \\ A \\ \hline O \end{array}$	(3) $\begin{array}{c} E \\ I \\ \hline O \end{array}$
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Let us take the first. The contradictories of **AO** are **OA**. But the conclusion will be the same as the original conclusion, not its contradictory.

Similarly taking the second, we find the contradictories of **OA** are **AO**. But the conclusion is again **O**. Taking the third we have **IE**. But from this combination no conclusion is possible. Thus we cannot, by replacing the premises by their contradictories, get a conclusion which is the contradictory of the original.

4. If the conclusion of a syllogism be a universal negative, determine the mood and figure. (March 1922)

The conclusion is universal negative, i.e., **S e P**. So both premises must be universal, one affirmative and one negative. We get the following combinations.

$\begin{array}{c} P a M \\ M e S \\ \hline \end{array}$	$\begin{array}{c} P a M \\ S e M \\ \hline \end{array}$	$\begin{array}{c} P e M \\ S a M \\ \hline \end{array}$	$\begin{array}{c} M e P \\ S a M \\ \hline \end{array}$
$\therefore S e P$	$\therefore S e P$	$\therefore S e P$	$\therefore S e P$

That is, **Camenes** in the fourth figure, **Camestres** and **Cesare** in the second figure and **Celarent** in the first figure.

We cannot have **M a P** as it would make for the fallacy of **Illicit Major**, nor **M a S** as it would give rise to the fallacy of **Illicit Minor**.

5. If the major premise and the conclusion of a valid syllogism agree in quantity but differ in quality, find the mood and figure. (1914)

The major premise and conclusion agree in quantity but differ in quality. Leaving the question of quantity for the moment, we may think of two combinations—one where the Major is affirmative and the conclusion negative—and the other where the reverse is the case. But the second alternative is manifestly absurd as we cannot get an affirmative conclusion from a negative major premise. So we must examine only those combinations where the conclusion is negative and the major premise affirmative. If the conclusion is negative, P is distributed and it must be distributed in its own premise. The major premise must be universal affirmative with the major term as its subject. So we have P a M. The conclusion and major premise are said to agree in quantity. So both must be universal. With the major as universal affirmative and the conclusion as universal negative we get the following combinations:—

$$\begin{array}{rcl} P & a & M \\ S & e & M \\ \hline \therefore & S & e & P \end{array} \qquad \begin{array}{rcl} P & a & M \\ M & e & S \\ \hline \therefore & S & e & P \end{array}$$

i.e., Camestres in the second figure and Camenes in the fourth figure.

6. Two valid syllogisms in the same figure have the same major, middle and minor terms, and their major premises are sub-contraries. Determine without reference to the mnemonic lines what the syllogisms must be.

The major, middle and minor terms are the same. But the major premises are related as sub-contraries to one another.

So we have I and O as the major premises.

Taking the second, we find that because the major is O, the conclusion also will be O. The major term is distributed in the conclusion and must be distributed in its premise. That is, the major must be M o P.

To distribute the middle term at least once, the minor must be M a S and the conclusion S o P.

$$\begin{array}{rcl} M & o & P \\ M & a & S \\ \hline \therefore & S & o & P \end{array}$$

The other syllogism is

$$\begin{array}{rcl} M & i & P \\ M & a & S \\ \hline \therefore & S & i & P \end{array}$$

If the major is $M \text{ i } P$, the middle term is not distributed. To distribute it in the minor we require an A proposition with the middle term as its subject.

Thus the syllogisms **Bocardo** and **Disamis** in the third figure.

QUESTIONS:

1. Explain the significance of the first line of the mnemonic formula:—Barbara, Celarent, Darii, Ferioque Prioris. (Sept. '26)

2. Why cannot O stand as a premise in the first, as a major in the second, as a minor in the third, or as a premise in the fourth figure? (Sept. '31)

3. Distinguish between a weakened and strengthened syllogism. Examine the following, and show whether they are weakened or strengthened:— AAI in Fig. I; AEO in Fig. II; EAO in Fig. III and AEO in Fig. IV. (March '29)

4. Explain why only negative conclusions can be obtained in the second figure, and only particular conclusions in the third figure? (April '30)

5. Why cannot an " A " conclusion be drawn in any but the first figure?

6. Why is EIO always valid while IEO is never valid? (Sept. '30)

7. Determine the Figure and Mood of a syllogism which has a particular negative for its major premise.

8. Prove that no syllogism in the fourth figure can be correct which has a particular negative as one of its premises or a universal affirmative for its conclusion. (March '31)

9. The middle term cannot be distributed twice when the conclusion is universal. (March '32)

10. When the middle term is the predicate of the premises of a syllogism, the major premise must be universal. (March '34)

11. Show that from a particular major and a negative minor no conclusion can be drawn. (Annamalai '40)

12. Prove that the minor premise in the first figure must be affirmative.

13. If in a syllogism the middle term is distributed twice, and if no other term is distributed in the premises, what is the figure and mood? (Annamalai '38)

14. In Fig. 4 if either premise be negative, the major must be universal; and if the major be affirmative, the minor must be universal. (Annamalai '37)

15. Determine with the help of the general rules of the syllogism the moods and figures in which a universal negative conclusion can be obtained.

16. Given that the major premise of a valid syllogism is affirmative and the major term is distributed in both premise and conclusion, while the minor term is undistributed in both, determine the syllogism. (Annamalai '39)

17. Prove:—

(i) If the middle term of a syllogism is distributed twice, the conclusion must be particular.

(ii) When the major term is predicate in its premise, the minor must be affirmative. (Annamalai '38)

XII

REDUCTION

1. Reduce Baroco and Bocardo to the first figure directly and indirectly. (1919)

Baroco P a M

S o M

∴ S o P

All those who deserve to be condemned are persons who do not repent for their wrongs.

Some liars are not those who do not repent for their wrongs.

∴ Some liars are not those who deserve to be condemned.

For direct reduction, we have the new mnemonic **Faksoko** (instead of **Baroco**). The first figure mood into which this should be reduced is **Ferio**.

K stands for obversion; **S** for simple conversion.

Faksoko

Ferio

P a M

M e P

S o M

S i M

∴ S o P

∴ S o P

No persons who repent for their wrongs are those who deserve to be condemned.

Some liars are persons who repent for their wrongs.

∴ Some liars are not those who deserve to be condemned.

Indirect Reduction: We have to prove that the conclusion is correct. We do this indirectly, i.e., by proving that it is not wrong. This we do by showing that the contradictory of the conclusion is not correct thus:

With the contradictory of the original conclusion substituted for the original minor premise we construct a new syllogism.

P a M

P a M

S o M

S a P

∴ S o P

∴ S a M

All those who deserve to be condemned are persons who do not repent for their wrongs.

All liars are those who deserve to be condemned.

∴ All liars are persons who do not repent for their wrongs.

The new conclusion contradicts the minor premise of the original syllogism. As the truth of the premise cannot be questioned it is

the new conclusion that must be wrong. This is due to assuming the contradictory of the original conclusion to be true. If the contradictory of the original conclusion is thus rejected, the original conclusion must be accepted as true.

$$\begin{array}{r} \text{Bocardo} \quad M o P \\ \quad \quad M a S \\ \hline \therefore S o P \end{array}$$

Some persons who hide the truth are not those who deserve to be condemned.

All persons who hide the truth are liars.

\therefore Some liars are not those who deserve to be condemned.

The new mnemonic is **Doksamosk** and it is to be reduced to **Darii**. **K** stands for obversion, **S** for simple conversion and **M** for transposition of premises.

$$\begin{array}{r} \text{Doksamosk} \quad \quad \text{Darii} \\ M o P \quad \quad M a S \\ M a S \quad \quad \overline{P} i M \\ \hline \therefore S o P \quad \quad \therefore \overline{P} i S \end{array}$$

The simple converse of $\overline{P} i S$ is $S i \overline{P}$. This, when obverted, gives us $S o P$.

All persons who hide the truth are liars.

Some people who do not deserve to be condemned are those who hide the truth.

Some people who do not deserve to be condemned are liars.

Some liars are people who do not deserve to be condemned.

Some liars are not people who deserve to be condemned.

Indirect Reduction. The principle followed here is the same except that instead of the minor, the major premise is displaced by the contradictory of the original conclusion.

$$\begin{array}{r} M o P \quad \quad S a P \\ M a S \quad \quad M a S \\ \hline \therefore S o P \quad \quad \therefore M a P \end{array}$$

All liars are those who deserve to be condemned.

All persons who hide the truth are liars.

\therefore All persons who hide the truth are those who deserve to be condemned.

This conclusion contradicts the original major premise. As the premise cannot be questioned, it is the conclusion which must be wrong. This wrong conclusion is drawn by assuming the original conclusion to be wrong. Hence the original conclusion must be right.

2. Reduce the following syllogism both directly and indirectly:—

Some men are virtuous.
All men are mortal.
∴ Some mortals are virtuous.

This is **Disamis** in the third figure. It must be reduced to **Darii** in the first figure.

Direct Reduction. S stands for simple conversion; M for transposition of premises.

Disamis	Darii
M i P	M a S
M a S	P i M
<hr/>	<hr/>
∴ S i P	∴ P i S
	∴ S i P

All men are mortal.
Some virtuous beings are men.
∴ Some virtuous beings are mortal.
∴ Some mortals are virtuous.

Indirect Reduction

M i P	S e P
M a S	M a S
<hr/>	<hr/>
∴ S i P	∴ M e P

No mortals are virtuous.
All men are mortal.
No men are virtuous.

The new conclusion contradicts the original major, which was replaced by the contradictory of the original conclusion. Hence the original conclusion must be correct.

3. Construct a syllogism in **Bocardo** to show that "Not all philosophers are failures in life," and reduce it directly and indirectly to the first figure. (Sept. '34)

The conclusion is given. It must be reduced to logical form.

L.F.—Some philosophers are not failures in life. We must construct a syllogism in **Bocardo**.

M o P
M a S
<hr/>
∴ S o P

S and P, i.e., the minor and major terms, are known to us. They are 'philosophers' and 'failures in life' respectively. We must find a suitable middle term. Let us take 'seekers after truth' as the middle term. This must occur as the subject of a particular negative proposition with 'failures in life' as predicate in the major premise; as the subject of a universal affirmative proposition with 'philosophers' as predicate in the minor.

The syllogism will be:

O Some seekers after truth are not failures in life.

A All seekers after truth are philosophers.

O \therefore Some philosophers are not failures in life.

Direct Reduction

Instead of Bocardo, we have the new mnemonic Doksamosk

Doksamosk	Darii
$M o P$	$M a S$
$M a S$	$P i M$
$\therefore S o P$	$\therefore P i S$
	$\therefore S i P$
	$\therefore S o P$

A All seekers after truth are philosophers.

I Some people who are not failures in life are seekers after truth.

I \therefore Some people who are not failures in life are philosophers.

\therefore Some philosophers are people who are not failures in life.

\therefore Some philosophers are not failures in life.

Indirect Reduction

$M o P$	$S a P$
$M a S$	$M a S$
$\therefore S o P$	$\therefore M a P$

All philosophers are failures in life.

All seekers after truth are philosophers.

\therefore All seekers after truth are failures in life.

The new conclusion contradicts the original major premise which was replaced by the contradictory of the original conclusion. Hence the original conclusion must be correct.

4. Construct a syllogism in Baroco to prove that some politicians are not fit to hold office, and reduce it directly and indirectly. (Mar. '35)

The conclusion is given. Its L.F. is:—

Some politicians are not persons fit to hold office. We must construct a syllogism in **Baroco** in the second figure. 'Politicians' and 'persons fit to hold office' are S and P, viz., the minor and major terms respectively.

$P a M$
$S o M$
$\therefore S o P$

We have to find a suitable middle term. Let us take 'statesmen' as the middle term. The middle term occurs as predicate in both premises—as the predicate of an A proposition in the major premise along with 'persons fit to hold office' the major term as the subject of the proposition; and as the predicate of an O proposition in the minor premise with 'politicians' for its subject.

The syllogism will be:

All persons fit to hold office are statesmen.

Some politicians are not statesmen.
 \therefore Some politicians are not fit to hold office.

Direct Reduction

The new mnemonic is Faksoko.

Faksoko

$P \ a \ M$

$S \ o \ M$

$\therefore S \ o \ P$

Ferio

$\overline{M} \ e \ P$

$S \ i \ \overline{M}$

$\therefore S \ o \ P$

No non-statesmen are fit to hold office.

Some politicians are non-statesmen.

\therefore Some politicians are not fit to hold office.

Indirect Reduction

$P \ a \ M$

$S \ o \ M$

$\therefore S \ o \ P$

$P \ a \ M$

$S \ a \ P$

$\therefore S \ a \ M$

All persons fit to hold office are statesmen.

All politicians are persons fit to hold office.

\therefore All politicians are statesmen.

The new conclusion contradicts the original minor premise which was replaced by the contradictory of the original conclusion. Hence the original conclusion must be correct.

QUESTION:

1. Reduce the following syllogism both directly and indirectly.
 Noor Jehan and Joan of Arc were courageous.
 Noor Jehan and Joan of Arc were women.
 \therefore Some women are courageous. (1917)
2. 'Not all despots are cruel'. Construct a syllogism in **Baroco** to prove the above statement and reduce it directly and indirectly. (1915)
3. Give a concrete example of 'OAO' in the third figure and reduce it directly and indirectly to the first figure. (Sept. '22)
4. What is the purpose of Reduction? Can this purpose be served in an easier way?
5. What are the distinctive characteristics of each of the four figures? (1917)
6. Construct a syllogism in **Baroco** to prove 'some industries do not require protection,' and reduce it directly. (Annamalai '38)
7. Construct a syllogism in **Bocardo** to prove that some politicians are not fit to hold office, and reduce it directly. (Mysore '43)
8. Construct a syllogism in **Baroco** to prove 'Not every one that sits for an examination comes out successful', and reduce it directly and indirectly.
9. Put the following in syllogistic form and reduce it directly:
 Some imperfect beings are not honest, for all men are imperfect and some men are not honest. (Mysore '44)

XIV

CONDITIONAL ARGUMENTS

MODEL EXERCISES:

Express the following arguments in their logical form and say whether they are valid or not:—

(1) **If Caesar was a tyrant, he deserved to die, but Caesar was not a tyrant; and so he did not deserve to die.**

If Caesar was a tyrant, he deserved to die.

Caesar was not a tyrant.

∴ Caesar did not deserve to die.

This is a fallacious hypothetical argument where the minor premise denies the antecedent. If reduced to the categorical form the argument will be seen to commit the fallacy of illicit major.

(2) **If all men were capable of perfection, some would have attained it; but none having done so, none are capable of it.**

If all men were capable of perfection, some would have attained it.

No one has attained perfection.

∴ No one is capable of perfection.

This is a hypothetical argument where the minor premise denies the consequent. The conclusion must therefore deny the antecedent, and it must be the contradictory of the antecedent. But here the conclusion 'No one is capable of perfection' is the contrary, and not the contradictory, of the antecedent. Hence the argument is fallacious.

(3) **If the cat is away the mice are everywhere; the cat must then be about, for the mice are nowhere.**

If the cat is away, the mice are everywhere.

The mice are not to be found anywhere.

∴ The cat must be about.

This is a valid hypothetical argument where the minor premise denies the consequent. *Modus tollens*.

(4) **If a country is prosperous, the people will be loyal. This country must be prosperous because its people are loyal.**

If a country is prosperous, the people will be loyal.

The people of this country are loyal.

∴ The people of this country are prosperous.

This is an invalid hypothetical argument where the minor premise affirms the consequent.

(5) **Logic is either a science or an art; it is a science; hence it is not an art.**

Logic is either a science or an art.

It is a science.

∴ It is not an art.

In form this is a disjunctive argument in which the minor premise affirms one alternative and the conclusion denies the other alternative. But 'science' and 'art' are not mutually exclusive; and

logic may well be regarded as both a science and an art. Some-logicians think that the alternatives of a disjunction need not be mutually exclusive; and they do not recognise the *modus ponendo tollens*.

(6) X is either intelligent or hard-working; but he is not intelligent; therefore he is hard-working.

X is either intelligent or hard-working.

X is not intelligent.

∴ X is hard-working.

This is a disjunctive argument, the *modus ponendo tollens*. As one cannot be sure that the alternatives given here are mutually exclusive, the argument is formally invalid.

(7) If there is censorship of the press, abuses which should be exposed will be hushed up; and if there is no censorship, truth will be sacrificed to sensation. But there must be either censorship or not. Therefore, either abuses which should be exposed will be hushed up, or truth will be sacrificed to sensation.

This is complex constructive dilemma and may be rebutted by the following counter-dilemma:—

If there is censorship of the press, truth will not be sacrificed to sensation; and if there is no censorship, abuses which should be exposed will not be hushed up.

Either there must be censorship or not.

∴ Either truth will not be sacrificed to sensation or abuses which should be exposed will not be hushed up.

This dilemma may be taken by the horns, i.e., we may admit the antecedents but deny the consequents. Censorship does not necessarily involve hushing up of abuses. Absence of censorship, likewise, need not involve sacrifice of truth to sensation.

(8) Moral exhortations are useless; for the good men do not need them, and the bad men will pay no heed to them.

If a man is good, moral exhortations are useless in his case as he does not require them; if a man is bad, moral exhortations are useless as he will not pay heed to them.

A man is either good or bad.

∴ Moral exhortations are useless.

This is a simple constructive dilemma and may be rebutted as follows:—

If a man is good, moral exhortations are not useless as he will pay heed to them; if he is bad, moral exhortations are not useless as he needs them.

A man is either good or bad.

∴ Moral exhortations are not useless.

Another way of meeting this dilemma is to escape between its horns. 'Good' and 'bad' do not exhaust the possibilities. Most men are neither absolutely good nor bad beyond recovery. In their case moral exhortations are certainly useful.

(9) If a body moves, it must move either where it is or where it is not. But a body cannot move where it is, nor can it move where it is not. Therefore motion is impossible.

If a body moves, it must move either in the place where it is or in the place where it is not.

But it can move neither in the place where it is nor in the place where it is not.

∴ It cannot move.

This is a simple destructive dilemma.

(See p. 114)

QUESTIONS:

1. How far can the hypothetical proposition be reduced to a categorical form? Reduce the following propositions to their categorical equivalents and discuss the adequacy of the categorical form to express their meanings:—

(a) If the weather is foggy, the train is late.

(b) If a metal is heated, it expands.

(c) If a tariff is introduced, the price of imported articles will rise.

(d) If you ask him, he is sure to refuse you.

(e) If you had come last night, you could have seen for yourself.

(Latta and Macbeath)

2. (a) In what sense is the disjunctive proposition an advance on the hypothetical?

(b) Express the following, as far as possible, in their equivalent hypothetical forms:—

(i) Either he has forgotten or he is deliberately lying.

(ii) A line is either straight or curved.

(iii) The man who did that was either drunk or stupid.

(iv) He is either a Protestant or a Catholic. (Ibid).

3. Discuss the relation between the categorical, the hypothetical, and the disjunctive proposition. What kind of knowledge is each form of proposition specially suited to express? (Ibid).

4. What is a hypothetical argument? Why is it so called? And what are the forms it may take?

5. What are your reasons for the rule 'Affirm the antecedent or deny the consequent'? To what fallacies of the categorical syllogism do the violations of this rule correspond?

6. Give an example of a valid syllogism involving denial of the antecedent, and show why you consider it valid.

7. Should the alternatives in a disjunctive major be exhaustive and exclusive? Illustrate your answer. (April '36)

8. Define a dilemma. Why is it said that dilemmatic arguments are more often fallacious than not? (April '37)

9. Comment on the statement, 'the practice of meeting a dilemma by another is a purely rhetorical device and has no logical efficacy.' (April '36)

10. Construct an argument in the form of a dilemma in favour of the freedom of the press. (March '31)

11. Explain with a suitable illustration the rules of a simple constructive dilemma, and the expressions—**taking a dilemma by the horns** and **rebutting a dilemma**. (Sept. '30)

12. Give an example of a complex destructive dilemma. How can a dilemmatic argument be met? (Sept. '28)

13. Make a dilemma to prove that a warrior must derive happiness from fighting valiantly in a religious war and rebut it. (1914)

14. State the following arguments in logical form and examine their validity:—

(a) If he had left the house a few minutes earlier, he would no doubt have caught the train; but as he was delayed he must have missed it.

(b) If all philosophical theories were sound, some would be accepted by a majority of thinkers; but as none are accepted by a majority of thinkers, none are sound.

(c) If man were not capable of progress, he would not differ from the brutes; but man does differ from the brutes; therefore he is capable of progress. (Latta and Macbeath)

(d) If all students had worked hard, some would have attained distinction; but as none has attained distinction, no one has worked hard.

(e) If it be fated that you recover from your present disease, you will recover, whether you call in a doctor or not; again if it be fated that you do not recover from your present disease, you will not recover, whether you call in a doctor or not; but one or other of these contradictories is fated, and therefore it can be of no service to call in a doctor.

(f) To speak of training people to be teachers is unmeaning. For either they have a natural gift for teaching and do not need to acquire it, or they lack the gift and no training can impart it.

15. Construct a dilemma to prove that 'advertisement is either needless or wicked.' (Annamalai '38)

16. Give examples of different forms of dilemma. What are the frequent sources of fallacy in them? Give illustrations. (Mysore '43)

17. What is meant by rebutting a dilemma? How will you meet the following:—It is useless to go on canvassing for either people intend to vote for your candidate, in which case canvassing is superfluous; or they do not intend to vote for your candidate, in which case canvassing is ineffective.

18. How will you meet the following dilemma?

If I keep the lamp burning, the rays of light physically disturb me; if I put it out, imaginary fears haunt me. I seem to be destined, therefore, to pass sleepless nights.

XV

ABRIDGED AND CONJOINED SYLLOGISMS

MODEL EXERCISES:

Express the following enthymemes in their complete form and give their order.

1. **You, as you are old and reverend, should be wise (Shakespeare).**

This is an enthymeme of the first order. The major premise must be supplied. The syllogism will be:

All old and reverend people are those who should be wise.

You are old and reverend.

∴ You are one who should be wise.

2. **Romeo is foolhardy because all lovers are foolhardy.**

This is an enthymeme of the second order. The minor premise is to be supplied and the syllogism stated thus:

All lovers are foolhardy.

Romeo is a lover.

∴ Romeo is foolhardy.

3. **No prophet is honoured in his own country but A is honoured.**

This is an enthymeme of the third order.

The conclusion has to be supplied and the syllogism completed thus:

No prophet is honoured in his own country.

A is honoured in his own country.

∴ A is not a prophet.

4. Identify the following arguments:—

(a) **All thieves are dishonest; all dishonest persons are immoral; some immoral persons are not punished; therefore, some thieves are not punished.**

This is an Aristotelian Sorites, and it may be expanded thus:

- (i) All dishonest persons are immoral.

All thieves are dishonest.

∴ All thieves are immoral.

- (ii) Some immoral persons are not punished.

All thieves are immoral persons.

∴ Some thieves are not punished.

In syllogism (ii) we have the fallacy of the undistributed middle. This is due to the violation of the rule that in an Aristotelian Sorites only the first premise may be particular. Here the last is particular and hence the fallacy.

(b) **A wise man always lives a life of hardship; for to make sacrifices is always a hardship, the industrious man has to make sacrifices, the man who seeks to gain knowledge must be industrious, and the wise man is one who seeks to gain knowledge.**

(Creighton)

This is a valid Goclenian Sorites, and the constituent syllogisms may be constituted thus:

- (i) All cases of sacrifices are cases of hardship.
All industrious men are those who have to make sacrifices.
∴ All industrious men are those who are subject to hardships.
- (ii) All industrious men are those who are subject to hardships.
All seekers after knowledge are industrious.
∴ All seekers after knowledge are subject to hardships.
- (iii) All seekers after knowledge are subject to hardships.
All wise men are seekers after knowledge.
∴ All wise men are subject to hardships.
- (c) All mirrors are fragile, since all glass is fragile.
This is mirror.
∴ This is fragile.

This is a single Epicheirema, as reason is given only for one of the premises. The major premise then would appear as the conclusion of the following syllogism:

- All glass is fragile.
- All mirrors are glass.
- ∴ All mirrors are fragile.

QUESTIONS:

- (1) 'Enthymemes do not constitute a separate class of logical arguments.' Explain and discuss.
- (2) (a) Prove that in the ordinary or Aristotelian Sorites no premise can be negative except the last.
(b) State and justify the rules of the Goclenian Sorites. (1911)
- (3) What is a double epicheirema? Give an example.
4. Identify, complete where necessary and test the following arguments:—
 - (a) You must have recorded your vote, for you are an official.
 - (b) Every war increases taxation and the popularity of anything that touches our pocket is shortlived.
 - (c) Gold is not a compound for no metal is a compound.
 - (d) My observation of life leads me to believe that lack of exercise is a serious limitation, for it tends to weaken the body. Any tendency in that direction interferes with clear thinking and everyone recognizes that lack of clearness in thinking is a serious handicap. (March '22)
 - (e) If trees are cut down extensively, the rainfall is affected; if there is not enough rain, there will be little water for cultivation; in the absence of sufficient water there will be a poor harvest, as was the case this year. Hence deforestation on a large scale must have taken place recently in this case. (March '28)

(f) All virtues are ennobling, for whatever exalts the soul is ennobling.

Temperance is a virtue for it keeps the mind calm.

∴ Temperance is ennobling.

(5) Give an example of your own of Aristotelian Sorites.

(6) What is a Sorites? Distinguish between the Aristotelian and Goclenian Sorites.

XVI

THE LIMITS OF SYLLOGISTIC REASONING

QUESTIONS:

1. Explain and examine the view that every syllogism involves the fallacy of **petitio principii**. (Latta and Macbeath)

2. What is meant by the 'paradox of inference'? Explain the conditions of inference to which the 'paradox' calls attention and indicate how they can be satisfied. (Ibid)

3. What is the general principle on which all a **fortiori** arguments proceed? How can you tell when an argument is of this type, and whether it is valid or not? (Creighton)

4. (a) What conditions must a valid relational inference satisfy?

(b) Distinguish carefully between the principle on which syllogism and that on which relational inference proceeds. (Ibid)

XVII

FALLACIES OF DEDUCTIVE REASONING

Detection and identification of fallacies require application of the knowledge of rules governing various types of reasoning. The student must remember that since 'error is infinite in its aberrations', a given argument may commit more than one fallacy.

He need point out, however, only the most glaring fallacy that an argument commits.

The following suggestions are given to aid the student to detect and identify fallacies:—

1. The student must first make sure, whether the unacceptability of the argument is due to ambiguity in language or errors in thinking itself. Both mediate and immediate inferences may be unacceptable on account of errors in interpretation.

2. If the fallacy is not due to language, the argument must be stated in logical form. It is advisable to decide, whether a given argument is mediate or immediate. Care should be taken to see in which of the three main forms of mediate reasoning, Categorical, Hypothetical and Disjunctive, the given argument may be stated naturally, without distortion of thought. After the argument is expressed in its proper logical form, it must be tested by reference to the rules governing that specific form.

3. An argument may be true to form and yet invalid. So, the matter or content of the argument must be examined. Two conditions must be satisfied: (i) that the terms used in an argument are unambiguous and well-defined; (ii) that what is proved is strictly derived from the premises and not presupposed or presumed.

MODEL EXERCISES:

Determine the type of reasoning the following arguments exemplify and test their validity.

1. Tomorrow afternoon at four o'clock the Rev. X will deliver the third and last address of a series of plain talks to young men about their perils at the—branch of the Y.M.C.A. (Creighton)

This is a case of the fallacy of Amphiboly. Owing to the ambiguous construction of the sentence, it may be misinterpreted that the talk will be about perils at the—branch of the Y.M.C.A., whereas what is meant is that the talk will be about perils, and that the talk will be given at a certain place.

2. Alexander is the son of Philip, and therefore, Philip is the father of Alexander.

This is valid Immediate Inference by Converse Relation. Arguments like these could be determined to be valid or invalid only if we have knowledge of the system to which they refer.

3. "I argue thus: the world agrees.

That he writes well, who writes with ease.

Then he, by sequence logical,

Writes best who never thinks at all."

The argument may be expressed thus:

He who writes with ease is one who writes well.

He who never thinks at all is one who writes with ease.

∴ He who never thinks at all is one who writes well.

It will be noticed here that the middle term is not the same in both the premises. The conclusion suggested by the verse is plausible only because the word "ease" is ambiguous.

4. This syllogism must be valid, for it has three terms.

(Sept. '38)

All valid syllogisms are those which have three terms.

This syllogism is one which has three terms.

∴ This syllogism is valid.

The middle term occurs as the predicate of affirmative premises and hence is not distributed. The syllogism commits the fallacy of undistributed middle.

5. All tulips are beautiful flowers.

No roses are tulips.

∴ No roses are beautiful flowers.

(Fowler)

The major term is distributed in the conclusion but is not distributed in its own premise.

Thus the argument commits the fallacy of illicit major.

6. Eleventh hour preparation is always bad, for it not infrequently leads to nervousness, confusion and offering wrong answers. (March '30)

This is an Enthymeme of the first order. The complete syllogism will be:

All things which lead to nervousness, confusion and offering wrong answers are bad.

Some eleventh hour preparation is that which leads to nervousness, confusion and offering wrong answers.

∴ All eleventh hour preparation is bad.

(not infrequently=some)

Here the minor term is used distributively in the conclusion while it is not so used in its own premise. Hence we have the fallacy of illicit minor.

7. He must be a Scotsman, for no Scotsman can see the point of a joke. (Sept. '32)

No Scotsman is one who sees the point of a joke.

He is not one who sees the point of a joke.

∴ He is a Scotsman.

Here the conclusion is sought to be derived from two negative premises. The syllogism is invalid as it violates the rule that no conclusion can be drawn from two negative premises.

8. Some men are good

Some men are wise.

∴ Some wise men are good. (Fowler)

This syllogism is invalid as it violates the rule that no conclusion can be drawn from two particular premises.

9. Two Negroes were discussing family trees.

"Yessuh, man," said Ambrose, "I can trace mah relations back to a family tree."

"Chase'em back to a family tree?" asked Mose.

"Now, man—trace'em—trace'em—get me?"

"Well, there ain't but two kinds of things that live in trees—birds and monkeys—and yo' sho' ain't got not feathas on you.

We could detect two fallacies in this amusing conversation. The first is the ambiguous and shifting use of the term 'tree.' The second is an imperfect disjunctive syllogism that could be stated thus:—

Things that live in trees are either birds or monkeys.

You are not a bird (since you do not have feathers).

∴ You are a monkey.

10. If prohibition is introduced, unemployment will increase. Unemployment has increased. Therefore prohibition has been introduced. (March '38)

This is an invalid hypothetical syllogism as it affirms the consequent.

If prohibition is introduced, unemployment will increase.
Unemployment has increased.

∴ Prohibition has been introduced.

11. The Government will not appoint you to the post, if you are not a graduate. But you are a graduate. Therefore the Government will appoint you to the post. (Sept. '39)

This is an invalid hypothetical argument as the antecedent is denied.

If you are not a graduate, the Government will not appoint you.

But you are a graduate.

∴ The Government will appoint you.

12. If a shop-keeper closes his shop for a day, he loses custom; so that if all the shop-keepers of the town do so on the Ayudha-puja day, they will lose custom. (March '34)

This argument commits the fallacy of composition. If one shop-keeper alone closes his shop on a certain day, he will lose custom. When all shop-keepers close on a certain day, there will be no transaction of business at all. The question of loss of custom does not arise.

13. He has no appreciation of beauty, for he has no taste for pictures. (Fowler)

This is the converse Fallacy of Accident—a dicto secundum quid ad dictum simpliciter. While a person may not have appreciation of Beauty of one type—or under certain circumstances, it need not follow that he will have no appreciation of Beauty as such or Beauty generally.

14. South Indians have astounding memories. A, B and C are South Indians. Therefore A, B and C have astounding memories. (Sept. '37)

This is the Fallacy of Division. What is true of South Indians generally, need not be true of particular South Indians.

15. He is a very bad marksman; hence it is safest to stand in front of the object he is aiming at. (Sept. '32)

This is the simple Fallacy of Accident—a dicto simpliciter, ad dictum secundum quid. It may be generally true that a person is a bad marksman. This does not prevent his hitting the mark on a given occasion.

16. I will not do this act, because it is unjust; I know that it is unjust, because my conscience tells me so, and my conscience tells me so, because the act is wrong. (Fowler)

This argument commits the Fallacy of Petitio Principii. It begs the question by arguing in a circle. (Circulus in probando). Each of the propositions is used in turn to prove the truth of the other.

17. Why are white bears more ferocious than black bears?

This argument commits the Fallacy of Complex Question. It assumes, without giving us reasons, that white bears are more ferocious than black bears; and requires us to give the reasons for their being so.

18. What fallacy may be involved in calling a certain age 'the Reformation.' (Creighton)

The fallacy will be *Petitio Principii*. We beg the question by using an expression which assumes instead of proving that a certain age could be characterised as Reformation.

19. In reply to the gentleman's arguments, I need only say that two years ago he advocated the very measure which he now opposes. (Creighton)

This argument commits the Fallacy of *Argumentum ad hominem*. Inconsistency between a former statement and the present one need be no bar to the acceptance of the latter, if it is based on logical grounds.

20. The soul is immortal, because Plato says that the soul exists for ever. (Sept. '39)

This argument commits two fallacies. The more glaring is *Argumentum ad verecundiam*. We are asked to accept the statement because a great man, Plato, has considered it to be true. The appeal to great names is no substitute for valid reasons. In the absence of reasons, authority of great men will have but doubtful value.

The second fallacy is *Petitio Principii*. 'Immortal' and 'exists for ever' are the same. A change in expression should not blind us to the fact that the reason for a position is yet to be given.

21. There can be no doubt that the black races are intellectually inferior; for have they not smaller brains? (March '27)

This is *Non-Sequitur* or the Fallacy of the Consequent. The conclusion does not follow from the premises supposed to support it. It may be a fact (which, again, must be proved) that the black races have smaller brains. The size of the brains is no indication of intelligence. Hence the conclusion does not follow.

22. Why does a ball, when dropped from the masthead of a ship in full sail, fall, not exactly at the foot of the mast but nearer to the stern of the vessel? (Quoted by Creighton)

This is a complex question. It assumes, without proof, that the ball falls nearer to the stern of the vessel and not at the foot of the mast. But it is not a fact that the ball falls nearer the stern. (vide *Intermediate Logic*: Welton and Monahan, p. 305).

23. If A is true, I is true; if I is true, O may be true; therefore, if A is true, O may be true.

This is an ingenious attempt to prove that A and O, which are contradictories, can both be true. Given A as true, it immediately follows that O, its contradictory must be false. Only where I is given as true (and nothing said about its sub-alternant A, which, therefore, is doubtful) is O doubtful, i.e., it may be true or not true.

This inference is plausible only if the law of non-contradiction is forgotten.

24. If it be true, as Mr. Spencer thinks, that the past experience of the race has produced innate ideas and feelings. Weis-

mann's denial of use-inheritance would be refuted. Certainly but it is just possible that Mr. Spencer's theory is not true.

(Creighton)
This argument commits the Fallacy of Objections. As Creighton says, "In any matter of dispute there will be objections to any solution offered; but this, of itself, is no disproof of the conclusion attacked, provided we have some positive grounds for it."

25. If man love not his brother whom he hath seen, how shall he love God whom he hath not seen? (Creighton)

This is an a fortiori argument. It would run thus: If it is difficult to love those whom we see, it is much more difficult to love God whom we have not seen.

26. Why should any one seek to learn? For learning is either a progress from knowledge to knowledge, in which case it is useless, or it is a progress from ignorance to knowledge in which case it is impossible.

This is a complex constructive dilemma which may be stated thus:

If learning is a progress from knowledge to knowledge, it is useless; if it is a progress from ignorance to knowledge, it is impossible.

Either learning is a progress from knowledge to knowledge or it is a progress from ignorance to knowledge.

∴ Either it is useless or it is impossible.

This dilemma may be met by escaping between the horns. The alternatives are not exhaustive. A third alternative is to treat learning as a progress from less knowledge to more knowledge, from what is incompletely known to what is completely known.

27. If we take in our hands any volume of divinity or school metaphysics, for instance, let us ask:—

Does it contain any abstract reasoning concerning quantity or number? No. Does it contain any experimental reasoning concerning matter of fact and experience? No. Commit it then to the flames, for it can contain nothing but sophistry and illusion.

(Sept. '23)

This argument assumes that any work worth studying must deal either with abstract reasoning or with experimental reasoning. A work which does not belong to either of these categories contains therefore nothing but sophistry and illusion. As a dilemma, the argument may be stated thus:

If a work contains abstract reasoning, it is worth reading; if it contains experimental reasoning, it is worth reading.

Metaphysical and theological works contain neither abstract reasoning nor experimental reasoning.

∴ Metaphysical and theological works are not worth reading.

It is clear that the above argument is fallacious as the minor premise denies the antecedent. Besides, some logicians take exception to both alternatives being denied in the minor premise. The argument is formally invalid.

It is possible to treat the argument as Non-Sequitur also. If a work does not contain abstract or experimental reasoning, it does not follow that it must contain only sophistry and illusion and that therefore it must be useless. In fact, Metaphysics is necessary as we require a discipline which, instead of losing itself in the details of the abstract and concrete sciences, goes to the ultimate principles and basic concepts relating to these sciences.

28. I am accused of inciting to sedition by the address which I delivered to the meeting. But there is not one man present at the meeting who, if my remarks had been addressed to him privately, would have been moved to disloyalty.

(Welton and Monahan)

This argument commits the fallacy of composition. What does not move individual persons may move them if they happen to constitute a group.

29. The king, by the constitution, can do no wrong, and therefore he is not rightly subject to praise or blame. (Ibid)

The term 'wrong' has a legal and a moral implication. In the above argument we have an ambiguous and shifting use of this term. While legally, the king can do no wrong, it does not follow that his deeds should not be judged from the point of view of Ethics.

30. If we have a wet summer there is always a good clover-crop. We shall therefore have a poor crop this year, for the summer has been very dry.

The argument may be stated thus:

If we have a wet summer, we shall have a good clover crop.

We did not have a wet summer.

∴ We shall not have a good clover crop.

This is an invalid Hypothetical argument as the antecedent is denied in the minor premise.

31. Your arguments against the philosophy of Hegel are of no value; for you uphold that of Schopenhauer, which is equally repugnant to common sense. (Creighton)

This is argumentum ad hominem—of the tu quoque variety. Instead of meeting the charge that Hegel's philosophy is repugnant to common sense, the same charge is levelled against the philosophy upheld by the opponent. This device of meeting a charge by hurling it back is logically invalid though it may silence the opponent.

32. I infer that this archaic statue must have been made about 525 B.C. because of the shape of the letters on the pedestal. The inscription on the statue belongs to the period about 525 B.C., for the statue shows the peculiar characteristics of that period.

(Sept. '41)

This argument commits the fallacy of *Petitio Principii*. It is an example of that variety of *Petitio*, known as *circulus in probando*—arguing in a circle. To determine the date of the statue, the proof given is the shape of the letters. To determine the date

of the particular type of letters, the date of the statue is used. Thus we use A to prove B, and B to prove A.

33. You are inconsistent with yourself, for you told me yesterday that there was a presumption of this man's guilt, and now, when I say that I may presume his guilt, you contradict me. (Fowler)

This is *argumentum ad hominem*. The charge of inconsistency is brought forward for rejecting an argument. The man accused of inconsistency might have had strong reasons for revising his opinion. So long as he can give valid reasons for his position, his character or previous utterances have nothing to do with his present position.

34. He talks with angels; for he himself says so, and a man who talks with angels cannot lie. (Sept. '34)

This is *Petitio Principii*. We have to prove that the man talks with angels. We do this by giving his own statement. In order to prove that his statement is true, we say that a man who talks with angels will not tell lies. That he talks with angels, is precisely what is to be proved. So, that cannot be assumed to validate his statement.

35. One can live without food; for one can live without bread or potatoes or porridge or any other kind of food.

(Latta and Macbeath)

This may be identified as the converse Fallacy of Accident. The fallacy consists in arguing that what is true of a thing under some condition or accident can be asserted of it simply or in its essential nature. While a man can certainly live without particular varieties of food, it does not follow that he can live without food as such or any food.

36. The great plea for increase of armaments is that any nation which makes itself strong makes itself respected and feared. (Ibid)

This is *Petitio Principii* involving a vicious circle. A nation has to arm itself in order to make itself respected and feared. But this starts the mischief. Other nations, seeing this particular nation arm itself feel that their security is threatened. Thus a race for armament begins. What is attempted as a solution of the problem really turns out to be an aggravation of it. Armament is for the sake of producing fear in others. Fear, thus started, makes the others arm themselves.

37. You say that this book is inaccurate; but I saw an excellent review of it. (Ibid)

Here we have *Argumentum ad verecundiam*. The respect felt for the critic who reviews a book should not prevent us from finding out for ourselves its merits and demerits.

38. How can you hold that self-government is good and deny that India should govern itself? (Ibid)

This argument commits the Simple Fallacy of Accident. The principle of self-government may be conceded in a general manner.

It does not follow that it must apply to a given case without regard to the particular conditions that govern it. This is not, however, to deny that there is a case for India's self-government. Each case has to be examined on its own merits.

39. My hand touches the pen; the pen touches the paper; therefore my hand touches the paper. (Fowler)

As a syllogism, it is invalid by having four terms. The argument may be treated as invalid on material grounds also. It will be one type of the Fallacy of Accident, 'what is true of the accidental qualities or relations of a thing is true of the thing itself.' (e.g. This dog is yours; this dog is a father; therefore this dog is your father.) Here the accidental relationship between pen and hand and pen and paper is brought forward to establish a relationship between hand and paper.

40. You must be a friend to my friend because you are a friend to me. (Creighton)

This is a non-syllogistic relational argument. The truth of the conclusion cannot be guaranteed. Non-syllogistic reasoning cannot be judged to be valid or otherwise by a reference to rules of logic. It can be accepted only if we have sufficient knowledge of the general nature of the system to which it refers and the relations prevalent therein.

41. When all is said, it remains true that there are but three ways of living possible in this world—by working, by robbing, or by begging. To beg is infamous; to rob is criminal; if a man will not work neither shall he eat. (Latta and Macbeath)

This is an improper disjunction. The alternatives are not exhaustive.

To live a man must either beg or steal or work. It must not be begging (as begging is infamous); it must not be stealing (as stealing is criminal).

∴ To live a man must work.

Of the three alternatives, two are denied, and therefore the third is forced upon us. But there may be other alternatives, e.g., a man may depend for his living on the property left him by his ancestors.

42. Only if you break the law are you imprisoned. That is why you are let free. (Ibid)

This is a hypothetical syllogism.

Only if you break the law are you imprisoned.

You have not broken the law.

∴ You are not imprisoned.

Though the minor premise denies the antecedent (which is normally a fallacy) as it is the only antecedent, there is no fallacy. Where we have more than one antecedent, we should not deny the antecedent.

43. Fallacies would only be excusable if they were unavoidable; but they are avoidable, therefore they are inexcusable. (Ibid)

Stated in the proper form, the argument will be:

Only if fallacies are unavoidable, are they excusable.

They are avoidable (i.e., not unavoidable).

∴ They are not excusable (i.e., inexcusable).

As in the previous example, here also though the antecedent is denied, as it is the only antecedent, this hypothetical syllogism is valid.

44. He did not take Greek in his Degree Course for all candidates must take either Latin or Greek, and he took Latin.

(Ibid)

This is a valid Disjunctive Syllogism—Modus ponendo tollens.

He must take either Latin or Greek.

He took Latin.

∴ He did not take Greek.

45. A child is a human being. AB is a big child. Therefore AB is a big human being. (March '35)

We have here a combination of Immediate and Mediate Inference.

A child is a human being.

∴ A big child is a big human being.

This is Immediate inference by Added Determinants. It is invalid as the Subject and the Predicate are differently qualified by the determinant. On the basis of this false inference, the following Syllogism rests:

A big child is a big human being

AB. is a big child

∴ AB is a big human being.

The conclusion is invalid as it is based on invalid premise.

46. "Then you should say what you mean," the March hare went on. "I do," replied Alice hastily, "at least, I mean what I say—that's the same thing you know." "Not the same thing a bit," said the Hatter. "Why you might just as well say that 'I see what I eat' is the same thing as 'I eat what I see'."

(From 'Alice in Wonderland')

In this amusing conversation the first mistake for which Alice is responsible, is due to the ambiguous use of the word "mean." In 'I say what I mean'—'mean' stands for the idea that the speaker wishes to convey. In the other statement 'I mean what I say'—'mean' introduces a confident assertion. The second mistake, for which the Hatter is responsible, occurs because the Hatter chooses a seemingly similar but essentially different statement. There is no ambiguity at all in the word "eat." Besides there is Illicit conversion when we substitute the statement "I see (all) things I eat" for "I eat (some) things I see."

47. Prodigality encourages industry, therefore parsimony discourages it. (Bartlet)

This is illogical obversion. The quality is not changed as it should be; instead of the contradictory of the predicate, the con-

trary is given. The subject is also replaced by its contrary while in a logical obverse, it will never be changed at all.

48. **Gold is one thing, heavy is another.—So it cannot be said that gold is heavy.** (Ibid)

We commit *Petitio Principii* involving two assumptions:

- (i) that gold can exist by itself,
- (ii) that if the two are different they must be completely alien to each other.

49. **Human beings are governed by primary human instincts, not by socialist theories; therefore socialist theories are wrong.**

(Stebbing)

A proposition to which no exception can be taken is stated first. As soon as it is accepted, something in no way related to it is forced upon us as though it really followed from the first statement. The conclusion does not follow and hence the argument is to be identified as *Non-Sequitur*.

50. **Women can never be man's equal; is she not the weaker vessel?** (Sept. 29)

The rhetorical question 'Is she not the weaker vessel' requires the answer 'she is the weaker vessel.' This is exactly the point at issue. So it cannot be given as a reason for accepting the statement that woman can never be man's equal. The argument is thus seen to commit the fallacy of *Petitio Principii*.

Note. The student is advised to explain briefly and not merely indicate by name the fallacies he detects in arguments. The following quotation from Miss Stebbing's *Logic in Practice* may help him in examining argument.

"It is useful to cultivate the habit of asking oneself whether a given statement is supported by the argument offered. If so, the premises must be consistent with the conclusion, and must provide some reason for it..... This reason will, we have seen, be valid in any other argument of the same form. The language used must be free from ambiguity; the point at issue must be definite. We do not **disprove** a proposition by showing that the argument offered in its support is unsound, but, unless we are offered another, and a sound argument in support of it, we have no reason for accepting it as true.

QUESTIONS:

1. Explain the nature of the following fallacies and give an example of each:—*Petitio Principii*, *Converse Fallacy of Accident*, and the *Fallacy of Composition*. (March '35)

2. Write short notes on: *Non-Sequitur*; *Amphibology*. (Sept. '38)

3. Write short notes on: *Ignoratio Elenchi*. (March '40)

4. Examine the following arguments (stating them in logical form wherever necessary):

(1) Some said Marner must have been in a fit. But the argumentative Mr. Macey shook his head, and asked if anybody was ever known to go off in a fit and not fall down. A fit was:

a stroke wasn't it? And it was in the nature of a stroke to partly take away the use of a man's limbs. (1911)

(2) A vacuum is impossible for if there is nothing between bodies they must touch. (1917)

(3) If the train is late, I shall miss my appointment; if it is not late, I shall miss the train; but either it will be late or not late; therefore in any case, I shall miss my appointment. (1921)

(4) Some mathematicians are logicians; no logicians are unacquainted with the work of Aristotle; therefore some mathematicians are not unacquainted with the work of Aristotle. (1921)

(5) Professor Stout in his Manual of Psychology says that some idiots have remarkable powers of memory; I ought to be thankful, therefore, that my memory is a bad one. (1921)

(6) Examine the following dilemma and rebut it:—If woman is like man and it is right for man to vote, it must be right for woman to do so. If woman is unlike man, he can never truly represent her and she ought to be allowed to represent herself. (Sept. '22)

(7) The reduction of railway fares is followed by an increase of pilgrim traffic. The increased pilgrim traffic to Madurai in April shows that fares must have been reduced. (April '30)

(8) There are two things of which a man ought not to fret: things which he can help, and things which he cannot help. (Dec. '21)

(9) All novels are false; therefore no novel reader is a lover of truth. (March '31)

(10) Good always triumphs and vice always fails. Therefore the victor cannot be wrong, nor the vanquished right. (March '31)

(11) Bodies subject to gravity descend, but smoke ascends. Therefore smoke is not subject to gravity. (March '31)

(12) The end of human life is perfection; death is the end of human life. Therefore death is perfection. (Sept. '37)

(13) Twenty is one number. Four and sixteen are twenty. Therefore four and sixteen are one number. (March '25)

(14) If all the accused were innocent, some at least would have been acquitted; but none was acquitted. Therefore none was innocent. (Sept. '25)

(15) All dogs have four legs. This table has four legs. Therefore this table is a dog. (1911)

(16) You are not what I am. I am a man. Therefore you are not a man. (1911)

(17) The Hindus are a religious nation; I am, therefore, religious. (April '27)

(18) If life is full of distraction, it is exhausting. Modern life is full of distraction. So it is exhausting. (Sept. '27)

(19) It is only philosophers who are truly religious. This man is a philosopher. Therefore he is truly religious. (March '39)

- (20) Humanity is bound to die out, for are not all men mortal? (March '40)
- (21) You say you do not believe in child marriage. Are you wiser than our ancestors who believed in it and advocated it? (March '41)
- (22) A man who is inoculated becomes immune from plague. I am not inoculated. Therefore I am not immune from plague. (March '41)
- (23) Ice is water; water is liquid. Therefore ice is liquid. (March '33)
- (24) All suffragettes are women and all suffragettes are progressive; hence all women are progressive. (March '36)
- (25) All amusements are irrational, as there is a purpose in everything reasonable. (Sept. '36)
- (26) A sprat is a fish, therefore a big sprat is a big fish.
- (27) This man must be deaf, for he talks loud. (March '35)
- (28) Food is a necessity of life; venison is food. Therefore venison is a necessity of life. (Sept. '35)
- (29) You cannot deny the advantage of a tariff, for it either keeps foreign goods out and benefits the home-producer, or lets them in and benefits the exchequer. (Sept. '35)
- (30) Cotton cannot be strong enough to make clothes of, for look, I can break this cotton thread quite easily. (Sept. '41)
- (31) Ill-managed business is unprofitable. Railways are never ill-managed. Therefore all railways are profitable. (Well-ton and Monahan.)
- (32) The governor of a country ought not to be blamed for using his influence to further his religious views, for every man has a right to inculcate his own opinions. (Ibid)
- (33) Where did you hide the goods you stole last night? (Ibid)
- (34) The Bill before the House is well calculated to elevate the character of education in the country, for the general standard of instruction in all the schools will be raised by it. (Ibid)
- (35) I have no hesitation in saying that the proposition, however good in theory, is in practice utterly absurd. (Fowler)
- (36) High technical advance and ruthlessness in warfare seem to be necessarily connected; look at the Germans. (Andhra University, March '40)
- (37) This measure would be destructive of the national prosperity, and I cannot adduce a more cogent argument than that, five years ago, you were yourself of the same opinion. (Fowler)
- (38) He must be a Mahammadan, for only Mohammadans hold these opinions. (Ibid)
- (39) He must be a Mohammadan, for all Mohammadans hold these opinions. (Ibid)
- (40) To reject this proposal would be unreasonable, and consequently to accept it is reasonable. (Ibid)
- (41) This event happened either at Rome, Naples, or

Florence; it did not happen at Rome or Naples, and consequently it must have happened at Florence. (Ibid)

(42) Logic is indeed worthy of being cultivated, if Aristotle is to be regarded as infallible; but he is not. Logic therefore is not worthy of being cultivated. (Ibid)

(43) If acquired variations are transmitted, there must be some unknown principle of heredity; if they are not transmitted, there must be some unknown factor of evolution. (Quoted by Creighton)

(44) If a man is educated, he does not want to work with his hands; consequently, if education is universal, industry will cease. (Creighton)

(45) AB is a great soul, for like some heroes of the world, Socrates and Bruno, AB is also persecuted.

(46) We cannot say Rama is good, simply because he is clever, for there are some clever men who are not good.

(47) Either Newton or Leibniz invented the Calculus. Newton invented it, therefore Leibniz did not. (Mellone)

(48) He must be mad to do such a thing. (Ibid)

(49) He speaks the truth, and a man who speaks the truth is always worth hearing. (Ibid)

(50) This author is certainly confused. If I understand his book rightly he is confused in his thinking, and if I do not understand it, he is confused in his writing. (Ibid)

(51) If the monsoon is very late, the crops will suffer. This year we may expect good crops as the monsoon was not late. (Sept. '42)

(52) A poet is a man. Therefore a good poet must be a good man. (Sept. '42)

(53) Patrons of arts and science are public benefactors. Only wealthy people can be patrons of arts and science. Therefore no poor men can be public benefactors. (Sept. '42)

(54) What is seen is visible; what is heard is audible; what is desired is desirable. (March '43)

(55) The accused must have stolen the watch, for he has had seven previous convictions for theft. (March '43)

(56) There is no harm in allowing boys to climb trees. If they are confident, they are safe; if they are nervous, they will not climb high enough to run a risk. (March '43)

(57) Matter does not exist, since it does not have consciousness; and whatever has consciousness exists. (March '43)

(58) The holder of some shares in a lottery is sure to gain a prize; I am the holder of some shares in a lottery and so I am sure to gain a prize. (March '43)

(59) Only industrious students pass their examinations. George cannot therefore be industrious, for he has failed. (Sept. '43)

(60) Men are generally lazy, and as Ramesh is a man, he must be lazy. (Sept. '43)

(61) We know that God exists, because our Scriptures tell us so: whatever our Scriptures say must be true, for they are of divine origin. (Sept. '43)

(62) No course of education is liberal which discourages the fine arts; and as the Wardha scheme does this, it is not liberal.

(63) Unlimited power corrupts human nature. Look at Hitler, Mussolini and Stalin!

(64) If the accused is guilty, it is morally wrong to defend him; if he is innocent, defence is unnecessary. So, no lawyer need undertake the defence of accused persons. (Annamalai '40)

(65) It is not true that wealth is the only thing that affords satisfaction, for conscience is not wealth.

(66) If people use too much alcohol, their health is impaired; if they abstain from it, they do not have any stimulus for work.

(67) If a nation increases its air-force, it will be better prepared to defend itself. Therefore, if all nations add to their airforce, all of them will be better prepared for defending themselves. (Annamalai '38)

(68) He that is of God heareth my words; you, therefore hear them not.

(69) If all voters had exercised their franchise, X would have succeeded; but he did not.

(70) If you win the battle, you will gain the kingdom; if you lose it, you will obtain the heroes' paradise. (Annamalai '37)

(71) Regulations and rules are of no avail; for, good men don't need them, the bad won't heed them.

(72) Gopal is, evidently, not honest, for only the honest are straightforward in their conduct; while my friend, Gopal, is not noted for his straightforwardness. (Annamalai '39)

(73) It acquires much intelligence and mechanical knowledge to fly well. Consequently, gnats must possess much intelligence and mechanical knowledge.

(74) Galileo preceded Newton; and Newton preceded Einstein. Therefore, Galileo preceded Einstein. (Annamalai '38)

(75) War is productive of evil, therefore, peace is productive of good.

(76) None of the evils of this world are to be feared, for they are all transitory.

(77) How can you say that he is not a careful examiner when he is seven in examining his papers, as careful examiners are known to be.

(78) Whoever believes this is a defeatist; so that you are no defeatist, for you do not believe this.

(79) If only the ignorant despise knowledge, this man cannot be ignorant, for he praises knowledge.

(80) He has been a politician for years, and is therefore not to be trusted.

(81) A classical education is worthless, for we make no use of the ancient languages in later life.

(82) Every candid man acknowledges merit in a rival; every learned man does not do so; therefore every learned man is not candid. (Mysore '43)

(83) Anything opposed to industrial prosperity is an evil; Wars are certainly evil. Therefore, wars are opposed to industrial prosperity.

(84) It hoots; therefore, it must be an owl.

(85) Blood is a colour, for it is red and red is a colour.

(86) When nations grow rich, they degenerate; from this we gather that the degenerate Republics of South America must be rich.

(87) High altitudes are bad for persons with high blood pressure; Kodaikanal does not agree with Gopal; hence Gopal's blood pressure must be high.

(88) As the learning of logical formulae does not give pleasure, it clearly has no value. (Mysore '44)

INDUCTION

XVIII

THE PROBLEM OF INDUCTION

QUESTIONS:

1. Define the problem of Induction, and distinguish it from that of Deduction. (Sept. '28)

2. Jevons says, "Induction is really the inverse process of Deduction." Explain the statement and state the relation between Deduction and Induction. (1922)

3. Explain what is meant by saying that the process of Induction might be represented in the form of a disjunctive syllogism. (March '36)

4. Explain "Induction through analysis." How does enumeration of instances help this? Can Induction be described as a process of elimination? (Dec. '21)

5. "Enumeration is the beginning rather than the end of the Inductive procedure." Explain clearly what enumeration contributes to the ends of scientific Induction, and briefly indicate what more is necessary to complete the inductive process. (Sept. '22)

6. No process of enumeration has any claim to the title of Perfect Induction. Explain the statement. (Sept. '37)

7. What is meant by the Scientific Method? Is it inductive or deductive?

8. Explain and illustrate colligation of facts. (March '30)

XIX

THE POSTULATE OF INDUCTION

1. What is the postulate of Induction? Can you derive it from the fundamental laws of thought? Is all explanation bound to be causal? (March '28)
2. The law of causation has been called the major premise of every induction. Explain what this means. (March '23)
3. State the principle of the Uniformity of Nature, and show in what sense it is the postulate of induction. (Sept. '25)
4. Mill says:—"The Law of Uniformity is established in precisely the same way as an induction by simple enumeration." Examine Mill's position. (Sept. '26)
5. Distinguish the Law of Uniformity of Nature, from the Law of Universal Causation and explain their relation to each other. (Sept. '23)
6. State and examine Mill's paradoxical claim that the principle of the Uniformity of Nature is at once both the presupposition of all induction and also the product of prior inductions.
7. Do you agree with the view that all induction is based on a *petitio principii*? (March '29)

XX

CAUSE

1. What are the different senses in which the word 'cause' may be used? In what sense is it used in Scientific Induction? (March '27)
2. "The cause of an event is its invariable antecedent"—Examine this. (1913)
3. Give some examples of cases in which cause and effect appear to be contemporaneous. In such cases how can you distinguish cause from effect? (March '23)
4. Mill says that it is not true that the same phenomenon is always produced by the same cause. Examine the truth of this statement. (March '25)
5. 'The same effect is always produced by the same cause'. 'Different effects may be produced by the same cause.' Illustrate these two statements, pointing out in what sense each is true. (March '27)
6. Explain and illustrate homogeneous intermixture of effects.
7. Is the doctrine of the plurality of causes consistent with your definition of cause? What is the value of the doctrine of plurality? (March '12)

XXI

OBSERVATION AND EXPERIMENT

1. Is there any fundamental distinction between Observation and Experiment? In what way is the scientific observer prone to err? (Sept. '27)

2. How does Experiment differ from mere observation as a scientific method? What are the errors to which each is liable?
(March '35)

3. 'Observation seems easy and simple and passive; we have as it were, simply to open our eyes and we see certain things.' With reference to this statement consider the question whether observation can be strictly separated from Explanation.
(March '31)

4. What is the place of Observation in Induction and what kind of Observation is necessary for a sound Induction?
(March '37)

5. Is there any difference between the methods of observation in the following cases? If so, distinguish them, and if possible, assign names to them:

- (i) I study the habits of ants in my garden.
- (ii) I study the stars every night by means of a telescope.
- (iii) I study the prominences in the sun every time there is a total solar eclipse.
- (iv) I study the actions of acids on minerals in my laboratory.
(1918)

6. Mention the conditions which should be observed in conducting an experiment. What methods will you use in those sciences in which experiment is impossible? Illustrate your answer.
(March '22)

7. Explain and illustrate:—(i) A crucial instance, (ii) A negative instance, (iii) Mal-observation, (iv) Non-observation.
(March '28)

8. Explain giving an illustration, what is meant by crucial experiment. Why is it difficult to arrange a crucial experiment?
(Sept. '31)

9. Consider the relation between description and explanation of facts. Would it be true to say that explanation is the ideal of science?
(March '38)

XXII

ENUMERATION—STATISTICS—PROBABILITY

1. Define Induction by Simple Enumeration and indicate its defects as a scientific method.
(March '27)

2. "Enumeration needs to be completed and transcended by a deeper method." Explain fully what this method is. (1918)

3. Has chance any place in Induction? If so, how would you explain its inconsistency with the postulate of Induction?
(March '35)

4. What is chance? How is it eliminated? Explain and illustrate.
(Sept. '19)

5. Can the calculation of chances lead to any reliable results?
(March '36)

6. 'The probability that a man of 21 will live till he is 57 is 2/3'. What is the meaning and value of such a statement?

7. For what purposes are Statistics employed? To what classes of phenomena are they applied? What are the principles to be observed in compiling useful statistics? (Dec. '21)

8. Discuss the worth of the Statistical method in relation to physical sciences and moral sciences. (March '29)

9. What is the value of Statistical Enumeration in inductive reasoning? Show, by means of illustrations, the manner in which this type of reasoning is likely to mislead. (March '28)

10. Explain the importance of Enumerative Induction with special reference to Statistical methods, showing the conditions favourable to their employment, the best methods of employing them, and the value of the results obtained. (1912)

XXIII

ANALOGY

1. Explain the chief differences between the popular and the scientific use of analogical reasoning. (March '36)

2. How will you distinguish between analogy and induction from simple enumeration? Describe and illustrate the nature and conditions of a valid analogical argument. (Sept. '26)

3. 'Many of the most important scientific laws were first suggested by Analogy.' Give illustrations of this. (Sept. '37)

4. 'What is deductively invalid may yet be suggestive inductively.' How far is the statement true of analogy? Upon what general conditions does the value of an analogical argument depend? (Sept. '22)

5. What is the general nature of an argument from analogy? Do you distinguish it from 'Example'? Illustrate your answer. (1915)

6. Show that the value of analogical reasoning depends upon the importance as well as upon the number of the points of resemblance. (Sept. '36)

7. Is analogical inference ever conclusive? Explain and illustrate the conditions which arguments based on Analogy should fulfil. (March '32)

8. What is an argument from Analogy? Into what figures of the syllogism does it naturally fall? Is the argument formally valid? Show with illustrations how it is useful in scientific research. (March '25)

9. (a) 'The criterion of value of an analogical inference must be found in the conception of purpose or end'—Explain this statement.

(b) Give an instance of false analogy and point out precisely where the fallacy in it lies. (1916)

XXIV

MILL'S METHODS

Inductive arguments bearing on Mill's methods are frequently given for examination. The student will do well, first of all, to

analyse the argument, bearing in mind the four principles formulated by Mill for discovering the cause or effect of a phenomenon. A given argument may reveal more than one method being employed to find causal relation. Analysis may suggest some factor as the cause or effect sought for. The student may determine whether complete certainty could be claimed for the conclusion or at any rate whether a sufficient degree of probability exists therefor.

MODEL EXERCISES:

1. Two small pieces of blanket, exactly alike in all respects, except that one is coloured white and the other black, are placed on a block of ice. After a certain time it is found that the black piece has sunk deeper into the ice than the white one. Therefore it is concluded that black absorbs more heat than white.

(March 1942)

In order to find whether black absorbs more heat than white, an experiment is made. Two instances are studied—instances similar in every respect save one, viz., that in one case we have black, in the other we do not have it. It is found that where black is present, more heat is absorbed; where it is not present, less. Therefore, the Method of Difference employed here suggests the conclusion that black absorbs more heat than white. The principle of elimination used here is: That is not the cause of a phenomenon which is present when the phenomenon is absent. With the exception of black, all the other circumstances are also present in the second instance; but, as the effect, viz., increased absorption of heat is co-absent with black, the two are suggested to be causally related.

2. The only cause of the diminution of crimes is the abundance of food supply, for crimes increase with the growing scarcity of food.

(March '42)

We notice here that increase in crime and scarcity of food go together; and diminution in crime and abundance of food go together. Bearing in mind the principle, that is not the cause of a phenomenon which varies when it is constant or is constant when it varies, or varies in the proportionate manner with it, we conclude that inasmuch as variation in increase or diminution of food is accompanied by a variation in a reverse manner in crime, crime and food are causally related. The Method of Concomitant Variations suggests this relationship. But the conclusion can only be probable. In the given argument, the cause is suggested as the only cause. For this we require more evidence than what is given here.

3. Overdriven cattle, if killed before recovery from fatigue, become rigid and putrefy in a surprisingly short time. A similar fact has been observed in the case of animals hunted to death; cocks killed during or shortly after a fight; and soldiers slain in the field of battle. These various cases agree in no circumstances

directly connected with the muscles, except that these have been subjected to exhausting exercise. (March '34)

The cause of quick putrefaction of dead bodies is sought here. Various cases of quick putrefaction agree in no circumstances except one,—that circumstance is the excessive strain caused to muscles by exhausting exercise like over-work, being chased, etc. Therefore exhausting exercise may be considered the cause of quick putrefaction. This conclusion is suggested by the Method of Agreement. To increase the probability of the conclusion, we should examine negative instances—cases where there is no quick putrefaction. If in such cases, we find absence of exhausting exercise of muscles, the conclusion arrived at by the Method of Agreement would be strengthened.

4. Jupiter gives out more light than it receives from the sun. What is the obvious conclusion and by what method is it reached? (Sept. '42)

If the light given out by Jupiter is in excess of what it receives from the sun, the source whence Jupiter derives its extra light must be some other heavenly body. This conclusion is suggested by the Method of Residues. "When any part of a complex phenomenon is still unexplained by the causes which have been assigned, a further cause for this remainder must be sought."

5. Poverty must be the cause of increase of population; for we find that all poor countries are thickly populated, while those that are rich have a scanty population. Further what is true of countries is also true of individuals. It is usually the poor who have big families. (March '43)

Several defects can be detected in this argument. Poverty and increase of population may both be co-effects of a common cause like ignorance. So poverty may then be a False Cause. Besides the analogy between country and individual is not sound.

The inductive method on which the conclusion is based is the Joint Method of Agreement and Difference.

	Antecedent	Consequent
Positive instances:	Poverty:	Increase of population.
	(i.e. all poor countries)	
Negative instances:	Absence of poverty:	Lack of increase in population.
	(Countries which are rich)	(Scanty population)

The probability of the conclusion that poverty is the cause of increase of population depends upon the thoroughness of analysis. As suggested already, it is possible that both are co-effects of a common cause.

6. Intermittent fever is found only in places where there are marshes even though they differ in every other respect. Therefore, marshes are the cause of intermittent fever. (March '43)

The phenomenon under investigation is intermittent fever. Instances of the phenomenon agree in having only one circum-

QUESTIONS AND MODEL EXERCISES

stance in common, viz., the circumstance of occurring in marshy places. Applying the Method of Agreement, we conclude that marshes are the cause of intermittent fever. The probability of the conclusion will increase if we examine negative instances—absence of intermittent fever in places which are not marshes.

Besides the antecedent 'Marshes' is a vague term. We must analyse it and find out precisely how it causes intermittent fever. It may cause intermittent fever, by being the breeding-ground of disease germs.

7. In all unhealthy countries the greatest risk of Malaria fever is run by sleeping on shore. Is that owing to the state of the body during sleep or to a great abundance of mosquitoes at such times? It appears certain that those who stay on board a vessel generally suffer less than those actually on shore. (1914)

Two causes are suggested for the incidence of malaria—one is the state of the body, the other is the abundance of mosquitoes. Taking cases of malaria, it is found that they are more numerous on land than at sea. Examining people on board a vessel and people living on shore, it is found that the latter are more prone to attacks of malaria than the former. The Method of Difference may be applied thus:

	Antecedent	Consequent
Positive Instance:	Mosquitoes (on land)	Malaria
Negative Instance:	No mosquitoes (at sea)	No Malaria

To strengthen the conclusion thus arrived at we may employ the Method of Concomitant Variations and find whether malaria increases with the increase of mosquitoes, decreases and disappears with the decrease and disappearance of mosquitoes.

As for the other cause, there is no reason for assuming that the state of the body during sleep differs either as between land and sea or as between mosquito infected areas and areas free from mosquitoes.

8. Worms do not possess any sense of hearing. They took not the least notice of the shrill notes of a metal whistle, which was repeatedly sounded near them; nor did they of the deepest and loudest notes of a bassoon. They were indifferent to shout if care were taken that the breath did not strike them. When placed on a table close to the keys of a piano, which was played as loudly as possible, they remained perfectly calm.. (Latta and Macbeath.)

The conclusion is that worms do not possess any sense of hearing. This is arrived at by producing sounds from different instruments and of different quality. Whether from an instrument or from the human throat, whether loud or not, so long as care was taken not to touch them, the worms remained unaffected and unperceptive. The conclusion that they do not possess any sense of hearing is arrived at by the Method of Agreement. Instances considered differ in every respect save one—of having sound. Differences relate to quality, source, etc. In all these

cases there is the lack of perception of sound. Sounds are produced—worms do not react to these. Therefore they have no sense of hearing.

This conclusion possesses, on the evidence adduced, a considerable degree of probability. It will be strengthened by being supplemented by other methods of investigation.

9. It is observed that dew is never copiously deposited on surfaces much screened from the open sky, and not at all in a cloudy night; but if the clouds withdrew even for a few minutes, and leave a clear opening, a deposition of dew presently begins, and goes on increasing. The inference, therefore, is that the presence or absence of an uninterrupted communication with the sky causes the deposition or non-deposition of dew. (Dec. 42)

The phenomenon studied here is the deposition of dew. It is found that in the presence of uninterrupted communication with the sky, there is deposition of dew (positive instance): in the absence of such communication, there is no deposition (negative instance). Thus, applying the Method of Difference, we conclude that uninterrupted communication with the sky is the cause of deposition of dew. This conclusion is strengthened by the fact that on the occasions when the sky is cloudy, deposition of dew varies according to the extent to which the surfaces are exposed. This is the Method of Concomitant Variations.

It may be noted here that Nature itself provides the positive and negative instances by withdrawing and introducing circumstances, like an experimenter. We have a natural experiment here.

10. We observe very frequently that very poor handwriting characterises the manuscripts of able men, while the best handwriting is as frequent with those who do little mental work, when compared with those whose penmanship is poor. We may, therefore, infer that poor penmanship is caused by the influence of severe mental labour. (Hyslop, quoted by Creighton)

The conclusion sought to be established here is that severe mental labour is the cause of bad handwriting. The Joint Method of Agreement and Difference is pressed into service. Two sets of instances are considered.

	Antecedent	Consequent
Positive instances:	Severe mental labour: (great men are involved)	Bad handwriting.
Negative instances:	Absence of severe mental labour	Absence of bad handwriting.

It is obvious that cases of great mental ability and good handwriting going together have not been considered here. Hence the conclusion lacks certainty. Besides, bad handwriting may be due to some other cause—early neglect or the like.

QUESTIONS:

1. Examine the following, identify the method or methods involved and assess the worth of the conclusions:—

(1) The length of the string determines the pitch of the note, for it is inversely proportional to the length of the vibrating string. (Sept. '38)

(2) Coconut trees best flourish in places which are not far removed from the sea. (March '40)

(3) "There is a disease called certinism which produces a stunted condition of body and mind. In cases where the symptoms of the disease are present, there is found to be an insufficient amount of secretion from the thyroid gland; and the less the secretion, the more pronounced the symptoms. When treatment with a preparation of thyroid is tried, the symptoms gradually disappear. If the treatment is stopped, as it has been in some cases, the symptoms reappear." (Latta and Macbeath)

(4) An eminent judge was in the habit of jocosely propounding after dinner, a theory that the cause of the prevalence of Jacobinism was the practice of bearing three names. He quoted, on one side, Charles James Fox, Richard Brinsley Sheridan, John Horne Tooke, John Philpot Curran, Samuel Taylor Coleridge, Theobald Wolfe Tone. On the other hand there were William Pitt, John Scott, William Windham, Samuel Horsley, Henry Dundas and Edmund Burke. Moreover, the practice of giving children three names has been a growing practice, and Jacobinism has also been growing. The practice of giving children three names is more common in America than in England. In England, we still have a King and a House of Lords; but the Americans are Republicans. Burke and Theobald Wolfe Tone are both Irishmen; therefore the being an Irishman is not the cause of Jacobinism. Horsley and Horne Tooke are both clergymen; therefore the being a clergyman is not the cause of Jacobinism. Fox and Windham were both educated at Oxford; therefore the being educated at Oxford is not the cause of Jacobinism. Pitt and Horne Tooke were both educated at Cambridge; therefore the being educated at Cambridge is not the cause of Jacobinism. The cause is, therefore, the having three names (Macaulay, quoted by Creighton).

(5) Some thirty years ago, a student of the Germanic languages, reading over an old English poem of considerable length, called the Genesis, was struck by the fact that five or six hundred lines, in the heart of the poem, seemed to differ in various respects from the lines which preceded and followed. Pursuing his inquiry further, and comparing the forms of these lines with those of a kindred language, he came to the conclusion that this section, which had always been supposed to be original old English, had in fact been translated from old Saxon, and was therefore led to believe in the existence of an old Saxon poem on this subject of Genesis, though he was obliged to confess that

he found no other trace of its existence. Some twenty years after, another scholar, at work in the Vatican Library, which had only recently rendered its treasures more accessible, discovered a fragment of the missing Old Saxon Genesis, of which probably no one had read a line for a thousand years. Yet such had been the faith of competent scholars in Sievers' processes that no one was surprised when the missing manuscript swam into sight, any more than astronomers were amazed when the telescope pointed to the quarter of the heavens indicated by Adams and Laverrier, and revealed the planet Neptune, which no human eye till then had ever seen. (Albert S. Cook, *The Higher Study of English*, quoted by Creighton.)

(6) In order to investigate the ability of insects to find their mates, Loeb arranged the following experiment. A female butterfly was placed in a closed and otherwise empty cigar box, which was then suspended from the ceiling of a room. The windows were opened. At the time, no other butterflies of this species were visible in the neighbourhood. During the course of a few hours, however, several males of this species entered the room and alighted on the box. Would you feel justified in drawing inferences from this result? (Creighton)

(7) Sachs maintained, in 1862, that starch is formed by the decomposition in chlorophyll of carbon-dioxide gas under the influence of light. He found that when all other conditions were constant, and light was excluded from a plant, no starch was formed; the single circumstance of readmitting light was accompanied by renewed formation of starch. Further, he found that if certain portions of the leaves of an illuminated plant were covered with black paper, no starch was found in these portions.

(Ibid)

(8) M. Arago, having suspended a magnetic needle by a silk thread, and set it in vibration, observed that it came much sooner to a state of rest when suspended over a plate of copper than when no such plate was beneath it. Now, in both cases there were two true causes. . . . why it should come to rest, viz., the resistance of the air, which opposes, and at length destroys, all motions performed in it; and the want of perfect mobility in the silk thread. But the effect of these causes being exactly known by the observation made in the absence of the copper, and being thus allowed for and subducted, a residual phenomenon appeared, in the fact that a retarding influence was exerted by the copper itself; and this fact, once ascertained, speedily led to the knowledge of an entirely new and unexpected class of relations.

(Mill)

(9) In decerebrate animals (or in man when as the result of injury, or disease, or anaesthesia the cord is cut off from the higher centres) certain responses may take place through the spinal cord alone. Thus the knee jerk, the scratch reflex and certain postural responses may continue, and the possibility of

these reflexes is taken to prove the integrity of the cord at this or the other level.
(Latta and Macbeath)

(10) In the summer of 1840, Mayer practising medicine in Java, was struck with the brighter red colour of the venous blood of his patients. Reasoning on this he conceived it possible that the brighter colour was due to less bodily oxidation being necessary to keep up the body temperature in hot climates. This drew his attention to animal heat, thence to heat production in relation to mechanical work, and finally, to all forms of force. From extensive researches along these lines he formulated the theory that throughout the universe, both in the inorganic and the organic world, there are forces which are convertible but are not destructible.
(Creighton)

2. What is the Method of Agreement and what are its limitations as a method of discovery.
(March '37)

3. Bring out the inter-relations of the Methods of Agreement, Analogy and Simple Enumeration.
(1920)

4. What is the difference between the Method of Agreement and the Method of Difference (1) as to the principle on which each proceeds, and (2) as to the character of the instances with which each deals?
(Sept. '27)

5. Compare the Method of Difference with the Joint Method of Agreement and Difference in respect of (a) facility of application, (b) certainty of the conclusion.
(March '34)

6. State the canon of the Joint Method of Agreement and Difference, and illustrate the method. Wherein is it superior to the methods of which it is a combination?
(March '22)

7. State and exemplify the canons of the Method of Residues and of Concomitant Variations. Exhibit them as applications of the Method of Difference.
(1914)

8. Almost all the greatest discoveries in Astronomy have resulted from the consideration of residual Phenomena. Explain the method and give an example.
(March '27)

9. Explain the Method of Residues. When can it prove that one event is the cause of another and when can it only suggest an inquiry into causation?
(March '34)

10. Mill's Methods are all complementary to one another. Explain.
(Sept. '30)

11. Can you show that amongst the five experimental Methods the Method of Agreement and the Method of Difference are fundamental and the others are derivative?
(March '36)

12. Arrange the five Inductive methods in the order of their conclusiveness. Give reasons for the order you adopt.
(1918)

13. Point out the defect in the symbolic representation of Mill's Experimental Methods.
(1916)

14. "The essence of inductive reasoning lies in the use of your facts to disprove erroneous theories of causal connections. It is a process of elimination. The reasoning is disjunctive." Explain and consider the validity of this view. (March '20)

XXV

HYPOTHESIS AND EXPLANATION

1. What is an hypothesis How do hypotheses arise in the mind? Explain the importance of hypotheses in induction. (March '28)

2. What are the characteristics of a good hypothesis? (March '36)

3. Does Analogy afford any help in the framing of hypotheses?

4. What are the requirements of a good hypothesis? Distinguish between 'false' and 'barren' hypothesis, and discuss if they are condemned to remain false or barren for ever. (Sept. '27)

5. Describe the process of verifying an hypothesis. Has the process formal certainty? (March '34)

6. Is it true that all Induction depends on hypothesis? What is the difference between Theory and Hypothesis? (Sept. '29)

7. Is an hypothesis necessarily true because all the rival hypotheses have been shown to be false? (Sept. '22)

8. Speaking of the relation between hypothesis and observation, it has been said, "No theorising apart from observation, and no observing save in the light of theory."—Discuss.

9. Trace the various stages through which an hypothesis has to pass before it becomes an established certainty. (March '21)

10. Distinguish between Hypothesis, Theory and Fact.

11. Examine the relation between Induction and Deduction in the use of hypotheses. When is it possible to speak of an hypothesis as having been proved? (Sept. '28)

12. 'The exception proves the rule', says the proverb. Explain the meaning of this paradoxical statement which seems to say that an exception confirms instead of refuting the general rule. How can this be? (March '26)

13. What is meant by Scientific Explanation? (March '29)

14. Explain and illustrate an empirical law. (March '30)

15. The apparent daily movement of the sun round the earth is explained as due to the earth's rotation (turning round its own axis). The Greek poets, however, explained it by saying that the sun god drives his bright chariot daily across the sky. Why do you call the former explanation scientific and not the latter? (March '25)

16. Construct an hypothesis to explain some fact of your experience, and explain how it may be either verified or overthrown. (Creighton)

17. Against what error in the formation of hypothesis was Laplace contending when, to Napoleon's observation that there was no mention of God in his work on celestial Mechanics, Laplace replied that he had no need of that hypothesis? (Ibid)

18. What do we mean by an *ad hoc* hypothesis? (March '43)

XXVI

CLASSIFICATION, NOMENCLATURE AND TERMINOLOGY

1. Discuss the importance of nomenclature and terminology in Induction.

XXVII

FALLACIES OF INDUCTIVE REASONING

In examining inductive arguments, the student is advised to bear in mind the nature of inductive process as such. Universal laws and causal connections are established by Induction. The student may consider whether one or more of Mill's methods have been applied to reach a given conclusion and assess the worth of the conclusion thus reached. The other methods of arriving at general principles—Simple Enumeration, and Analogy may be found in an argument. Such an argument should be tested with special reference to the fallacies incident to these methods—hasty generalisation, insufficient enumeration, inadequate analysis and unsound analogy.

Attention may be paid to the two main stages of Induction—Observation and Explanation. The nature of Observation, whether it is aided by experiment or not, and the soundness of observation, whether it is free from errors of mal and non-observation are to be considered in examining the first stage. At the stage of explanation care should be taken to avoid hasty generalization, resulting from insufficient or incorrect observation, *post hoc ergo propter hoc*, mistaking mere succession for causal connection, improper application of general principles, forgetting the conditions under which they hold, etc. General principles should not be taken as overthrown if exceptions thereto are met with. In explaining causal connection, the totality of positive and negative conditions must be kept in view and not the last or exciting condition alone. Phenomena which are complex or which involve reciprocal relations must be carefully analysed and explained. Individual prepossessions and popular theories of the age should not usurp the place of reasoning.

The logical worth of arguments, and ways of increasing it, may, also be stated.

MODEL EXERCISES:

Examine the following arguments:

1. A buttercup leaf, a blade of grass, a fern, a moss, a volvox, and a protococcus, all contain green colouring matter. I

infer that all the members of the vegetable kingdom contain green colouring matter. (Creighton)

The inference here is based on simple enumeration of some members of the vegetable kingdom. A characteristic that is true of them is predicated of all the members of the vegetable kingdom. There is no analysis to reveal any connection between green colouring matter and being a member of the vegetable kingdom. Until analysis reveals such a connection, the conclusion cannot be accepted as certain.

2. The flood was due to the wrath of the goddess, for it appeared immediately after she had been slighted, and it disappeared immediately after the performance of a propitiatory sacrifice. (March '39)

Cause of the flood is sought here. To suggest that the flood appeared immediately after the goddess was slighted and that therefore that it must be due to her wrath, and to seek to confirm this by referring to the disappearance of the flood immediately after the performance of propitiatory sacrifice is to commit the fallacy of *post hoc ergo propter hoc*. Popular superstitions predispose people to attach importance to irrelevant but antecedent circumstances.

3. 'I am a Jew. Hath not a Jew eyes? Hath not a Jew hands, organs, dimensions, senses, affections, passions? Fed with the same food, hurt with the same weapons, subject to the same diseases, healed by the same means, warmed and cooled by the same winter and summer, as a Christian is? If you prick us, do we not bleed? If you tickle us, do we not laugh? If you poison us, do we not die? And if you wrong us, shall we not revenge? If we are like you in the rest, we will resemble you in that?'

(*'Merchant of Venice,'* quoted by Fowler)

It is obvious that the argument here is from analogy: if a Jew resembles a Christian in a certain number of points, he will resemble him in a point known to be present in the latter but ignored or forgotten in the former.

Analogy never proves but merely suggests and so the conclusion suggested by it must be tested before it is accepted. The points of resemblance are not superficial. Yet, the desire for revenge need not necessarily be present in a Jew who resembles a Christian in having certain other qualities.

4. That town must be unhealthy, for I know three people who live there, and none of them are keeping good health.

(March '39)

Three cases of indifferent health are not sufficient to warrant the generalization that the town as a whole is unhealthy. This is hasty generalization resulting from incomplete enumeration and inadequate analysis.

5. Some comets have been observed to have the same orbit as a certain meteoric showers. The hypothesis is suggested that all meteoric showers may represent the debris of disintegrated

comets. Biela's comet having been missing for sometime, it was accordingly predicted that when next due it would be replaced by a meteoric shower. This prediction was verified by observation.

(Creighton)

Analogy suggests a hypothesis—because some meteoric showers are seen to be the debris of disintegrated comets (since the former have the same orbit as the latter); therefore all meteoric showers are the debris of disintegrated comets. To test this hypothesis, Biela's comet was studied. As it was missing, when next due, a meteoric shower was expected if the hypothesis was true. Observation confirmed the expectation. Thus the hypothesis was verified. But though the hypothesis is verified, to prove it we must show that no other hypothesis is satisfactory.

6. All the great empires that ever flourished in the past have dwindled and lost their eminence; hence no great empire can maintain its supremacy in future.

(March '37)

A universal principle is sought to be established on the basis of enumeration. Unless analysis can reveal a causal connection between being an empire and losing eminence, nothing can be said about the future empires.

7. Democracy cannot succeed in India; look at the condition of China.

(March '37)

This is an argument from analogy. The points of similarity implied but not mentioned may be largeness of population, high percentage of illiteracy, vastness of territory, etc. But it does not follow therefore that if democracy did not succeed in China, it will not succeed in India. Unless we are assured that the conditions indispensable for the successful functioning of democracy are absent in India, we cannot conclude anything in advance.

8. Drink must be the cause of poverty, for most poor people drink.

(Sept. '32)

Because most poor people drink, it is suggested that their poverty is due to drink. We have reciprocity of phenomena here. It may be that a man drinks because he is poor and would like to forget bodily and mental pain by drinking; again, because he drinks, spending money instead of saving it, he may be poor. Each may cause and be caused by the other. It is possible, however, that neither is the cause of the other but both are co-effects of a common cause.

9. The naturalist gets his bone or his tooth and from it he can build up limb upon limb, muscle upon muscle, organ upon organ, the unknown animal of which these things were parts.

(March '43)

The inter-dependence of deduction and induction is seen here. As the student is aware, all inference—whether deductive or inductive—presupposes and proceeds upon the basis of system. Whether we work up our way from the members to the system or down from the system to the members, it is the presence of the system that makes either procedure possible. Though the

naturalist builds up from a bone or tooth the entire animal, he is able to do it only because he has, whether vaguely or clearly, a knowledge of the system to which the bone or tooth belongs.

10. What inductive fallacy may David be said to have committed when he said in his haste that all men are liars. (Sept. '42)

David's experience was not with regard to all men. Yet, in haste, he generalizes. He is guilty of hasty generalization. The statement cannot be true, if it is not made after analysing instances prompting it.

QUESTIONS:

1. Explain and illustrate the Fallacies of Induction.

(March '25)

2. What fallacies arise from carelessness in the use of words?

(Sept. '31)

3. What were Bacon's Four Idols and how far do they serve as a classification of the fallacies of Induction? (March '37)

4. What are the fallacies of (a) *post hoc, ergo propter hoc* and (b) false analogy? Give illustrations.

5. Explain and illustrate by examples some of the fallacies due to hasty generalization and individual prepossessions.

(March '41)

6. Describe and give an example of (a) *mal-observation* (b) *non causa pro causa*.

(March '35)

7. Examine the following:—

(1) War is a blessing, not an evil. Show me a nation that has ever become great without waging wars. (Sept. '42)

(2) I am sure to pass my B.A. Degree examination in the First Class, because my brother passed his B.A. in the First Class.

(Sept. '42)

(3) "No body can be healthful without Exercise, neither Natural Body, nor politique: And certainly, to a Kingdom or Estate, a Just and Honourable War is the true Exercise. A Civil War, indeed, is like the Heat of a Fever; but a Foreign War is like the Heat of Exercise, and serveth to keep the Body in Health."

(Bacon)

(4) All the Ambitious men I have come across are selfish: why should I not then infer that all ambitious men are selfish?

(March '42)

(5) England, which is governed by the British is a wealthy and prosperous country; there is every reason to think that India which is also governed by the British should be prosperous too.

(March '43)

(6) Judge:—"Five witnesses have shown that they have seen you stealing the purse, what have you to say?"

Culprit:—"I can produce sixteen witnesses who can all honestly swear that they never saw me stealing the purse."

(7) Why should any but professional moralists trouble themselves with the solution of moral difficulties? For, as we resort to a physician in case of any physical disease, so, in the

case of any moral doubt or any moral disorganization, it seems natural that we should rely on the judgment of some man specially skilled in the treatment of such subjects. (Fowler)

(8) Children are bright and interesting, but adults are dull and uninteresting. What has happened in the middle? Education. (March '32)

(9) Lord Curzon, arguing for the continued hereditary chamber, says:—"The hereditary principle is established in every branch and aspect of national life. We have hereditary bankers, lawyers, and cotton spinners. Why should it be a blot and offence when applied to the House of Lords?" (March '34)

(10) The metropolises of a country is like the heart of an animal body; therefore the increased size of a metropolis is a disease. (March '39)

(11) The waving of the juggler's wand was the cause of the appearance of the snake, because the snake appeared the moment the juggler waved his wand. (Sept. '39)

(12) After Franklin had investigated the nature of electricity for some time, he began to consider how many of the effects of the thunder and lightning were the same as those produced by electricity. Lightning travels in a zigzag line, and so does an electric spark; electricity sets things on fire, so does lightning; electricity melts metals, so do lightning. Animals can be killed by both and both cause blindness. Pointed bodies attract the electric spark, and in the same way lightning strikes spires, and trees, and mountain tops. Is it not likely then that lightning is nothing more than electricity passing from one cloud to another, just as an electric spark passes from one substance to another? (Creighton)

(13) For many generations the people of the Isle of St. Kilda believed that the arrival of a ship in the harbour inflicted on the islanders epidemic colds in the head, and many ingenious reasons were devised why the ship should cause colds. At last it occurred to somebody that the ship might not be the cause of the cold, but that both might be effects of some other common cause, and it was then remembered that a ship could only enter the harbour when there was a strong north-east wind blowing. (Ibid)

(14) Mars must be inhabited, because it resembles the Earth in possessing water and a moderate temperature. (Sept. '39)

(15) If I am not justified in general in inferring that *d* is a good book because *a*, *b*, and *c* are good books, why may I nevertheless conclude with some probability that Guy Mannering is a good book because Waverley, Ivanhoe, and Rob Roy are? What bearing has this on the question of induction by simple enumeration and the assertion that all inference is by means of a Universal?

(16) Before it was known that light travelled in waves, it was known that sound did so. Light and sound were both capable of being reflected, and the direction of their reflection obeyed the same law. From these facts it was inferred that light, like sound, travelled in waves. (Dec. '42)

(17) The wicked woman's evil eye fell upon the child; therefore the child fell ill. (Dec. '42)

(18) A child has come to know that, when the dog is pleased, he wags his tail. On this he argues that, when the cat wags its tail, it must be pleased. (Dec. '42)

(19) An enterprise started on a Tuesday failed; therefore all enterprises started on Tuesdays will fail.

(Annamalai University, Sept. '41)

(20) Dreams go by contraries; it has been repeatedly seen that dreams of filth are followed by financial gain.

(Andhra University, March '40)

(21) Since lightning invariably precedes thunder, it must be a cause of thunder. (Annamalai University, Sept. '41)

(22) A certain school had 150 pupils on its roll. One morning the attendance suddenly fell to 50. The average attendance was 130, the 20 absentees being accounted for by slight ailments, engagements and indifference. There was no festival nor epidemic in the neighbourhood to account for the sudden drop. There had been, however, a big political meeting; so the Headmaster concluded that this accounted for the unusual number of absentees. (Sept. '43)

(23) Having constantly observed the full moon in a clear sky, I assert that the weather is always fine when the moon is full. (Sept. '43)

(24) I have gone carefully through the list of members, and I find that they are all Hindus. (March '44)

11. What is the purpose of the "Data" section in a research paper? to present the results of the study